Class – XII
Subject – MATHEMATICS

M.M. 100 **(TIME: 3 HOURS)**

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start working during this time.)

Section A: Answer **Question 1** (compulsory) and **five** other questions.
Section B and Section C-Answer **two** questions from **either** Section B **or** Section C.
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables and graph papers are provided.

SECTION A

Question 1

(i) If $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A' A^{-1}=\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$. [3]

(ii) Given two regression lines 4x + 3y + 7 = 0 and 3x + 4y + 8 = 0, determine : [3]

(a) The regression line of y on x.
(b) Mean of x and y.
(c) The coefficient of correlation**.**

(iii) Solve: $\cos^{-1}[\sin(\cos^{-1} x)]=\dfrac{\pi}{3}$ [3]

(iv) Find the value of k, if the equation $8x^2-16xy+ky^2-22x+34y=12$ represents an ellipse. [3]

(v) Evaluate: $\lim_{y\to 0}\frac{y-\tan^{-1}y}{y-\sin y}$ [3]

(vi) Evaluate: $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ [3]

(vii) Solve the differential equation : y dx – (x +2y^2) dy = 0 . [3]

(viii) Evaluate: $\int_2^3 \frac{x^3+1}{x(x-1)}dx$ [3]

(ix) If 1 , ω , ω^2 are the cube roots of unity , then show that : [3]

$$\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^8\right)\left(1-\omega^8+\omega^{16}\right).........to\ 2n\ factors = 2^{2n}$$

(x) Two women, Sonia and Maya, play a game in which they take it in turns to throw an unbiased six-sided die. The first woman to throw a "6" wins the game. Sonia is the first to throw. Find the probability that [3]

(a) Maya wins on her first throw;

(b) Sonia wins on her second throw.

Question 2

(a) Using properties of determinants, prove that: [5]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(b) Using matrices, solve the following system of equations: [5]

$$x+y+z=6, x-y+z=2, 2x+y-z=1$$

Question 3

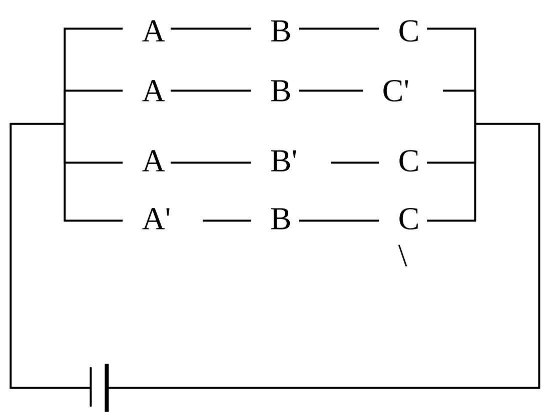
(a) Show that the function $f(x) = x^2 - 6x + 1$ satisfies the Lagrange's Mean Value Theorem. Also find the co-ordinate of a point at which the tangent to the curve represented by the above function is parallel to the chord joining A(1, – 4) and B(3, – 8). [5]

(b) Find the equation of ellipse whose latus rectum is 5 and whose eccentricity is $\frac{2}{3}$. [5]

Question 4

(a) Prove that : $2(\tan^{-1} 1 + \tan^{-1} 1/2 + \tan^{-1} 1/3) = \pi$. [5]

(b) Write the statement for the adjoining switching circuit. Simplify the statement. Construct the switching circuit for the simplified form. [5]



Question 5

(a) Evaluate: $\int \cos 2x \log(\sin x)\, dx$ [5]

(b) Calculate the area of the figure bounded by the curve y = log x, the straight line x = 2 and the x-axis. [5]

Question 6

(a) If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 2 = 0.$ [5]

(b) A figure consists of a semicircle with a rectangle on its diameter. Given that the perimeter of the figure is 20 m, find its dimensions in order that its area may be maximum. [5]

Question 7

(a) The data for marks in Physics and History obtained by a ten students are given below : [5]

Marks in Physics	15	12	8	8	7	7	7	6	5	3
Marks in History	10	25	17	11	13	17	20	13	9	15

Using this data, compute Karl Pearson's coefficient of correlation between the marks in Physics and History obtained by 10 students.

(b) Find the regression coefficients b_{yx} *and* b_{xy} for the following data : [5]

$n = 6, \sum x = 30, \sum y = 42, \sum xy = 199, \sum x^2 = 184, \sum y^2 = 318$

Question 8

(a) A candidate is selected for interview of management trainees for 3 companies. For the first company there are 12 candidates. For the second there are 15 candidates and for the third there are 10 candidates. Find the probability that he is selected at least one of the companies [5]

(b) In a group of 18 students, eight are females. What is the probability of choosing five students [5]

(i) With all girls? (ii) with three girls and two boys? (iii) with at least one boy?

Question 9

(a) Solve the following differential equation : [5]

$$\sin x \frac{dy}{dx} - y = \cos^2 x . \sin x . \tan \frac{x}{2}.$$

(b) Using De Moivre's theorem, find the least value of $n \in N$ for which the expression

$(1+i)^n + (1-i)^n$ is equal to $-(2)^{\frac{n+2}{2}}$. [5]

SECTION B

Question 10

(a) In any ΔABC , prove by vector method that $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$. [5]

(b) Show that the four points A, B, C and D, whose position vectors are [5]

$6i - 7j, 16i - 19j - 4k, 3j - 6k \text{ and } 2i - 5j + 10k$ respectively, are coplanar.

Question 11

(a) Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ units from the [5]

point (1, 2, 3).

(b) Find the equation of the plane passing through the point (1, -1, -1) and perpendicular [5]

to each of the planes $2(x-2y)+2(y-z)-(6z+x)=0$ and $2(y-x)+3(y+z)+4(x-z)=0$.

Question 12

(a) Only two international airlines fly daily into an airport. UN Air has 70 flights a day and [5]

IS Air has 65 flights a day. Passengers flying with UN Air have an 18% probability of

losing their luggage and passengers flying with IS Air have a 23% probability of losing their luggage. You overhear someone in the airport complain about her luggage being lost.

Find the probability that she travelled with IS Air.

(b) The probability that a bulb produced by a factory will fuse in 90 days of use is 0.06. Find the probability that out of 10 such bulbs, after 90 days use: [5]

(i) None fuse.

(ii) Not more than one fuses.

(iii) More than one fuses.

(iv) At least one fuses.

SECTION C

Question 13

(a) Find the Banker's discount and the discounted value of a bill worth ₹ 600 drawn on May 15, 2005 for 3 months and discounted on July 20, 2005 at 5% per annum. [5]

(b) Suppose the demand per month for a commodity is 24 if the price is ₹ 16 and 12 if the price is ₹ 22. Assuming that the demand curve is linear, determine. [5]

(i) The demand function (ii) the total revenue function

(ii) the marginal revenue function.

Question 14

(a) A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and two hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 per trunk on the first type of trunk and ₹ 25 per trunk on the second type. Formulate a Linear Programming Problem to find out how many trunks of each type he must make each day to maximize his profit. [5]

(b) A man borrowed some money and paid it back in three quarterly installments of ₹ 12,450 each. If the first installment is to be paid one year after the date of borrowing and rate of interest charged was 24% p.a. compounded quarterly, find the sum he borrowed. Find also total interest charged. [5]

Question 15

(a) The number of traffic offences committed in a certain city over a period of 3 years is given in the following table: [5]

	January - March	April - June	July - September	October - December
1980	74	56	48	69
1981	83	52	49	81
1982	94	60	48	79

Draw a graph illustrating these figures. Calculate the 4-quartely moving averages and plot them on the same graph.

Comment briefly on a local politician's claim that traffic offences were on the increase.

(b) Taking 1965 as the base year, with an index number of 100, calculate an index number for 1972, based on weighted average of price relatives derived from the following: [5]

Commodities	A	B	C
Weights	34	26	40
Price per unit in 1965	16	20	32
Price per unit in 1972	24	19	36

The weights are now changed so that the weight for A is 40 and the total weight is 100. If the value of the index number for 1972 is now 120.5, calculate the weights applied to B and C.