Sample Paper 13

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. **If the sum of the zeros of the polynomial $2x^2 + 3kx + 3$ is 6, then the value of k is:**

(a) 5 (b) 2
(c) – 4 (d) 6 1

2. **The roots of the quadratic equation $(3x - 5)(x + 3) = 0$ is:**

(a) $-3, \frac{5}{3}$ (b) $3, \frac{-5}{3}$
(c) $-2, \frac{5}{3}$ (d) $1, \frac{3}{5}$ 1

3. **The sum of natural numbers from 51 to 100 is:**

(a) 3005 (b) 2895
(c) 375 (d) 3775 1

4. **If A(6, 2), B(4, 2) and C(6, 4) are the vertices of ΔABC, then the length of the median through C is:**

(a) $\sqrt{2}$ units (b) $\sqrt{3}$ units
(c) 5 units (d) $\sqrt{5}$ units 1

5. **The base PQ of two equilateral triangles PQR and PQR' with side '$2a$' lies along y – axis such that the mid – point of PQ is at the origin. The coordinates of the vertices R and R' of the triangles is:**

(a) $(\sqrt{3}a, 0)(\sqrt{3}a, 0)$ (b) (1, 0)(– 1, 0)
(c) (2, 0) (– 2, 0) (d) $(\sqrt{3}a, 0)(-\sqrt{3}a, 0)$ 1

6. **In a Δ ABC, if DE is parallel to BC, $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, then the length AE is:**

(a) 45 (b) $\frac{23}{7}$
(c) 1 (d) $\frac{45}{7}$ 1

7. **If a right circular cylinder with a height of 7 cm has a volume of 448π cm^3, then the radius is:**

(a) 8 cm (b) 10 cm
(c) 11 cm (d) 15 cm 1

8. **If the bisector of an angle of a triangle bisects the opposite side, the triangles is:**

(a) isosceles (b) scalene
(c) equilateral (d) right angled triangle 1

9. **Two concentric circles are of radii 5 cm and 3 cm. The length of the chord of the larger circle which touches the smaller circle is:**

(a) 5 cm (b) 3 cm
(c) 8 cm (d) 4 cm 1

10. The area of a sector of angle θ (in degrees) of a circle with radius 'r' is:

(a) $\frac{\theta}{360°} \times \pi r^2$ (b) πr^2

(c) $2\pi r^2$ (d) $\frac{\theta}{360°} \times 2\pi r$ 1

11. The perimeter of a semi – circular protractor is 36 cm. Find its diameter.

(a) $\frac{72}{2+\pi}$ (b) $\frac{72}{\pi}$

(c) $\frac{36}{2+\pi}$ (d) 0 1

12. The modal class for the frequency distribution given below is:

Class interval	0-20	20-40	40-60	60-80	80-100
Number of workers	15	18	21	29	17

(a) 20 – 40 (b) 0 – 20
(c) 60 – 80 (d) 80 – 100 1

13. If P(E) = 0.005, then the probability of "not E" is:
(a) 0.002 (b) 0.95
(c) 0.995 (d) 0.095 1

14. The upper limit of the median class of the following frequency distribution is:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

(a) 17.5 (b) 18.5
(c) 19 (d) 19.5 1

15. An unbiased die is rolled once. The probability of getting an even prime number is:

(a) $\frac{6}{5}$ (b) $\frac{1}{5}$

(c) $\frac{1}{6}$ (d) 1 1

16. If sin B = 0.5, the value of $3 \cos B - 4 \cos^3 B$ is:
(a) 2 (b) 4
(c) –3 (d) 0 1

17. When two hemispheres with the same radius are connected at their bases, then we get a:
(a) cone (b) cylinder
(c) sphere (d) cuboid 1

18. The value of $\text{cosec}^2 30° \sin^2 45° - \sec^2 60°$ is:
(a) –2 (b) 1
(c) 2 (d) 3 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).
Choose the correct option as:

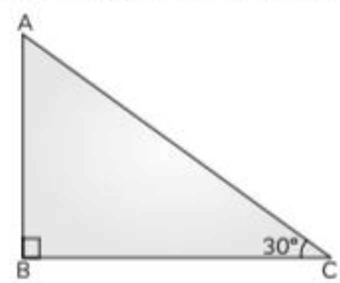
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** In the figure, if BC = 20 m, then height AB is 11.56 m.



Statement R (Reason): $\tan\theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{base}}$ where θ is the angle ∠ACB. 1

20. **Statement A (Assertion):** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.

Statement R (Reason): If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$. 1

SECTION - B **10 marks**

(Section B consists of 5 questions of 2 marks each.)

21. Using prime factorisation, find the HCF and LCM of 150 and 240.

OR

Show that $3 + \sqrt{5}$ is an irrational number, assuming that $\sqrt{5}$ is an irrational number. 2

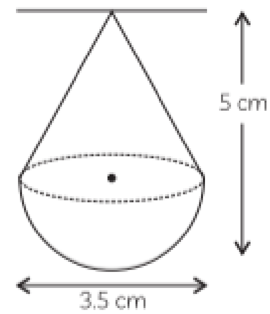
22. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

OR

Using distance formula, show that the points A(1, –1), B(5, 2) and C(9, 5) are collinear. 2

23. If $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$, then find the value of θ. 2

24. Raju got a playing lattu as his birthday present but it has surprisingly no colour on it. He wants to colour it with his crayons in a smart way . It is shaped like a cone surmounted by a hemi-sphere as shown. The entire latto is 5 cm in height and the diameter of the hemi-sphere base is 3.5 cm.



Find:

(A) the surface area of the hemi-sphere.

(B) the area that he has to colour. 2

25. For the following frequency distribution, determine the mean:

Class	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

2

SECTION - C **18 marks**

(Section C consists of 6 questions of 3 marks each.)

26. In an A.P., the last term is 28 and the sum of all the 9 terms of the A.P. is 144. Find the first term. 3

27. In what ratio does the x-axis is divide the line segment showing the points (–4, –6) and (–1, 7)? Find the coordinates of the point of division.

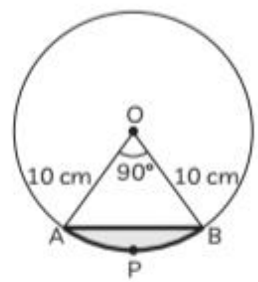
OR

Prove that the parallelogram circumscribing a circle is a rhombus. 3

28. In ΔABC right angles at B, if $\tan A = \dfrac{1}{\sqrt{3}}$, find value of:

$$\sin A \cos C + \cos A \sin C$$ 3

29. In the centre of a circle with a radius of 10 cm, a chord subtends a right angle. Find The area of the corresponding minor and major segment.



OR

A hollow cone is cut by a plane parallel to the base and upper portion is removed. If the curved surface area of the remainder is $\dfrac{8}{9}$ of the curved surface area of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane. 3

30. A number 'x' is selected from the numbers 1, 2, 3 and then second number 'y' is selected from the numbers 1, 4, 9. Find the probability that the product 'xy' of the two numbers is less than 9. 3

31. If the median of the following data is 32.5, find the missing frequencies.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

3

SECTION - D

20 marks

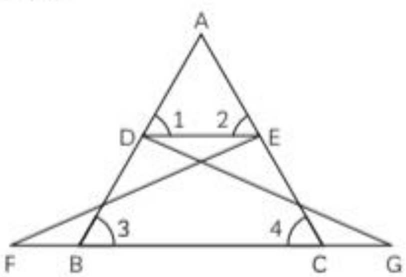
(Section D consists of 4 questions of 5 marks each.)

32. Two pipes together can start to fill a tank in $9\frac{3}{8}$ hours. The larger diameter pipe fills the tank separately in 10 hours less time than the smaller one. Determine the time at which each pipe can fill the tank on its own.

OR

A 2- digit number is four times the sum and three times the product of its digits. Find the number. 5

33. In the given figure, ΔFEC is congruent to ΔGDB and $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$



5

34. If an isosceles triangle ABC in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle. 5

35. Prove that '$\sqrt{a}$' is not a rational number, if 'a' is not perfect square.

OR

From an aeroplane vertically above a horizontal plane, the angles of depression of two consecutive kilometre stones on the opposite sides of the aeroplane are found to be α and β. Show that the height of the aeroplane is $\frac{\tan\alpha . \tan\beta}{\tan\alpha + \tan\beta}$. 5

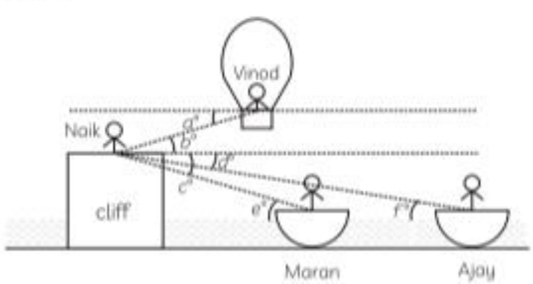
SECTION - E

12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *Mr. Naik is a paramilitary Intelligence Corps officer who is tasked with planning a coup on the enemy at a certain date. Currently he is inspecting the area standing on top of the cliff. Agent Vinod is on a hot air balloon in the sky. When Mr. Naik looks down below the cliff towards the sea, he has Ajay and Maran in boats positioned to get a good vantage point.*



The main goal is to scope out the range and angles at which they should train their soldiers.

On the basis of the above information, answer the following questions:

(A) Write one pair of 'angle of elevation' and one pair of 'angle of depression'. 1

(B) If the vertical height of the balloon from the top of the cliff is 12 m and $\angle b = 30°$, then find the distance between the Naik and vinod. 1

(C) Ajay's boat is 25 m away from the base of the cliff. If $\angle d = 30°$. What is the height of the cliff? (use $\sqrt{3}$ = 1.73)

OR

If the height of the cliff is 30 m , $\angle c = 45°$ and $\angle d = 30°$, then find the horizontal distance between the two boats (use $\sqrt{3}$ = 1.73) 2

37. *Prime Minister's National Relief Fund (PMNRF) was established to help the families of earthquake affected village. The allotment officer is trying to come up with a method to calculate fair division of funds across various affected families so that the fund amount and amount received per family can be easily adjusted based on daily revised numbers.*

The total fund allotted is formulated by the officer as:

$$x^3 - 5x^2 - 2x - 6$$

The officer has also divided the fund equally among families of the village and each family receives an amount of $x^2 + 2x + 1$. After distribution, an amount of $11x + 1$ should be left to have some buffer for future disbursements.

On the basis of the above information, answer the following questions:

(A) If an amount of ₹ 540 is left after distribution, what is value of x? 1

(B) How much amount (In rupees) does each family receive? 1

(C) What is the amount of fund (In rupees) allocated ?

OR

If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, then find the value of k. 2

38. *To celebrate Diwali festival among senior citizens of an old age home, four friends of a society, Rohan, Amar, Saran and Madhukar decided to pool some money to gift packs to every old man/woman staying in the neighbouring old-age home. They pooled money in the ratio 2 : 3 : 4 : 5. With the pooled money of ₹ 3500, they start preparing gift packs. In the preparation of one gift pack, Rohan, Amar, Saran and Madhukar spend 7, 6, 8 and 9 minutes respectively.*

On the basis of the above information, answer the following questions:

(A) How much time (in minutes) was spent on one gift? 1

(B) If each gift costs to them ₹ 70, how many senior citizens were given the cards? 1

(C) How much amount was pooled by Rohan and Madhukar together for giving the gifts and how much time (in hours) was spent in preparing all the gift packs?

OR

How much amount was pooled by Saran? 2

SOLUTION

SECTION - A

1. (c) −4

Explanation: Here, $p(x) = 2x^2 + 3kx + 3$

On comparing it with $ax^2 + bx + c = 0$, we get $a = 2$, $b = 3k$ and $c = 3$

$$\text{sum of zeros} = \frac{-b}{a}$$

$$= \frac{-3k}{2}$$

And, sum of zeros = 6 (given)

$$\therefore \quad \frac{-3k}{2} = 6$$

$$\Rightarrow \quad k = -4$$

2. (a) $-3, \frac{5}{3}$

Explanation: The two roots of the given equation $(3x - 5)(x + 3) = 0$ are -3 and $\frac{5}{3}$.

3. (d) 3775

Explanation: Sum of numbers from 51 to 100

$$= \frac{50}{2}[2 \times 51 + 49(1)]$$
$$= 25[102 + 49]$$
$$= 25 \times 151 = 3775$$

4. (d) $\sqrt{5}$ units

Explanation: The coordinates of Z (the mid-point of AB) are $\left(\frac{6+4}{2}, \frac{2+2}{2}\right)$ i.e. (5, 2)

So, length of CZ $= \sqrt{(5-6)^2 + (2-4)^2}$
$$= \sqrt{1+4} = \sqrt{5} \text{ units}$$

5. (d) $(\sqrt{3}a, 0)\left(-\sqrt{3}a, 0\right)$

Explanation: Since, the mid-point of PQ is the origin and PQ = 2a.

$\therefore$ OP = OQ = a

Hence, the coordinates of P and Q are $(0, a)$ and $(0, -a)$ respecively.

Since, ΔS PQR and PQR' are equilateral triangle

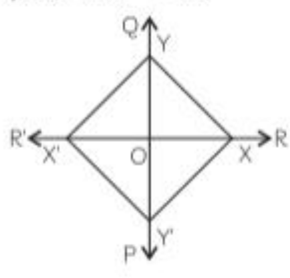
$\therefore$ Their third vertices R and R' lie on the ⊥r bisector of base PQ.

X' OX is the perpendicular bisector of base PQ.

Thus, R and R' lies on X-axis.

$\therefore$ Their Y - coordinates are 0.

In ΔROP, $OR^2 + OP^2 = PR^2$



$\Rightarrow \quad OR^2 + a^2 = (2a)^2$

$\Rightarrow \quad OR^2 = 3a^2$

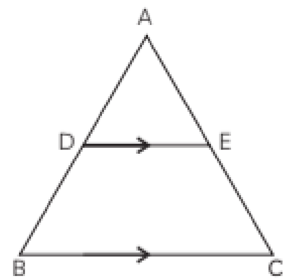
$\Rightarrow \quad OR = \sqrt{3}a$

Similarly, $OR' = \sqrt{3}a$

Thus, the coordinates of vertices R and R' are $(\sqrt{3}a, 0)$ an $(-\sqrt{3}a, 0)$ respectively.

6. (d) $\frac{45}{7}$ cm

Explanation: Since, DE || BC,



By thales theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{AE}{EC} = \frac{3}{4}$$

$\Rightarrow \quad \frac{AE}{AC - AE} = \frac{3}{4}$

$\Rightarrow \quad \frac{AE}{15 - AE} = \frac{3}{4}$

$\Rightarrow \quad 7AE = 45$

$\Rightarrow \quad AE = \frac{45}{7}$ cm

7. (a) 8 cm

Explanation: Let the radius be r cm.

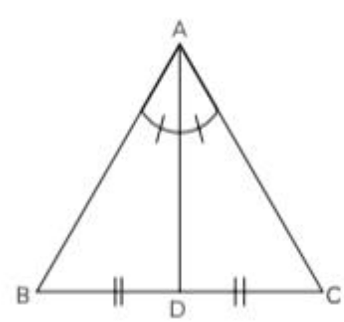
We know that,

Volume of cylinder $= \pi r^2 h$

$$448\pi = \pi r^2 h$$
$$\frac{448\pi}{\pi} = r^2 \times 7$$
$$448 = r^2 \times 7$$
$$\frac{448}{7} = r^2$$
$$64 = r^2$$
$$\sqrt{64} = r$$
$$8 = r$$

8. (a) isosceles

Explanation: Given, ΔABC, AD bisects ∠A and BC.



In ΔABD and ΔACD.

$\angle DAB = \angle DAC$ (AD bisects $\angle A$)

$AD = AD$ (common)

$BD = CD$ (AD bisects BC)

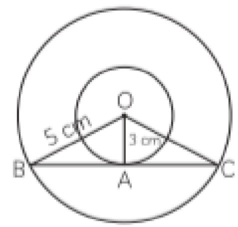
Thus, $\Delta ABD \sim \Delta ACD$ (by SAS rule)

Thus, $AB = AC$ (by cpct)

Hence, ΔABC is an isosceles triangle.

9. *(c) 8 cm*

Explanation: Here, radius of circles are 3 cm and 5 cm i.e., OA = 3 cm and OB = 5 cm



Now, OA is ⊥r on BC and bisects BC

As, tangent is ⊥r to the radius and ⊥r from the centre bisects the chord.

∴ In ΔOAB, by pythagoras theorem

$$OB^2 = AB^2 + OA^2$$

$$\Rightarrow \quad 5^2 = AB^2 + 3^2$$

$$\Rightarrow \quad AB^2 = 25 - 9 = 16$$

$$\Rightarrow \quad AB = 4 \text{ cm}$$

and $BC = 2AB$

$= 2 \times 4$

$= 8$ cm

10. *(a)* $\frac{\theta}{360°} \times \pi r^2$

Explanation: Area of sector of angle (θ) with radius (r) is = $\frac{\theta}{360°} \times \pi r^2$

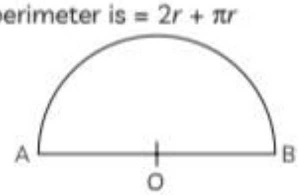
Caution

➥ *Students usually confused between area of sector and length of sector, formula used for area of sector is* $\frac{\theta°}{360°} \times \pi r^2$ *while length of sector is* $\frac{\theta°}{360°} \times 2\pi r$.

11. *(a)* $\frac{72}{2+\pi}$

Explanation: Let, the radius of protractor be 'r'

Then, its perimeter is = $2r + \pi r$



and $36 = 2r + \pi r$

$$\Rightarrow \quad r = \frac{36}{2+\pi}$$

$$\therefore \quad \text{Diameter} = 2r = \frac{2 \times 36}{2+\pi}$$

$$= \frac{72}{2+\pi} \text{ cm}$$

12. *(c) 60 – 80*

Explanation: Since, the frequency of class 60-80 is maximum as 29.

Then, 60 – 80 is the modal class.

13. *(c) 0.995*

Explanation: P (not E) = 1 – P (E)

P (not E) = 1 – 0.005 = 0.995

14. *(a) 17.5*

Explanation: The classes in exclusive form are:

(– 0.5) – 5.5; 5.5 – 11.5 11.5 – 17.5; 17.5 – 23.5; 23.5 – 24.5 with cumulative frequencies of 13, 23, 38, 46 and 57.

Here, $N = 57$. So, $\frac{N}{2} = 28.5$

Cumulative frequency just greater than 28.5 is 38, which belongs to class 11.5 – 17.5.

Thus, median class is 11.5 – 17.5 whose upper limit is 17.5.

15. *(c)* $\frac{1}{6}$

Explanation: Possible outcomes on rolling a die are 1, 2, 3, 4, 5 and 6.

Out of these six numbers, only 2 is the even prime number.

So, the required probability is $\frac{1}{6}$.

16. *(d) 0*

Explanation: Here, $\sin B = 0.5 = \frac{1}{2}$

Then, $\cos^2 B + \sin^2 B = 1$

$$\cos^2 B = 1 - \left(\frac{1}{2}\right)^2$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore \quad \cos B = \frac{\sqrt{3}}{2}$$

Then, $3 \cos B - 4 \cos^3 B$

$$= 3 \times \frac{\sqrt{3}}{2} - 4 \times \left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8}$$

$$= 0$$

17. (c) Sphere

Explanation: If we join two hemispheres of same radius along their bases, then we get a sphere.

18. (a) -2

Explanation: Here, $\text{cosec}^2 30° \sin^2 45° - \sec^2 60°$

$$= (2)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2$$

$$= 4 \times \frac{1}{2} - 4 = 2 - 4 = -2$$

Caution

➥ *Learn the table of trigonometric ratios for specific angles properly for solving such types of questions.*

19. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Explanation: $\tan 30° = \frac{AB}{BC} = \frac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20$$

$$= \frac{20}{1.73} = 11.56 \text{ m}$$

20. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Explanation: Total surface area $= 2\pi rh + 2\pi r^2$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 14(30 + 14)$$

$$= 88(44)$$

$$= 3872 \text{ cm}^2$$

Caution

➥ *It is important to know the formula of total surface area of cylinder. i.e., ($2\pi rh + 2\pi r^2$).*

SECTION - B

21. Prime factorisation of 150 and 240.

2	150
3	75
5	25
5	5
	1

2	240
2	120
2	60
2	30
3	15
5	5
	1

Then, $150 = 2 \times 3 \times 5 \times 5$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{HCF} = 2 \times 3 \times 5 = 30$$

$$\text{LCM} = 2 \times 3 \times 5 \times 5 \times 2 \times 2 \times 2$$

$$= 1200$$

Caution

➥ *While calculating prime factors, start with the lowest prime number.*

OR

If possible, let us assume that $3 + \sqrt{5}$ be a rational number. So, there exist positive integers a and b such that, $3 + \sqrt{5} = \frac{a}{b}$, where a and b are integers having no common factor other than 1.

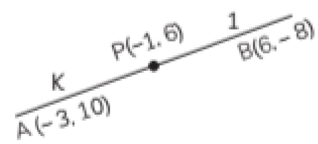
$$\Rightarrow \quad \sqrt{5} = \frac{a}{b} - 3$$

$$\Rightarrow \quad \sqrt{5} = \frac{a - 3b}{b}$$

Since, $\frac{a-3b}{b}$ is a rational number, so $\sqrt{5}$ must a rational number (L.H.S. = R.H.S.) which is a contradiction to the fact that "$\sqrt{5}$ is irrational".

Hence, $3 + \sqrt{5}$ is an irrational number.

22. Let P(–1, 6) divide the join of A(–3, 10) and B(6, –8) in the ratio K : 1.



Then,

$$P(-1, 6) = \left(\frac{6K - 3}{K + 1}, \frac{-8K + 10}{K + 1}\right)$$

$$\Rightarrow \quad \frac{6K - 3}{K + 1} = -1 \quad ; \quad \frac{-8K + 10}{K + 1} = 6$$

$$\Rightarrow \quad 6K - 3 = -K - 1 \; ; -8K + 10 = 6K + 6$$

$$\Rightarrow \quad 7K = 2 \quad ; \quad 14K = 4$$

$$\Rightarrow \quad K = \frac{2}{7}$$

Thus, the required ratio is 2 : 7.

OR

Given, points are A(1, – 1), B(5, 2) and C(9, 5)

Distance between AB $= \sqrt{(5 - 1)^2 + (2 + 1)^2}$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{25} = 5 \text{ units}$$

Distance between BC $= \sqrt{(9 - 5)^2 + (5 - 2)^2}$

$$= \sqrt{4^2 + 3^2} = 5 \text{ units}$$

Distance between AC = $\sqrt{(9-1)^2 + (5+1)^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$ = 10 units

Then, AC = AB + BC = 10 units

Hence, the point A, B and C are collinear.

Caution

The distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

It gives the same answer.

23. We have,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing numerator and denominator of L.H.S. by cos θ,

$$\Rightarrow \frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

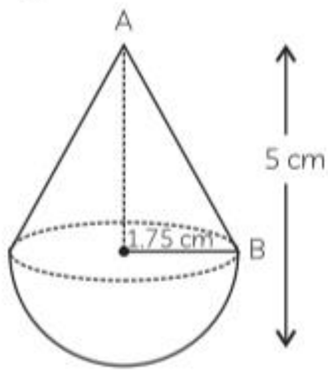
On comparing, we get $\tan\theta = \sqrt{3}$

$\Rightarrow \qquad \theta = 60°.$

Caution

Apply deduction of trigonometric Identities, wherever necessary.

24. (A) Hemisphere is of radius 1.75 cm (*i.e.,* 3.5/2)



So, its curved surface area

$= 2\pi r^2$

$= \left(2 \times \frac{22}{7} \times 1.75 \times 1.75\right) \text{ cm}^2$

$= 19.25 \text{ cm}^2.$

(B) For Cone r = 1.75 cm

$h = 5 - 1.75 = 3.25$ cm

and $l = \sqrt{r^2 + h^2}$

Area of the whole lattu

= curved surface area of cone + curved surface area of hemisphere

$= \left[\pi(1.75)\sqrt{(1.75)^2 + (3.25)^2} + 19.25\right] \text{cm}^2$

$= \left[\frac{22}{7} \times 1.75 \times 3.691 + 19.25\right] \text{ cm}^2$

$= (20.30 + 19.25) \text{ cm}^2$

$= 39.55 \text{ cm}^2.$

25.

Class	Frequency (f_i)	Mid Point (x_i)	$x_i f_i$
100-120	12	110	1320
120-140	14	130	1820
140-160	8	150	1200
160-180	6	170	1020
180-200	10	190	1900
Total	50		7260

Then, Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7260}{50} = 145.2$

SECTION - C

26. Let 'a' be the first term of AP and 'd' be the common difference. Here, total number of terms of AP is 9, *i.e.* $n = 9$

n^{th} term = last term = $a_n = a + 8d = 28$ ...(i)

Also,

$$S_n = S_9 = \frac{9}{2}[2a + (9-1)d] = 144$$

$\Rightarrow \quad 9(a + 4d) = 144$

$\Rightarrow \quad 9a + 36d = 144$ or $a + 4d = 16$ ...(ii)

Subtracting (ii) from (i), we get

$(a + 8d) - (a + 4d) = 28 - 16$

$\Rightarrow \quad 4d = 12$

$\Rightarrow \quad d = 3$

Putting value of d in (i).

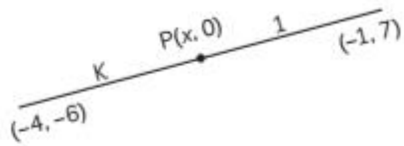
$a + 8 \times 3 = 28$

$\Rightarrow \quad a = 28 - 24 = 4$

So, $\quad a = 4$ and $d = 3$

Thus, the required first term is 4.

27. Let the join of (–4, –6) and (–1, 7) be divided by a point P on x-axis in the ratio K : 1.



Then,

$$P(x, 0) = P\left(\frac{-K-4}{K+1}, \frac{7K-6}{K+1}\right)$$

$$\Rightarrow \quad \frac{7K-6}{K+1} = 0$$

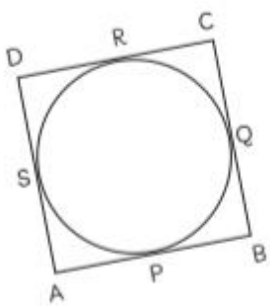
$$\Rightarrow \quad K = \frac{6}{7}$$

Hence, the required ratio is 6 : 7.

With $K = \frac{6}{7}$, the point P is $P\left(\frac{-\frac{6}{7}-4}{\frac{6}{7}+1}, 0\right)$ or P $\left(\frac{-34}{13}, 0\right)$

OR

Let ABCD be a parallelogram circumscribing a circle.



Let P, Q, R, S be the points where the circle touches the sides AB, BC, CD and DA respectively.

Now,

$$AB = DC \quad \text{and} \quad AB \parallel DC$$

Also,

$$AD = BC \quad \text{and} \quad AD \parallel BC$$

(∵ ABCD is a parallelogram) ...(i)

From the figure, we have:

AP = AS; BP = BQ; CR = CQ and DR = DS.

∵ AP + PB + CR + RD = AS + DS + CQ + BQ

$\Rightarrow$ AB + DC = AD + CB

$\Rightarrow$ 2 AB = 2 AD

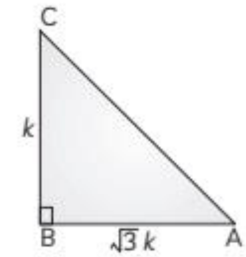
$\Rightarrow$ AB = AD

Using (i), we have:

AB = BC = CD = DA

Thus, ABCD is a rhombus.

28. Consider a triangle ABC in which $\angle B = 90°$.



Let BC = k and AB = $\sqrt{3}k$

Then, using Pythagoras theorem,

$$AC = \sqrt{(BC)^2 + (AB)^2}$$

$$= \sqrt{(k)^2 + (\sqrt{3}k)^2}$$

$$= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

For $\angle C$, Base = BC, Perpendicular = AB and Hypotenuse = AC

$$\therefore \quad \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

sin A cos C + cos A sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

29. In the mentioned circle,

O is the centre and AO = BO = Radius = 10 cm

AB is a chord which subtents 90° at centre O, i.e., $\angle AOB = 90°$

Area of minor segment APB (Shaded region)

= Area of sector ΔAOB – Area of ΔAOB

$$= \left(\frac{\pi \times 10 \times 10}{4}\right) - (0.5 \times 10 \times 10)$$

$$= 78.5 - 50$$

$$= 28.5 \text{ cm}^2$$

Area of major sector = Area of circle – Area of sector AOB

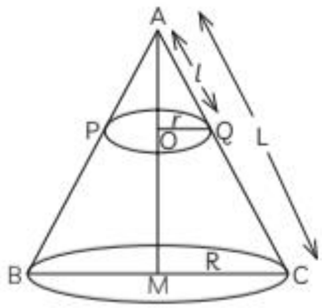
$$= (\pi \times 10 \times 10) + \left(\frac{\pi \times 10 \times 10}{4}\right)$$

$$= 314 - 78.5$$

$$= 235.5 \text{ cm}^2$$

OR

In the figure, the smaller cone APQ has been cut off through the plane PQ || BC. Let r and R be the radii of the smaller and bigger cones and l and L be their slant heights respectively.



Here,

$$OQ = r,\ MC = R,\ AQ = l \text{ and } AC = L$$

Now, $\Delta AOQ \sim \Delta AMC$

$$\Rightarrow \quad \frac{OQ}{MC} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{l}{L}$$

$$\Rightarrow \quad r = \frac{Rl}{L} \quad \text{...(i)}$$

Since, curved surface area of the remainder $= \frac{8}{9}$, of the curved surface area of the whole cone therefore, we get

CSA of smaller cone $= \frac{1}{9}$ of the CSA of the whole cone

$$\therefore \quad \pi rl = \frac{1}{9}\pi RL$$

$$\Rightarrow \quad \pi\left(\frac{Rl}{L}\right)l = \frac{1}{9}(\pi RL) \quad \text{[Using (i)]}$$

$$\Rightarrow \quad l^2 = \frac{L^2}{9}$$

$$\Rightarrow \quad \frac{l}{L} = \frac{1}{3}$$

Now, again in similar triangles, AOQ and AMC, we have

$$\frac{AO}{AM} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{AO}{AM} = \frac{l}{L} = \frac{1}{3}$$

$$\Rightarrow \quad AO = \frac{AM}{3}$$

$$\Rightarrow \quad OM = AM - OA = AM - \frac{AM}{3} = \frac{2}{3}AM$$

$$\therefore \quad \frac{AO}{OM} = \frac{\frac{AM}{3}}{\frac{2\,AM}{3}} = \frac{1}{2}$$

Hence, the required ratio of the heights = 1 : 2

30. Number 'x' can be selected in 3 ways and corresponding to each, such way there are 3 ways of selecting 'y'.

Therefore, 2 numbers can be selected in 9 ways as listed below :

(1, 1) (1, 4) (1, 9) (2, 1) (2, 4) (2, 9) (3, 1) (3, 4) (3, 9)

Total numbers of outcomes = 9

The product xy will be less than 9, if x and y are chosen in one of the following ways.

(1, 1) (1, 4) (2, 1) (2, 4) (3, 1)

Number of favourable outcomes = 5

$$\therefore \quad P(\text{product less than } 9) = \frac{5}{9}$$

31. Median of data = 32·5

sum of frequency = 40

i.e., $f_1 + 31 + f_2 = 40$

$$\therefore \quad f_1 + f_2 = 9$$

Then, median class is 30-40

Class Interval	f_i	c.f.
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
Total	40	

Then, $$M_e = \frac{l + \left(\frac{N}{2} - cf\right) \times h}{f}$$

$$\Rightarrow \quad 32.5 = 30 + \frac{(20 - 14 - f_1)}{12} \times 10$$

$$\Rightarrow \quad 2.5 \times 6 = (6 - f_1) \times 5$$

$$\Rightarrow \quad 15.0 = 30 - 5f_1$$

$$\Rightarrow \quad 5f_1 = 15$$

$$\Rightarrow \quad f_1 = 3$$

$$\text{and} \quad f_2 = 6$$

Hence, the values of f_1 and f_2 are 3 and 6 respectively.

SECTION - D

32. Let time taken by pipe of smaller diameter to fill the tan $k = x$ hours

Let time taken by pipe of larger diameter to fill the tank = $(x - 10)$ hours

In 1 hour, the pipe with a smaller diameter can fill $\frac{1}{x}$ part of the tank.

In 1 hour, the pipe with larger diameter can fill $\frac{1}{(x-10)}$ part of the tank.

The tank is filled up in $\frac{75}{8}$ hours.

Thus, in 1 hour the pipe fill $\frac{8}{75}$ part of the tank.

$$\frac{1}{x}+\frac{1}{x-10} = \frac{8}{75}$$

$$\frac{(x-10)+x}{x(x-10)} = \frac{8}{75}$$

$$\frac{2x-10}{x^2-10x} = \frac{8}{75}$$

$$75(2x - 10) = 8(x^2 - 10x)$$

by cross multiplication

$$150x - 750 = 8x^2 - 80x$$

$$8x^2 - 230x + 750 = 0$$

$$4x^2 - 115x + 375 = 0$$

$$4x^2 - 100x - 15x + 375 = 0$$

$$4x(x - 25) - 15(x - 25) = 0$$

$$(4x - 15)(x - 25) = 0$$

$$4x - 15 = 0 \text{ or } x - 25 = 0$$

$$x = \frac{15}{4} \text{ or } x = 25$$

Case 1: When $x = \frac{15}{4}$

Then $x - 10 = \frac{15}{4} - 10$

$\Rightarrow$ $= \frac{15-40}{4}$

$\Rightarrow$ $= -\frac{25}{4}$

Time can never be negative so $x = 15/4$ is not possible.

Case 2: When $x = 25$ then

$$x - 10 = 25 - 10 = 15$$

$\therefore$ The pipe of smaller diameter can separately fill the tank in 25 hours, and the time taken by the larger pipe to fill the tank = (25 – 10) = 15 hours.

OR

Let, the number be $10x + y$ i.e., digit at units's place is 'y' and digit at ten's place is 'x'.

According to the question,

$$10x + y = 4(x + y)$$

$\Rightarrow$ $10x + y = 4x + 4y$

$\Rightarrow$ $6x - 3y = 0$

$\Rightarrow$ $2x = y$ ...(i)

and $10x + y = 3xy$

From (i)

$$10x + 2x = 3xy$$

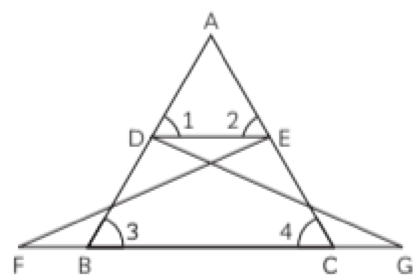
$\Rightarrow$ $12x = 3xy$

$\Rightarrow$ $y = 4$

and $x = 2$

$\therefore$ then, the numbers is 24.

33. Given : $\Delta FEC \cong \Delta GDB$



and $\angle 1 = \angle 2$

To prove: $\Delta ADE \sim \Delta ABC$

Proof: Since, $\Delta FEC \cong \Delta GDB$

Then, $EC = BD$ (by cpct) ...(i)

and $\angle 1 = \angle 2$

$\therefore$ $AE = AD$ ...(ii)

Then, $\frac{AE}{EC} = \frac{AD}{BD}$ (from equation (i) & (ii)]

$\therefore$ DE || BC (by converse of thales theorem)

$\therefore$ $\angle 1 = \angle ABC$ and $\angle 2 = \angle ACB$ (corresponding pair of angles)

In ΔADE and ΔABC,

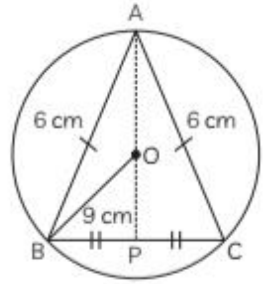
$\angle A = \angle A$ (common)

$\angle 1 = \angle ABC$ (proved above)

and $\angle 2 = \angle ACB$ (proved above)

$\therefore$ $\Delta ADE \sim \Delta ABC$ [by AA similarity]

34. Let O be the centre of the circle and P be the mid-point of BC. Then, OP ⊥ BC.



Since, ΔABC is isosceles and P is the mid-point of BC.

Therefore, AP ⊥ BC as median from the vertex in an isosceles triangle is perpendicular to the base.

Let, AP = x and PB = CP = y

Applying pythagoras theorem in Δs APB and OPB, we have

$$AB^2 = BP^2 + AP^2 \text{ and}$$
$$OB^2 = OP^2 + BP^2$$

$\Rightarrow \quad 36 = y^2 + x^2 \quad$ ...(i)

and $\quad 81 = (9 - x)^2 + y^2 \quad$ ...(ii)

$\Rightarrow \quad 81 - 36 = (9 - x)^2 + y^2 - y^2 + x^2$

(subtracting (i) from (ii))

$\Rightarrow \quad 45 = 81 - 18x$

$\Rightarrow \quad x = 2$ cm

Put $x = 2$ in equation (i), we get

$$36 = y^2 + 4$$

$\Rightarrow \quad y^2 = 32$

$\Rightarrow \quad y = 4\sqrt{2}$ cm

$\therefore \quad BC = 2BP = 2y = 8\sqrt{2}$ cm

Hence, area of ΔABC = $\frac{1}{2}$ BC × AP

$$= \frac{1}{2} \times 8\sqrt{2} \times 2 \text{ cm}^2$$

$$= 8\sqrt{2} \text{ cm}^2$$

35. Let $\sqrt{a}$ be a rational number

$\therefore \quad \sqrt{a} = \frac{p}{q}$, where p and q are co-prime integers.

On squaring both side, we get $q \neq 0$.

$$a = \frac{p^2}{q^2}$$

$\Rightarrow \quad p^2 = aq^2 \quad$ ...(i)

$\Rightarrow a$ divides p^2

$\Rightarrow a$ divides p

Let p be a prime number. If p divides n^2, then p divides n, where n is a positive integer.

Let $p = am$, where m is any integer.

$$p^2 = a^2m^2$$

$\Rightarrow \quad aq^2 = a^2m^2 \quad$ [Using (i)]

$\Rightarrow \quad q^2 = am^2$

$\Rightarrow a$ divides q^2

$\Rightarrow a$ divides $q \quad$ ...(iii)

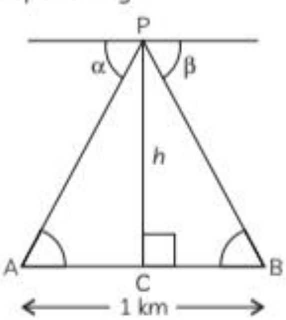
From (ii) and (iii), a is a common factor of both p and q which contradicts the assumption that p and q are co-prime integers.

So, our supposition is wrong.

Hence, $\sqrt{a}$ is an irrational number.

OR

Let P be the position of plane, A and B be the positions of two stones one kilometre apart. Angles of depression of stones A and B are α and β respectively.



Let PC = h

In right-angled ΔACP, we have

$$\tan \alpha = \frac{PC}{AC} \Rightarrow h = AC \tan \alpha$$

or $\quad AC = \frac{h}{\tan \alpha} \quad$ ...(i)

In right-angled ΔPCB, we have

$$\tan \beta = \frac{PC}{BC} \Rightarrow h = BC \tan \beta$$

or $\quad BC = \frac{h}{\tan \beta} \quad$ ...(ii)

From (i) and (ii), we have

$$AC + BC = \frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$$

$$\Rightarrow h\left(\frac{\tan \beta + \tan \alpha}{\tan \alpha \ \tan \beta}\right) = 1$$

$$\Rightarrow \quad h = \frac{\tan \alpha \ . \ \tan \beta}{\tan \alpha + \tan \beta}$$

Thus, the height of the aeroplane is $\frac{\tan \alpha \ . \ \tan \beta}{\tan \alpha + \tan \beta}$.

SECTION - E

36. (A) One pair of angle of elevation is $\angle b°$ and $\angle e°$ and one pair of angle of depression is and $\angle c°$ and $\angle d°$

(B) Then, sin 30°

$$= \frac{\text{Vertical height}}{\text{Distance between Naik and Vinod}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{12}{D_{N and V}}$$

$\Rightarrow$ Distance = 24 m

(C) Here, $\angle d° = \angle f° = 30°$

Then, $\dfrac{\text{Height of cliff.}}{\text{Distance of Ajay's boat from the base of cliff}} = \tan 30°$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{25}$$

$$\Rightarrow \quad h = \frac{25}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{25}{3} \times \sqrt{3}$$

$$= 14.45 \text{ m}$$

Caution

Learn the table of trigonometric ratios for specific angles properly for solving such types of questions.

OR

Here, height of cliff = 30 m

Then, $\angle c = \angle e = 45°$

$$\therefore \tan 45° = \frac{\text{height of cliff}}{\text{Distance of Maran's boat}}$$

$$\Rightarrow \quad 1 = \frac{30}{\text{Distance of Maran's boat}}$$

$\Rightarrow$ Distance of maran's boat = 30 m

And $\angle d = \angle f = 30°$

$$\therefore \tan 30° = \frac{\text{height of cliff}}{\text{Distance of Ajay's boat}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30}{D_A}$$

$$\Rightarrow \quad D_A = 30\sqrt{3}$$

$\therefore$ Distance between boats = $30\sqrt{3} - 30$

$= 30\,(1.73 - 1)$

$= 21.9$ m

37. (A) Amount lift = $11x + 1$

$\therefore 11x + 1 = 540$

$$x = \frac{539}{11} = 49$$

(B) Since, $x = 49$

$\therefore$ Amount received by each family is

$x^2 + 2x + 1 = (49)^2 + 2(49) + 1$

$= 2401 + 98 + 1$

$= 2500$

(C) Since, $x = 49$

$\therefore$ Fund alloted is—

$x^3 - 5x^2 - 2x - 6$

$= (49)^3 - 5(49)^2 - 2(49) - 6$

$= 117649 - 12005 - 98 - 6$

$= 1,05,540$

OR

If α and β are the zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c$,

$a \neq 0$, then $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Here, $\alpha + \beta = 8$; $\alpha\beta = k$

It is given that

$\alpha^2 + \beta^2 = 40$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$40 = (8)^2 - 2k$

$40 = 64 - 2k$

$2k = 24$

$k = 12$

38. (A) Total time spent = 7 + 6 + 8 + 9

= 30 minutes

(B) Total money pooled = ₹ 3500

Cost of 1 gift = ₹ 70

$\therefore$ No. of gifts = $\dfrac{3500}{70} = 50$

(C) Total money pooled = ₹ 3500

Money given by Rohan and Madhukar

$$= \left(\frac{2x + 5x}{14x}\right) \times 3500$$

$$= \frac{7}{14} \times 3500$$

= ₹ 1750

No. of gifts are 50

Time taken for one gift = 30 minutes

$\therefore$ Total time taken = 50 × 30

= 1500 minutes

$$= \frac{1500}{60}$$

$$= 25\ h$$

OR

Total money polled = ₹ 3500

Let, the money contributed be $2x$, $3x$, $4x$, $5x$

$\therefore \quad 2x + 3x + 4x + 5x = 3500$

$\Rightarrow \quad 14x = 3500$

$\Rightarrow \quad x = \frac{500}{2} = 250$

Then, contribution of saran = 4 × 250 = 1000