Pair of Linear Equations in two Variables and Quadratic Equation

Linear Equation in Two Variables

A linear equation in two variables is an equation which contains a pair of variables which can be graphically represented in xy-plane by using the coordinate system. For example ax + by=c and dx+ ey=f, is a pair of linear equations in two variables. Solutions of the linear equation in two variables are the pair of values of the variables that satisfies the given equation. In other words, we can say that a system of linear equation is nothing but two or more linear equations that are being solved simultaneously. Mostly, the system of equations are used in the business purposes by predicting their future events. They model a real life situation in two system of equations to find the solution and manage their business. We can make an accurate prediction by using system of equations. The solution of the system of equations in two variables is an ordered pair that satisfies each equation.

Graphical Representation of a Pair of Linear Equations in Two Variables

If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are a pair of linear equations in two variables such that:

- If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then pair of linear equations is consistent with a unique solution.
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent and dependent and having infinitely many solutions.
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent and have no solution.

Unique Solution

If the lines represented by a pair of linear equations are intersecting each other at one point, then the system is said to have unique solution. The point at which the two lines intersect each other is called the solution of the system of equation.

No Solution

If the graph of the system of equation is parallel and does not intersect each other at any point, then it is said to have no solution.

Infinitely Many Solutions

If the lines represented by the pair of linear equations in two variables coincides each other, then it is said to have infinitely many solution.

Solving the System of Equations

There are different algebraic methods for solving the system of linear equations. The three different methods are:

- Elimination Method
- Substitution Method
- Cross Multiplication Method

- **Example:**

Find the relation between m and n for which the system of equations

$4x+6y=7 and (m+n)x+(2m-n)y=21$, has unique solution.

(a) 2m=3n (b) m = 5n

(c) $2m \neq 3n$ (d) $m \neq 5n$

Answer (d)

Explanation:

We have, the system of equations $4x+6y=7$ and $(m+n)x+(2m-n)y=21$

For a unique solution, the required condition is

$$\frac{4}{m+n} \neq \frac{6}{2m-n}$$

$\Rightarrow 8m-4n \neq 6m+6n \Rightarrow 8m-6m \neq 6n+4n \Rightarrow 2m \neq 10n \Rightarrow m \neq 5n$

Quadratic Equation

Quadratic equation is a type of polynomial of degree two. The general form of a quadratic equation is $ax^2+bx+c=0$, where a, b, c are the constants and $a \neq 0$. The quadratic equation which contains both second and first powers of the variable is called a complete quadratic equation and the equation in which first power is missing is called pure quadratic equation.

Roots of a Quadratic Equation

The values of variable x which satisfy the quadratic equation $ax_2+bx+c=0$ $(a \neq 0)$ is called the roots of a quadratic equation.

Nature of Roots of a Quadratic Equation

The general form of a quadratic equation is $ax^2+bx+c=0$. This equation can be solved by using the Discriminant method. In this method first we find the discriminant $D=b^2-4ac$ of the given quadratic equation.

- If D > 0, then the given equation will have real and distinct roots and we can find the roots of the given equation.
- If D = 0, then the equation will have real and equal roots.
- If D < 0, then the given equation will have no real roots. In this case roots will be imaginary.

In case of real roots we can find the roots by using the formula, $x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ or $\frac{-b \pm \sqrt{D}}{2a}$

A quadratic equation can have maximum of two roots.

Relations between the Roots of a Quadratic Equation

If α and β are the roots of the equation $ax^2+bx+c=0$ then the relation between the roots of the quadratic equation is given by:

Sum of the roots $=\alpha+\beta=-\frac{b}{a}$

Product of the roots $=\alpha\beta=\frac{c}{a}$

Formation of Quadratic Equations

If α and β are the roots of the quadratic equation, and S denotes its sum and P denotes its product, then the quadratic equation is given by: $x_2-Sx+P=0$

- **Example:**

The value of k for which the equation $k^2x^2-2(2k-1)x+4=0$ have real and equal roots is:

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Answer (c)

Explanation: The given equation is $k^2x^2-2(2k-1)x+4=0$

Discriminant $(D)=4(2k-1)^2-4.4.k^2$

$=4(4k^2+1-4k)-16k^2 = 16k^2+4-16k-16k^2 = 4-16k$