Exponents and Powers

Exponents or Indices

The term ‘exponent’ refers to the number of times a quantity is multiplied with itself. It is also called ‘index’ or ‘power’. The exponential form of any number is x^y, where ***x* is the base and *y* is the exponent**. It is read as ‘y^{th} power of *x*’ or ‘*x* raised to the power *y*’.

Till now, we have studied the laws of exponents for numbers having non-zero integers as the base and the **integral exponent.**

For example, consider the expression $11^5 \div 11^3$. It can be simplified using the law of exponents $a^p \div a^q = a^{(p-q)}$ as follows: $11^5 \div 11^3 = 11^{(5-3)} = 11^2$

But what about numbers whose base is any real number or whose exponent is any rational number?

Can we simplify them using the same laws? Go through this lesson to find out how to simplify such expressions.

Numbers with Fractional Indices

While studying about the exponents, we come across few numbers having fractional indices. For example, $8^{\frac{1}{3}}$, $16^{\frac{1}{2}}$, $81^{\frac{1}{4}}$ etc.

Do you know what these numbers mean?

We know that the square root of a number is represented with the index $\frac{1}{2}$ as well as with the symbol $\sqrt{\ }$.

For example, the square root of 16 can be represented as $16^{\frac{1}{2}}$ and $\sqrt{16}$.

Similarly, the square root of any real number *a* can be represented as $a^{\frac{1}{2}}$ and $\sqrt{a}$.

So, there are two ways to represent the square roots of numbers.

In the same manner, we can represent the **cube root, fourth root, fifth root,..., *n*th root** of a real number with the symbol $\sqrt{\ }$ as well as with fractional indices.

For example, the cube root of 8 can be represented as $\sqrt[3]{8}$ and $8^{\frac{1}{3}}$. The fourth root of 81 can be represented as $\sqrt[4]{81}$ and $81^{\frac{1}{4}}$.

Similarly, the *n*th root of a number can also be represented as $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$.

Solved Examples

Easy

Example 1:

Find the values of the following index numbers.

i. $27^{\frac{1}{3}}$

ii. $256^{\frac{1}{4}}$

iii. $576^{\frac{1}{2}}$

iv. $4096^{\frac{1}{6}}$

Solution:

i. $27^{\frac{1}{3}}$ means the cube root of 27.

Therefore,

$$27^{\frac{1}{3}} = \sqrt[3]{27}$$
$$\Rightarrow 27^{\frac{1}{3}} = \sqrt[3]{3 \times 3 \times 3}$$
$$\Rightarrow 27^{\frac{1}{3}} = 3$$

ii. $256^{\frac{1}{4}}$ means fourth root of 256.

Therefore,

$$256^{\frac{1}{4}} = \sqrt[4]{256}$$
$$\Rightarrow 256^{\frac{1}{4}} = \sqrt[4]{4 \times 4 \times 4 \times 4}$$
$$\Rightarrow 256^{\frac{1}{4}} = 4$$

iii. $576^{\frac{1}{2}}$ means square root of 576.

Therefore,

$$576^{\frac{1}{2}} = \sqrt{576}$$
$$\Rightarrow 576^{\frac{1}{2}} = \sqrt{24 \times 24}$$
$$\Rightarrow 576^{\frac{1}{2}} = 24$$

iv. $4096^{\frac{1}{6}}$ means sixth root of 4096.

Therefore,

$$4096^{\frac{1}{6}} = \sqrt[6]{4096}$$
$$\Rightarrow 4096^{\frac{1}{6}} = \sqrt[6]{4 \times 4 \times 4 \times 4 \times 4 \times 4}$$
$$\Rightarrow 4096^{\frac{1}{6}} = 4$$

Laws of Exponents for Real Numbers

Consider two real numbers *a* and *b* and two rational numbers *m* and *n*. The laws of exponents involving these real bases and rational exponents can be written as follows:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$
- $(a^m)^n = a^{mn} = (a^n)^m$
- $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- $a^m \times b^m = (ab)^m$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

Solved Examples

Easy

Example 1:

Simplify the following expressions.

1. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$

2. $\sqrt[3]{(512)^{-2}}$

Solution:

i) $\left(\frac{27}{125}\right)^{\frac{2}{3}}$

$$=\left[\frac{(3)^3}{(5)^3}\right]^{\frac{2}{3}}$$

$$=\left[\left(\frac{3}{5}\right)^3\right]^{\frac{2}{3}} \qquad \left[\because \frac{a^m}{b^m}=\left(\frac{a}{b}\right)^m\right]$$

$$=\left(\frac{3}{5}\right)^{3\times\frac{2}{3}} \qquad \left[\because \left(a^m\right)^n = a^{m\times n}\right]$$

$$=\left(\frac{3}{5}\right)^2$$

$$=\frac{(3)^2}{(5)^2} \qquad \left[\because \left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\frac{9}{25}$$

ii) $\sqrt[3]{(512)^{-2}}$

$$= \left[(512)^{-2}\right]^{\frac{1}{3}}$$

$$= (512)^{\frac{-2}{3}} \qquad \left[\because (a^m)^n = a^{mn}\right]$$

$$= (8^3)^{\frac{-2}{3}}$$

$$= (8)^{3\times\frac{-2}{3}} \qquad \left[\because (a^m)^n = a^{mn}\right]$$

$$= (8)^{-2}$$

$$= \frac{1}{8^2} \qquad \left[\because a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{1}{64}$$

Example 2:

Simplify the expression $\left(\sqrt{\frac{2}{3}}\right)^{\frac{3}{5}} \times \left(\sqrt{\frac{1}{7}}\right)^{\frac{3}{5}}$.

Solution:

$$\left(\sqrt{\frac{2}{3}}\right)^{\frac{3}{5}} \times \left(\sqrt{\frac{1}{7}}\right)^{\frac{3}{5}}$$

$$= \left(\sqrt{\frac{2}{3}} \times \sqrt{\frac{1}{7}}\right)^{\frac{3}{5}} \qquad [\because a^m \times b^m = (ab)^m]$$

$$= \left(\sqrt{\frac{2}{21}}\right)^{\frac{3}{5}}$$

Example 3:

Simplify $\sqrt[3]{2^4} \times \sqrt[3]{3^4}$.

Solution:

$$\sqrt[3]{2^4} \times \sqrt[3]{3^4}$$
$$= \left(2^4\right)^{\frac{1}{3}} \times \left(3^4\right)^{\frac{1}{3}}$$
$$= (2)^{\frac{4}{3}} \times (3)^{\frac{4}{3}} \quad \left[\because \left(a^m\right)^n = a^{mn}\right]$$
$$= (2 \times 3)^{\frac{4}{3}} \quad \left[\because a^m \times b^m = (ab)^m\right]$$
$$= 6^{\frac{4}{3}}$$

Medium

Example 1:

Find the values of *x* and *y* in the expression $\mathbf{3^{x+1} \times 7^{2y-1} = 189}$.

Solution:

It is given that $3^{x+1} \times 7^{2y-1} = 189$

$$\Rightarrow 3^{x+1} \times 7^{2y-1} = 3 \times 3 \times 3 \times 7$$
$$\Rightarrow 3^{x+1} \times 7^{2y-1} = 3^3 \times 7^1$$

On equating the exponents of 3 and 7 on both sides of the above equation, we get:

$x + 1 = 3$ and $2y - 1 = 1$

$\Rightarrow x = 3 - 1 = 2$ and $2y = 1 + 1 = 2$

$\Rightarrow x = 2$ and $y = 1$

Thus, the values of *x* and *y* are 2 and 1 respectively.

Hard

Example 1:

Simplify the expression $\left(\frac{243}{32}\right)^{\frac{-4}{5}} \times \left[\left(\frac{625}{81}\right)^{\frac{-3}{4}} \div \left(\frac{25}{4}\right)^{\frac{-3}{2}}\right]$.

Solution:

$$\left(\frac{243}{32}\right)^{\frac{-4}{5}} \times \left[\left(\frac{625}{81}\right)^{\frac{-3}{4}} \div \left(\frac{25}{4}\right)^{\frac{-3}{2}}\right]$$

$$= \left[\left(\frac{3}{2}\right)^{5}\right]^{\frac{-4}{5}} \times \left[\left\{\left(\frac{5}{3}\right)^{4}\right\}^{\frac{-3}{4}} \div \left\{\left(\frac{5}{2}\right)^{2}\right\}^{\frac{-3}{2}}\right]$$

$$= \left(\frac{3}{2}\right)^{5\times\frac{-4}{5}} \times \left\{\left(\frac{5}{3}\right)^{4\times\frac{-3}{4}} \div \left(\frac{5}{2}\right)^{2\times\frac{-3}{2}}\right\} \qquad \left[\because \left(a^{m}\right)^{n} = a^{mn}\right]$$

$$= \left(\frac{3}{2}\right)^{-4} \times \left\{\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{2}{3}\right)^{4} \times \left\{\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right\} \qquad \left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}\right]$$

$$= \left(\frac{2}{3}\right)^{4} \times \left(\frac{3}{5} \div \frac{2}{5}\right)^{3} \qquad \left[\because a^{m} \div b^{m} = \left(a \div b\right)^{m}\right]$$

$$= \left(\frac{2}{3}\right)^{4} \times \left(\frac{3}{5} \times \frac{5}{2}\right)^{3}$$

$$= \left(\frac{2}{3}\right)^{4} \times \left(\frac{3}{2}\right)^{3}$$

$$= \left(\frac{2}{3}\right)^{4} \times \left(\frac{2}{3}\right)^{-3} \qquad \left[\because \left(\frac{b}{a}\right)^{m} = \left(\frac{a}{b}\right)^{-m}\right]$$

$$= \left(\frac{2}{3}\right)^{4-3} \qquad \left[\because a^{m} \times a^{n} = a^{m+n}\right]$$

$$= \frac{2}{3}$$

Example 2:

Prove that $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^{b}}{x^{a}}\right)^{c} = 1$.

Solution:

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c$$

$$= \frac{x^{(ab-ac)}}{x^{(ab-bc)}} \div \left(x^{b-a}\right)^c \qquad \left[\because \left(a^m\right)^n = a^{mn} \text{ and } \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= x^{(ab-ac)-(ab-bc)} \times \frac{1}{x^{(b-a)c}} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= x^{ab-ac-ab+bc} \times \frac{1}{x^{bc-ac}} \qquad \left[\because \left(a^m\right)^n = a^{mn}\right]$$

$$= x^{-ac+bc} \times x^{ac-bc} \qquad \left[\because \frac{1}{a^m} = a^{-m}\right]$$

$$= x^{-ac+bc+ac-bc} \qquad \left[\because a^m \times a^n = a^{m+n}\right]$$

$$= x^0$$

$$= 1$$

Identification of Perfect Squares

To understand the concept of perfect squares, look at the following video.

Thus, perfect squares can be defined as follows.

"The result of the product of any natural number with itself is a perfect square or a square number".

In this way, we can write

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

From the above list of numbers, we can say that the numbers 1, 4, 9, 16 25, 36, 49, 64, 81, and 100 are perfect squares, as these are the squares of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 respectively.

A perfect square can also be defined as follows.

"A number x is said to be a perfect square, if x can be expressed as y^2 where y is a natural number". Also, y^2 is read as "the square of y" or "y square".

For example, 9^2 is read as "the square of 9" or "9 square".

Let us discuss some examples based on perfect squares to understand the concept better.

Example 1:

Find the perfect square number between 30 and 85.

Solution:

The squares of 5, 6, 7, 8, 9, and 10 are as follows.

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

The perfect squares between 30 and 85 are 36, 49, 64, and 81.

Example 2:

Find the perfect squares between 70 and 90.

Solution:

We know that for three consecutive natural numbers 8, 9, and 10.

$8^2 = 64$, $9^2 = 81$, and $10^2 = 100$

64 and 100 does not lie between 70 and 90. But 81 lies between 70 and 90. Therefore, the only perfect square between 70 and 90 is 81.

Example 3:

Write how the following numbers can be read. Also, find their values.

(a) 5^2 (b) 12^2 (c) 108^2 (d) 111^2

Solution:

(a) 5^2 can be read as "the square of 5" or "5 square".

The value of 5^2 can be obtained as follows:

$5^2 = 5 \times 5 = 25$

(b) 12^2 can be read as "the square of 12" or "12 square".

The value of 12^2 can be obtained as follows:

$12^2 = 12 \times 12 = 144$

(c) 108^2 can be read as "the square of 108" or "108 square".

The value of 108^2 can be obtained as follows:

$108^2 = 108 \times 108 = 11664$

(d) 111^2 can be read as "the square of 111" or "111 square".

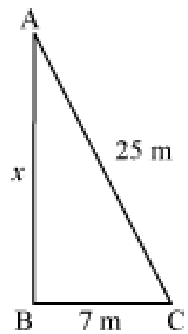
The value of 111^2 can be obtained as follows:

$111^2 = 111 \times 111 = 12321$

Prime Factorisation Method of Finding Square Roots

A park is in the shape of a right triangle. The length of the hypotenuse and a side of this park are 25 m and 7 m respectively. What is the length of the third side?

Let us consider the following triangle ABC as the right triangular park whose hypotenuse AC = 25 m and side BC = 7 m.



Let AB be *x.*

Using Pythagoras theorem for triangle ABC,

$AB^2 + BC^2 = AC^2$

$x^2 + (7)^2 = (25)^2$

$x^2 + 49 = 625$

$x^2 = (625 - 49)$

$x^2 = 576 \text{ m}^2$

To find the value of x, we require a number whose square is 576.

For this, we will find the square root of 576. Mathematically, we write it as

$x = \sqrt{576} \text{ m}$

Here, $\sqrt{\ }$ represents the square root. To find the square root of 576, we follow a method, which is known as prime factorization method.

Let us discuss this method and find the square root of 576 with the help of the given video.

In this way, we can find the square root of a given number by prime factorization method and solve problems related to it.

Let us discuss some more examples to understand the concept better.

Example 1:

Find the square roots of the following numbers.

(i) 324

(ii) 676

(iii) 1225

(iv) 3136

Solution:

(i) The prime factorization of 324 is

$$\begin{array}{r|l} 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\therefore 324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

$$\therefore \sqrt{324} = 2 \times 3 \times 3 = 18$$

(ii) The prime factorization of 676 is

$$\begin{array}{r|l} 2 & 676 \\ \hline 2 & 338 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$\therefore 676 = \underline{2 \times 2} \times \underline{13 \times 13}$$

$\therefore \sqrt{676} = 2 \times 13 = 26$

(iii) The prime factorization of 1225 is

$$\begin{array}{r|l} 5 & 1225 \\ \hline 5 & 245 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore 1225 = \underline{5 \times 5} \times \underline{7 \times 7}$

$\therefore \sqrt{1225} = 5 \times 7 = 35$

(iv) The prime factorization of 3136 is

$$\begin{array}{r|l} 2 & 3136 \\ \hline 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

$\therefore \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

Example 2:

Is 504 a perfect square? If not,

(i) find the smallest number multiplied to this number, so that the product would be a perfect square

(ii) find the smallest number by which 504 must be divided, so that the quotient is a perfect square

Solution:

The prime factorization of 504 is

$$\begin{array}{r|l} 2 & 504 \\ \hline 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$504 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 7$$

Here, the prime factors 2 and 7 do not occur in pair. Therefore, 504 is not a perfect square.

(i) In order to obtain a perfect square, each factor of 504 must be paired. Therefore, we have to make pairs of 2 and 7. For this, 504 should be multiplied by 2 × 7 i.e., 14.

$$\text{Now,} \quad 504 \times (2 \times 7) = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$
$$504 \times (14) = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Therefore, 14 should be multiplied with 504 to make it a perfect square.

(ii) In order to obtain a perfect square, each factor of 504 must be paired. Therefore, 504 should be divided by 2 × 7 i.e., 14.

$$\text{Now,} \quad 504 \div (2 \times 7) = \underline{2 \times 2} \times \underline{3 \times 3}$$
$$504 \div (14) = \underline{2 \times 2} \times \underline{3 \times 3}$$

Therefore, 504 should be divided by 14 so that the quotient is a perfect square.

Example 3:

Find the smallest perfect square which is a multiple of the numbers 8, 9, and 20.

Solution:

To find the smallest number which is a multiple of 8, 9, and 20, first of all, we have to find the least common multiple (L.C.M.) of these numbers and then, we find the required perfect square.

Now,

$$
\begin{array}{r|l}
2 & 8,\ 9,\ 20 \\ \hline
2 & 4,\ 9,\ 10 \\ \hline
2 & 2,\ 9,\ 5 \\ \hline
5 & 1,\ 9,\ 5 \\ \hline
3 & 1,\ 9,\ 1 \\ \hline
3 & 1,\ 3,\ 1 \\ \hline
 & 1,\ 1,\ 1
\end{array}
$$

Now, L.C.M. of 8, 9, and 20 $= 2\times2\times2\times3\times3\times5 = 360$

Prime factorization of 360 $= \underline{2\times2}\times2\times\underline{3\times3}\times5$

It can be seen that 2 and 5 are not in pairs. Therefore, 360 is not a perfect square. In order to obtain a perfect square, each factor of 360 must be paired. Therefore, we have to make pairs of 2 and 5. Therefore, 360 should be multiplied by 2 × 5 i.e., 10.

Hence, required perfect square = 360 × 10 = 3600

Example 4:

A farmer had 11300 plants. He planted them in such a way that each row contains as many plants as two times the number of rows. At last, he found that 50 plants were not planted. Find the number of rows and the number of plants in each row.

Solution:

Let the number of rows be x.

Number of plants in each row = $2x$

Therefore, number of plants planted by the farmer = $(x)(2x) = 2x^2$

The farmer had 11300 numbers of plants. 50 plants were not planted among these. Therefore, number of plants planted by the farmer = 11300 − 50 = 11250

Now, $2x^2 = 11250$

$$\Rightarrow x^2 = \frac{11250}{2} = 5625$$

$$\therefore x = \sqrt{5625}$$

The prime factorization of 5625 is

$$\begin{array}{r|l} 3 & 5625 \\ \hline 3 & 1875 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\text{Now, } 5625 = \underline{3\times3}\times\underline{5\times5}\times\underline{5\times5}$$

$$\therefore\ x = \sqrt{5625} = 3\times5\times5 = 75$$

$$2x = 2\times75 = 150$$

Therefore, the number of rows is 75 and the number of plants in each row is 150.

Square Roots of Perfect Squares by Division Method

What is the square root of 6241?

We know how to find the square root of 6241 by prime factorization method. For this, first of all, we have to find the prime factors of 6241.

- The smallest prime number is 2. But 6241 is not divisible by 2. Therefore, we can say that 2 is not a prime factor of 6241.
- Similarly, we will find that the prime numbers 3, 5, 7, 11, 13, 17, 19, etc. are not factors of 6241.

Actually 6241 is the square of 79 and it is a prime number. We have already taken a lot of time to find the prime factors of 6241 and have not been able to obtain it. If we keep on doing this to obtain such a large prime number, 79, then it will be a time consuming process and there might be some errors in our calculation.

In order to overcome such situations, we follow another method to find the square root of a number. This method is known as **division method**.

Let us understand the division method of finding the square root of a number by taking 6241 as example in the following video.

Using division method, we can also find the square root of a decimal. Let us learn this method and find the square root of a decimal number, lets say 51.84.

To understand the method of finding the square root of 51.84 by division method, look at the following video.

In this way, we can find the square root of a number by division method and solve problems related to it.

Let us discuss some examples to understand the concept better.

Example 1:

Find the least number that must be added to 7523 so as to obtain a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

By finding the square root of 7523 by division method, we obtain

$$\begin{array}{r|l} & \quad 86 \\ \hline 8 & \overline{75}\,\overline{23} \\ & -\ 64 \\ \hline 166 & \ 1123 \\ & -\ \ 996 \\ \hline & \ \ \ 127 \end{array}$$

When we found $\sqrt{7523}$ by long division method, we obtained the remainder as 127.

This shows, $86^2 < 7523$

The next perfect square to 86^2 is 87^2.

Since, $87^2 = 7569$

Therefore, 7569 is the perfect square which is greater than 7523.

7569 – 7523 = 46

Therefore, 46 should be added to 7523 so that the result is a perfect square.

Now, $\sqrt{7569} = 87$

Therefore, the square root of the perfect square is 87.

Example 2:

Find the least number that must be subtracted from 3241 so as to obtain a perfect square. Also, find the square root of the perfect square so obtained.

Solution:

By finding the square root of 3241 by division method, we obtain

When we found $\sqrt{3241}$ by long division method, we obtained 105 as the remainder.

This shows that 56^2 is less than 3241 by 105.

This means, if we subtract 105 from 3241, then the remainder we obtain is a perfect square.

Therefore, required perfect square = 3241 – 105 = 3136

Now, $\sqrt{3136} = 56$

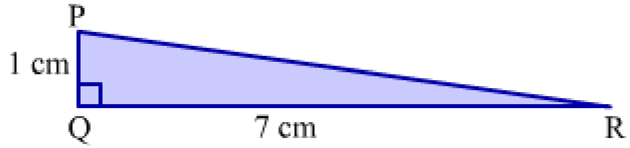
Therefore, the square root of the perfect square is 56.

Square Roots of Non-perfect Square Numbers

Square Roots of Non-perfect Square Numbers

We have studied about the square roots of numbers that are perfect squares. But what about non-perfect square numbers? Can we find their square roots as well? If yes, how?

Have a look at the given triangle.



In ΔPQR, PQ = 1 cm, QR = 7 cm, ∠Q= 90°.

By Pythagoras theorem:

$(PR)^2 = (PQ)^2 + (QR)^2$

$\Rightarrow (PR)^2 = (1)^2 + (7)^2$

$\Rightarrow (PR)^2 = 1 + 49$

$\Rightarrow (PR)^2 = 50$

$\Rightarrow PR = \sqrt{50}$ cm

Thus, the length of line segment PR is $\sqrt{50}$ cm.

It can be observed that 50 is a non-perfect square number. Therefore, to find the length of PR, we need to find the square root of the non-perfect square number 50.

Let us try to calculate the square root of 50 using division method.

We can write 50 as 50.000000… to simplify our division.

The method of finding the square root of decimal numbers is same as that of integers.

Firstly, the digits in the integer part are paired off starting from the right and then the digits in the decimal part (i.e. .0000…) are paired off starting from the left.

	7.071
7	$\overline{50}.\overline{00}\ \overline{00}\ \overline{00}...$
+ 7	– 49
1407	1 00 00
+ 7	– 98 49
14141	1 51 00
+ 1	–1 41 41
	9 59

The value of $\sqrt{50}$ has been calculated up to the third decimal place. As 50 is not a perfect square, we will get more decimal places if we continue the process. So, this is a never ending process.

∴ $\sqrt{50}$ = 7.071…

Here, the dots on the right side of the decimal number denote the fact that there will be an infinite number of digits.

Similarly, with the help of this method, we can find the square roots of other decimal numbers.

For example, let us find the square root of 231.79.

Here, 231.79 is paired as $\overline{2}\,\overline{31}.\overline{79}$.

	15.2
1	$\overline{2}\,\overline{31}.\overline{79}$
+ 1	−1
25	1 31
+ 5	− 1 25
302	6 79
+ 2	− 6 04
	75

∴ $\sqrt{231.79}$ = 15.2

Approximate value of the square root of a non-perfect square number

There are many problems in cases where square roots of non-perfect square numbers are involved. Since their values are non-terminating decimal numbers, we cannot consider them to simplify problems and thus, we take their approximate values.

For example, in ΔPQR (discussed above), the length of hypotenuse PR, when measured using a ruler, is obtained to be 7.1 cm. So, we can say that 7.1 is the approximate value of $\sqrt{50}$.

Using division method, the approximate value of the square root of a non-perfect square number can be calculated up to one, two, three or any number of decimal places, as shown below:

• Find the square root of the given number using division method up to one place more than the required decimal place.

For example, to determine the approximate value of $\sqrt{27}$ up to the second decimal place, the value of $\sqrt{27}$ is found up to the third decimal place.

• If the digit at the additional decimal place is 5 or more than 5, then increase the digit at the previous place by 1 and drop the digit at the additional place and all further places (if any).

For example, the value of $\sqrt{27}$ cup to the third decimal place is obtained as 5.196. Here, the digit at the third place, i.e. 6, is greater than 5. Thus, we drop 6 and increase the digit before it, i.e. 9, by 1. Hence, we get the approximate value of $\sqrt{27}$ up to the second decimal place as 5.20.

• If the digit at the additional decimal place is less than 5, then we drop that digit (and all further digits) by keeping the remaining number as it is.

For example, the value of $\sqrt{50}$ up to third decimal place is obtained as 7.071. Here, the digit at the third place, i.e. 1, is less than 5. Thus, we drop 1 and keep the remaining number as it is. Hence, the value of $\sqrt{50}$ up to the second decimal place is obtained as 7.07.

Similarly, the approximate value of the square roots of non-perfect square numbers up to the required decimal place can be obtained.

Let us go through some examples for a better understanding of the concept.

Example 1:

Find the square root of 3 up to three decimal places using division method.

Solution:

The square root of 3 up to three decimal places can be calculated using division method as follows:

	1.732	
1	$\overline{3}.\overline{00}\ \overline{00}\ \overline{00}$	
+ 1	− 1	
27	2 00	
+ 7	− 1 89	
343	11 00	
+ 3	− 10 29	
3462	71 00	
+ 2	− 69 24	
	1 76	

∴ $\sqrt{3}$ = 1.732

Example 2:

Find the square root of 58.315 up to two decimal places.

Solution:

The square root of 58.315 up to two decimal places can be calculated using division method as follows:

	7.63
7	$\overline{58}.\overline{31}\ \overline{50}$
+ 7	−49
146	9 31
+ 6	− 8 76
1523	55 50
+ 3	− 45 69
	9 81

∴ $\sqrt{58.315} = 7.63$

Example 3:

Find the approximate value of the square root of 31.25 up to the first decimal place by division method.

Solution:

Since we have to find the approximate value of the square root of 31.25 up to the first decimal place, we must find its square root up to the second decimal place.

Let us write 31.25 as 31.2500 and find its square root.

	5.59
5	$\overline{31}.\overline{25}\ \overline{00}$
+ 5	− 25
105	6 25
+ 5	− 5 25
1109	1 00 00
+ 9	− 99 81
	19

∴ $\sqrt{31.25}$ = 5.59

In 5.59, the digit at the second place after decimal, i.e. 9, is greater than 5. Thus, we drop 9 and increase the digit before it, i.e. 5, by 1.

Hence, the approximate value of $\sqrt{31.25}$ up to the first decimal place is 5.6.

Properties of Perfect Squares

We know how to find the square of a number. On multiplying the same number two times, we obtain the square of the number. While calculating the squares of numbers, we come across various properties of perfect squares.

In order to understand the properties of perfect squares, look at the given video.

Here we have another interseting property about the number of digits in a square of a number.
The number of digits in a square of a number with n digits is either $2n$ −- 1 or $2n$.

Let us discuss some examples to understand the concept better.

Example 1:

Are 20124598 and 900000 perfect squares? Why?

Solution:

20124598 is not a perfect square because a perfect square never ends with 8.

900000 is also not a perfect square because it has odd number of zeroes.

Example 2:

The squares of which of the following numbers are even numbers?

9012, 3375, 1024, 378, 87

Solution:

We know that the squares of even numbers are even.

∴ Squares of 9012, 1024, and 378 are even numbers.

Example 3:

What would be the unit digit of the squares of the following numbers?

8754, 967, 35120

Solution:

The square of 8754 ends with 6.

We know that if a number ends with 4, then its square ends with 6. Therefore, the unit place digit of the square of 8754 is 6.

The square of 967 ends with 9.

We know that if a number ends with 7, then its square ends with 9. Therefore, the unit place digit of the square of 967 is 9.

The square of 35120 ends with 0.

We know that if a number ends with 0, then its square also ends with 0. Therefore, the unit place digit of the square of 35120 is 0.

Identification of Perfect Cubes by Prime Factorisation

The cube of an integer is the number obtained on multiplying that integer with itself three times.

Also, the cube of an integer is written with the index 3 taking the integer as the base.

For example, cube of 2 is written as 2^3.

Thus, $2^3 = 2 \times 2 \times 2 = = 4 \times 2 = 8$

Similarly, cubes of few more integers are as follows:

$(-3)^3 = (-3) \times (-3) \times (-3) = 9 \times (-3) = -27$

$11^3 = 11 \times 11 \times 11 = 121 \times 11 = 1331$

$(-15)^3 = (-15) \times (-15) \times (-15) = 225 \times (-15) = -3375$

$20^3 = 20 \times 20 \times 20 = 400 \times 20 = 8000$

From the above examples, it can be observed that:

- **The cube of a positive number is always positive.**
- **The cube of a negative number is always negative.**

The cube of an integer is known as perfect cube.

In the above examples, 8, –27, 1331, –3375 and 8000 all are perfect cubes or cube numbers.

The following table lists the cubes of natural numbers from 1 to 20.

Number	**Cube of the Number**	**Number**	**Cube of the Number**
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

We can thus define a perfect cube as follows:

A number is said to be a perfect cube or a cube number, if it is obtained by multiplying a natural number with itself three times.

From the above calculations, we can see that 64 and 125 are the cubes of two consecutive natural numbers (4 and 5 respectively). We know that there is no natural number between 4 and 5. Therefore, any number between 64 and 125, say 100, 73, etc., are not perfect cubes.

This method can also be used to identify whether a given number is a perfect cube or not. Let us take the example of the number 1224. Now, we know that $10^3 = 1000$ and $11^3 = 1331$. Now, the given number 1224 lies between two consecutive perfect cubes 1000 and 1331. Thus, it cannot be a perfect cube. However, this is not a very convenient method to use, especially in the case of large numbers.

Therefore, let us go through the following video to understand the method used for identifying a perfect cube.

Let us discuss one more example based on this property.

Example 1:

Check whether the numbers 5832 and 6400 are perfect cubes or not.

Solution:

The number 5832 can be expressed as a product of its prime factors as

$5832 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

Here, each of the prime factors occurs in groups of three.

Hence, 5832 is a perfect cube.

The number 6400 can be expressed as a product of its prime factors as

$6400 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2 \times 2 \times 5 \times 5$

Here, the prime factors 2 and 3 do not occur in groups of three.

Hence, 6400 is **not** a perfect cube.

Prime Factorisation Method of Finding Cube Roots

We find the cube of an integer by multiplying the same integer with itself three times. For example, we can find the cube of 6 as $6^3 = 6 \times 6 \times 6 = 216$

We can write this expression, i.e. “216 is the cube of 6”, in another way as “cube root of 216 is 6”. Mathematically, we can express it as $\sqrt[3]{216} = 6$.

Here, the symbol $\sqrt[3]{\ }$ denotes the cube root.

Similarly,

1. $8^3 = 512 \Rightarrow \sqrt[3]{512} = 8$
2. $15^3 = 3375 \Rightarrow \sqrt[3]{3375} = 15$
3. $(-7)^3 = -343 \Rightarrow \sqrt[3]{-343} = -7$
4. $(-12)^3 = -1728 \Rightarrow \sqrt[3]{-1728} = -12$

Hence, we can say that finding the cube root is the inverse operation of finding the cube.

The cube root of a perfect cube can also be found by the method of prime factorization. Let us find the cube root of 21952 by this method.

Now, let us find the cube root of –74088 by the method of prime factorization.

For the same, we will first find the cube root of 74088.

74088 can be factorized as follows:

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$\therefore$ 74088 = (2 × 2 × 2) × (3 × 3 × 3) × (7 × 7 × 7)

$\Rightarrow$ 74088 = $(2 \times 3 \times 7)^3$

$\Rightarrow \sqrt[3]{74088} = 2 \times 3 \times 7$

$\Rightarrow \sqrt[3]{74088} = 42$

$\therefore \sqrt[3]{-74088} = -42$

Let us discuss more examples based on the cube root of a perfect cube by the method of prime factorization.

Example 1:

Find the cube root of 287496 by prime factorization method.

Solution:

The prime factorization of 287496 can be done as follows:

$$\begin{array}{r|l} 2 & 287496 \\ \hline 2 & 143748 \\ \hline 2 & 71874 \\ \hline 3 & 35937 \\ \hline 3 & 11979 \\ \hline 3 & 3993 \\ \hline 11 & 1331 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

Thus, the number 287496 can be expressed as a product of its prime factors as

287496 = (2 × 2 × 2) × (3 × 3 × 3) × (11 × 11 × 11)

$\Rightarrow$ 287496 = $(2 \times 3 \times 11)^3$

$\Rightarrow \sqrt[3]{287496} = 2 \times 3 \times 11$

$\Rightarrow \sqrt[3]{287496} = 66$

Example 2:

Find the cube root of –1157625 by prime factorization method.

Solution:

To find the cube root of –1157625, we will first find the cube root of 1157625.

The prime factorization of 1157625 can be done as follows:

3	1157625
3	385875
3	128625
5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

$\therefore 1157625 = (3 \times 3 \times 3) \times (5 \times 5 \times 5) \times (7 \times 7 \times 7)$

$\Rightarrow 1157625 = (3 \times 5 \times 7)^3$

$\Rightarrow \sqrt[3]{1157625} = 3 \times 5 \times 7$

$\Rightarrow \sqrt[3]{1157625} = 105$

$\therefore \sqrt[3]{-1157625} = -105$