Introduction to Three Dimensional Geometry

Question 1.
The projections of a directed line segment on the coordinate axes are 12, 4, 3. The DCS of the line are
(a) 12/13, -4/13, 3/13
(b) -12/13, -4/13, 3/13
(c) 12/13, 4/13, 3/13
(d) None of these

Answer: (c) 12/13, 4/13, 3/13
Let AB be the given line and the DCs of AB be l, m, n. Then
Projection on x-axis = AB . l = 12 (Given)
Projection on y-axis = AB . m = 4 (Given)
Projection on z-axis = AB . n = 3 (Given)
⇒ $(AB^2)(l^2 + m^2 + n^2) = 144 + 16 + 9$
⇒ $(AB^2) = 169$ {since $l^2 + m^2 + n^2 = 1$}
⇒ AB = 13
Hence, DCs of AB are 12/13, 4/13, 3/13

Question 2.
The angle between the planes $r . n_1 = d_1$ and $r . n_1 = d_2$ is
(a) $\cos \theta = \{|n_1| \times |n_2|\} / (n_1 . n_2)$
(b) $\cos \theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}^2$
(c) $\cos \theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$
(d) $\cos \theta = (n_1 . n_2)^2 / \{|n_1| \times |n_2|\}$

Answer: (c) $\cos \theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$
The angle between the planes $r . n_1 = d_1$ and $r . n_2 = d_2$ is defined as
$\cos \theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$

Question 3.
For every point $P(x, y, z)$ on the xy-plane
(a) $x = 0$
(b) $y = 0$
(c) $z = 0$
(d) None of these

Answer: (c) $z = 0$
The perpendicular distance of $P(x, y, z)$ from xy-plane is zero.

Question 4.
The locus of a point $P(x, y, z)$ which moves in such a way that $x = a$ and $y = b$, is a
(a) Plane parallel to xy-plane
(b) Line parallel to x-axis
(c) Line parallel to y-axis
(d) Line parallel to z-axis

Answer: (b) Line parallel to x-axis
Since $x = 0$ and $y = 0$ together represent x-axis, therefore $x = a$ and $y = b$ represent a line parallel to x-axis.

Question 5.
The equation of the plane containing the line $2x – 5y + z = 3$, $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$ is
(a) $x + 3y + 6z + 7 = 0$
(b) $x + 3y – 6z – 7 = 0$
(c) $x – 3y + 6z – 7 = 0$
(d) $x + 3y + 6z – 7 = 0$

Answer: (d) $x + 3y + 6z – 7 = 0$
Let the equation of the plane is
$(2x – 5y + z – 3) + \lambda(x + y + 4z – 5) = 0$
$\Rightarrow (2 + \lambda)x + (\lambda – 5)y + (4\lambda + 1)z – (3 + 5\lambda) = 0$
Since the plane is parallel to $x + 3y + 6z – 1 = 0$
$\Rightarrow (2 + \lambda)/1 = (\lambda – 5)/3 = (1 + 4\lambda)/6$
$\Rightarrow 6 + 3\lambda = \lambda – 5$
$\Rightarrow 2\lambda = -11$
$\Rightarrow \lambda = -11/2$
Again,
$6\lambda – 30 = 3 + 12\lambda$
$\Rightarrow -6\lambda = -33$

⇒ $\lambda = -33/6$
⇒ $\lambda = -11/2$
So, the required equation of plane is
$(2x - 5y + z - 3) + (-11/2) \times (x + y + 4z - 5) = 0$
⇒ $2(2x - 5y + z - 3) + (-11) \times (x + y + 4z - 5) = 0$
⇒ $4x - 10y + 2z - 6 - 11x - 11y - 44z + 55 = 0$
⇒ $-7x - 21y - 42z + 49 = 0$
⇒ $x + 3y + 6z - 7 = 0$

Question 6.
The coordinate of foot of perpendicular drawn from the point A(1, 0, 3) to the join of the point B(4, 7, 1) and C(3, 5, 3) are
(a) (5/3, 7/3, 17/3)
(b) (5, 7, 17)
(c) (5/3, -7/3, 17/3)
(d) (5/7, -7/3, -17/3)

Answer: (a) (5/3, 7/3, 17/3)
Let D be the foot of perpendicular and let it divide BC in the ration m : 1
Then the coordinates of D are $\{(3m + 4)/(m + 1), (5m + 7)/(m + 1), (3m + 1)/(m + 1)\}$
Now, AD ⊥ BC
⇒ AD . BC = 0
⇒ $-(2m + 3) - 2(5m + 7) - 4 = 0$
⇒ $m = -7/4$
So, the coordinate of D are (5/3, 7/3, 17/3)

Question 7.
The coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane is
(a) (0, 17/2, 13/2)
(b) (0, -17/2, -13/2)
(c) (0, 17/2, -13/2)
(d) None of these

Answer: (c) (0, 17/2, -13/2)
The line passing through the points (5, 1, 6) and (3, 4, 1) is given as
$(x - 5)/(3 - 5) = (y - 1)/(4 - 1) = (z - 6)/(1 - 6)$
⇒ $(x - 5)/(-2) = (y - 1)/3 = (z - 6)/(-5) = k$(say)
⇒ $(x - 5)/(-2) = k$
⇒ $x - 5 = -2k$
⇒ $x = 5 - 2k$

$(y - 1)/3 = k$
$\Rightarrow y - 1 = 3k$
$\Rightarrow y = 3k + 1$
and $(z - 6)/(-5) = k$
$\Rightarrow z - 6 = -5k$
$\Rightarrow z = 6 - 5k$
Now, any point on the line is of the form $(5 - 2k, 3k + 1, 6 - 5k)$
The equation of YZ-plane is $x = 0$
Since the line passes through YZ-plane
So, $5 - 2k = 0$
$\Rightarrow k = 5/2$
Now, $3k + 1 = 3 \times 5/2 + 1 = 15/2 + 1 = 17/2$
and $6 - 5k = 6 - 5 \times 5/2 = 6 - 25/2 = -13/2$
Hence, the required point is $(0, 17/2, -13/2)$

Question 8.
If P is a point in space such that OP = 12 and OP inclined at angles 45 and 60 degrees with OX and OY respectively, then the position vector of P is
(a) $6i + 6j \pm 6\sqrt{2}k$
(b) $6i + 6\sqrt{2}j \pm 6k$
(c) $6\sqrt{2}i + 6j \pm 6k$
(d) None of these

Answer: (c) $6\sqrt{2}i + 6j \pm 6k$
Let l, m, n be the DCs of OP.
Then it is given that $l = \cos 45 = 1/\sqrt{2}$
$m = \cos 60 = 1/2$
Now, $l^2 + m^2 + n^2 = 1$
$\Rightarrow 1/2 + 1/4 + n^2 = 1$
$\Rightarrow n^2 = 1/4$
$\Rightarrow n = \pm 1/2$
Now, $r = |r|(li + mj + nk)$
$\Rightarrow r = 12(i/\sqrt{2} + j/2 \pm k/\sqrt{2})$
$\Rightarrow r = 6\sqrt{2}i + 6j \pm 6k$

Question 9.
The image of the point P(1,3,4) in the plane $2x - y + z = 0$ is
(a) (-3, 5, 2)
(b) (3, 5, 2)

(c) (3, -5, 2)
(d) (3, 5, -2)

Answer: (a) (-3, 5, 2)
Let image of the point P(1, 3, 4) is Q in the given plane.
The equation of the line through P and normal to the given plane is
$(x – 1)/2 = (y – 3)/-1 = (z – 4)/1$
Since the line passes through Q, so let the coordinate of Q are (2r + 1, -r + 3, r + 4)
Now, the coordinate of the mid-point of PQ is
(r + 1, -r/2 + 3, r/2 + 4)
Now, this point lies in the given plane.
$2(r + 1) – (-r/2 + 3) + (r/2 + 4) + 3 = 0$
$\Rightarrow 2r + 2 + r/2 – 3 + r/2 + 4 + 3 = 0$
$\Rightarrow 3r + 6 = 0$
$\Rightarrow r = -2$
Hence, the coordinate of Q is (2r + 1, -r + 3, r + 4) = (-4 + 1, 2 + 3, -2 + 4)
= (-3, 5, 2)

Question 10.
There is one and only one sphere through
(a) 4 points not in the same plane
(b) 4 points not lie in the same straight line
(c) none of these
(d) 3 points not lie in the same line

Answer: (a) 4 points not in the same plane
Sphere is referred to its center and it follows a quadratic equation with 2 roots. The mid-point of chords of a sphere and parallel to fixed direction lies in the normal diametrical plane.
Now, general equation of the plane depends on 4 constants. So, one sphere passes through 4 points and they need not be in the same plane.

Question 11.
The points on the y- axis which are at a distance of 3 units from the point (2, 3, -1) is
(a) either (0, -1, 0) or (0, -7, 0)
(b) either (0, 1, 0) or (0, 7, 0)
(c) either (0, 1, 0) or (0, -7, 0)
(d) either (0, -1, 0) or (0, 7, 0)

Answer: (d) either (0, -1, 0) or (0, 7, 0)
Let the point on y-axis is O(0, y, 0)
Given point is A(2, 3, -1)

Given OA = 3
⇒ $OA^2 = 9$
⇒ $(2 – 0)^2 + (3 – y)^2 + (-1 – 0)^2 = 9$
⇒ $4 + (3 – y)^2 + 1 = 9$
⇒ $5 + (3 – y)^2 = 9$
⇒ $(3 – y)^2 = 9 – 5$
⇒ $(3 – y)^2 = 4$
⇒ $3 – y = \sqrt{4}$
⇒ $3 – y = \pm 4$
⇒ 3 – y = 4 and 3 – y = -4
⇒ y = -1, 7
So, the point is either (0, -1, 0) or (0, 7, 0)

Question 12.
The coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane is
(a) (0, 17/2, 13/2)
(b) (0, -17/2, -13/2)
(c) (0, 17/2, -13/2)
(d) None of these

Answer: (c) (0, 17/2, -13/2)
The line passing through the points (5,1,6) and (3,4,1) is given as
(x – 5)/(3 – 5) = (y – 1)/(4 – 1) = (z – 6)/(1 – 6)
⇒ (x – 5)/(-2) = (y – 1)/3 = (z – 6)/(-5) = k(say)
⇒ (x – 5)/(-2) = k
⇒ x – 5 = -2k
⇒ x = 5 – 2k
(y – 1)/3 = k
⇒ y – 1 = 3k
⇒ y = 3k + 1
and (z-6)/(-5) = k
⇒ z – 6 = -5k
⇒ z = 6 – 5k
Now, any point on the line is of the form (5 – 2k, 3k + 1, 6 – 5k)
The equation of YZ-plane is x = 0
Since the line passes through YZ-plane
So, 5 – 2k = 0
⇒ k = 5/2
Now, 3k + 1 = 3 × 5/2 + 1 = 15/2 + 1 = 17/2

and $6 - 5k = 6 - 5 \times 5/2 = 6 - 25/2 = -13/2$
Hence, the required point is $(0, 17/2, -13/2)$

Question 13.
he equation of plane passing through the point $i + j + k$ and parallel to the plane $r . (2i - j + 2k) = 5$ is
(a) $r . (2i - j + 2k) = 2$
(b) $r . (2i - j + 2k) = 3$
(c) $r . (2i - j + 2k) = 4$
(d) $r . (2i - j + 2k) = 5$

Answer: (b) $r . (2i - j + 2k) = 3$
The equation of plane parallel to the plane $r . (2i - j + 2k) = 5$ is
$r . (2i - j + 2k) = d$
Since it passes through the point $i + j + k$, therefore
$(i + j + k) . (2i - j + 2k) = d$
$\Rightarrow d = 2 - 1 + 2$
$\Rightarrow d = 3$
So, the required equation of the plane is
$r . (2i - j + 2k) = 3$

Question 14.
The cartesian equation of the line is $3x + 1 = 6y - 2 = 1 - z$ then its direction ratio are
(a) 1/3, 1/6, 1
(b) -1/3, 1/6, 1
(c) 1/3, -1/6, 1
(d) 1/3, 1/6, -1

Answer: (a) 1/3, 1/6, 1
Give $3x + 1 = 6y - 2 = 1 - z$
$= (3x + 1)/1 = (6y - 2)/1 = (1 - z)/1$
$= (x + 1/3)/(1/3) = (y - 2/6)/(1/6) = (1 - z)/1$
$= (x + 1/3)/(1/3) = (y - 1/3)/(1/6) = (1 - z)/1$
Now, the direction ratios are: 1/3, 1/6, 1

Question 15.
Under what condition does the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d$ represent a real sphere
(a) $u^2 + v^2 + w^2 = d^2$
(b) $u^2 + v^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$
(d) $u^2 + v^2 + w^2 < d^2$

Answer: (b) $u^2 + v^2 + w^2 > d$
Equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d$ represent a real sphere if
$u^2 + v^2 + w^2 - d > 0$
$\Rightarrow u^2 + v^2 + w^2 > d$

Question 16.
The locus of a first-degree equation in x, y, z is a
(a) sphere
(b) straight line
(c) plane
(d) none of these

Answer: (c) plane
In an x-y-z cartesian coordinate system, the general form of the equation of a plane is
$ax + by + cz + d = 0$
It is an equation of the first degree in three variables.

Question 17.
The image of the point P(1,3,4) in the plane $2x - y + z = 0$ is
(a) (-3, 5, 2)
(b) (3, 5, 2)
(c) (3, -5, 2)
(d) (3, 5, -2)

Answer: (a) (-3, 5, 2)
Let image of the point P(1, 3, 4) is Q in the given plane.
The equation of the line through P and normal to the given plane is
$(x - 1)/2 = (y - 3)/-1 = (z - 4)/1$
Since the line passes through Q, so let the coordinate of Q are $(2r + 1, -r + 3, r + 4)$
Now, the coordinate of the mid-point of PQ is
$(r + 1, -r/2 + 3, r/2 + 4)$
Now, this point lies in the given plane.
$2(r + 1) - (-r/2 + 3) + (r/2 + 4) + 3 = 0$
$\Rightarrow 2r + 2 + r/2 - 3 + r/2 + 4 + 3 = 0$
$\Rightarrow 3r + 6 = 0$
$\Rightarrow r = -2$
Hence, the coordinate of Q is $(2r + 1, -r + 3, r + 4) = (-4 + 1, 2 + 3, -2 + 4)$
$= (-3, 5, 2)$

Question 18.
The distance of the point P(a, b, c) from the x-axis is
(a) $\sqrt{(a^2 + c^2)}$
(b) $\sqrt{(a^2 + b^2)}$
(c) $\sqrt{(b^2 + c^2)}$
(d) None of these

Answer: (c) $\sqrt{(b^2 + c^2)}$
The coordinate of the foot of the perpendicular from P on x-axis are (a, 0, 0).
So, the required distance = $\sqrt{\{(a - a)^2 + (b - 0)^2 + (c - 0)^2\}}$
= $\sqrt{(b^2 + c^2)}$

Question 19.
The vector equation of a sphere having centre at origin and radius 5 is
(a) |r| = 5
(b) |r| = 25
(c) |r| = $\sqrt{5}$
(d) none of these

Answer: (a) |r| = 5
We know that the vector equation of a sphere having center at the origin and radius R
= |r| = R
Here R = 5
Hence, the equation of the required sphere is |r| = 5

Question 20.
The ratio in which the line joining the points(1, 2, 3) and (-3, 4, -5) is divided by the xy-plane is
(a) 2 : 5
(b) 3 : 5
(c) 5 : 2
(d) 5 : 3

Answer: (b) 3 : 5
Let the points are P(1, 2, 3) and Q(-3, 4, -5)
Let the line joining the points P(1, 2, 3) and Q(-3, 4, -5) is divided by the xy-plane at point R in the ratio k : 1
Now, the coordinate of R is
$\{(-3k + 1)/(k + 1), (4k + 2)/(k + 1), (-5k + 3)/(k + 1)\}$
Since R lies on the xy-plane.
So, z-coordinate is zero
$\Rightarrow (-5k + 3)/(k + 1) = 0$

⇒ $k = 3/5$
So, the ratio = $3/5 : 1 = 3 : 5$
