Solved Paper 2005

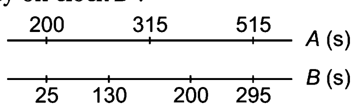
AMU

Engineering Entrance Exam

Physics

1. How far must you travel in degrees of latitude until your watch must be reset by 2 h ?
(a) 15° (b) 30°
(c) 45° (d) 60°

2. Two digital clocks A and B run at different rates and do not have simultaneous readings of zero as shown in the figure. If two events are 800 s apart on clock A, how far apart in second are they on clock B ?



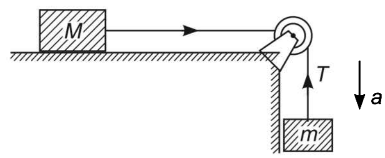
(a) 795 (b) 350
(c) 815 (d) 660

3. A man walks for 2 s at speed of 2.5 m/s and then runs for 4 s at a speed of 3.5 m/s. What is the average speed ?
(a) 3.05 m/s (b) 2.75 m/s
(c) 3.16 m/s (d) 3.25 m/s

4. A car is travelling at 72 km/h and is 20 m from a barrier when the driver puts on the brakes. The car hits the barrier 2 s later. What is the magnitude of the constant deceleration ?
(a) 7.2 m/s^2 (b) 10 m/s^2
(c) 36 m/s^2 (d) 15 m/s^2

5. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, where t is
(a) $\frac{a}{\sqrt{v^2 + v_1^2}}$ (b) $\sqrt{\frac{a^2}{v^2 - v_1^2}}$
(c) $\frac{a}{v - v_1}$ (d) $\frac{a}{v + v_1}$

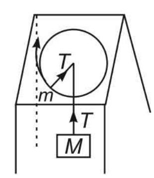
6. Two blocks of masses M and m are connected by a string passing over a pulley as shown in the figure. The downward acceleration of the block with mass m is



(a) $\frac{M}{(m + M)g}$ (b) $\frac{mg}{(m + M)}$
(c) $\frac{(m + M)}{mg}$ (d) $\frac{(m + M)}{Mg}$

7. A 80 kg person is parachuting and is experiencing a downward acceleration of 2.8 m/s^2. The mass of the parachute is 5 kg. The upward force on the open parachute is (Take $g = 9.8$ m/s^2)
(a) 595 N (b) 675 N
(c) 456 N (d) 925 N

8. A particle of mass m is rotating in a horizontal circle of radius R and is attached to a hanging mass M as shown in the figure. The speed of rotation required by the mass m to keep M steady is



(a) $\sqrt{\frac{mgR}{M}}$ (b) $\sqrt{\frac{MgR}{m}}$
(c) $\sqrt{\frac{mg}{MR}}$ (d) $\sqrt{\frac{mR}{Mg}}$

9. A force of 1200 N acts on a 0.5 kg steel ball as a result of collision lasting 25 ms. If the force is in a direction opposite to the initial velocity of 14 m/s, then the final speed of the steel ball would be
(a) 24 m/s (b) 35 m/s
(c) 12 m/s (d) 46 m/s

10. A 1.5 kg ball drops vertically on a floor hitting with a speed of 25 m/s. It rebounds with an initial speed of 15 m/s. If the ball was in contact for only 0.03 s, the force exerted on the floor by the ball is

(a) 2000 N (b) 3000 N
(c) 3500 N (d) 4000 N

11. A thin uniform circular disc of mass m and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to the plane with an angular velocity ω. Another disc of same dimensions but of mass $\frac{1}{4}m$ is placed gently on the first disc co-axially. The angular velocity of the system is

(a) $\sqrt{2}\,\omega$ (b) $\frac{4}{5}\omega$
(c) $\frac{3}{4}\omega$ (d) $\frac{1}{3}\omega$

12. What is the magnitude of torque acting on a particle moving in the xy-plane about the origin if its angular momentum is $4.0\sqrt{t}$ kg-m^2/s ?

(a) $8t^{3/2}$ (b) $4.0/\sqrt{t}$
(c) $2.0/\sqrt{t}$ (d) $3/2\sqrt{t}$

13. An asteroid of mass m is approaching earth, initially at a distance of 10 R_e with speed v_i. It hits the earth with a speed v_f (R_e and M_e are radius and mass of earth), then

(a) $v_f^2 = v_i^2 + \frac{2Gm}{M_eR}\left(1 - \frac{1}{10}\right)$

(b) $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 + \frac{1}{10}\right)$

(c) $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$

(d) $v_f^2 = v_i^2 + \frac{2Gm}{R_e}\left(1 - \frac{1}{10}\right)$

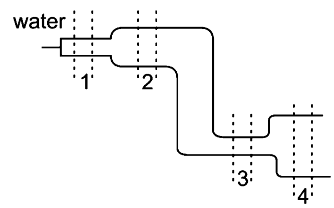
14. A rocket is sent vertically up with a velocity v less than the escape velocity from the earth. Taking M and R as the mass and radius of earth, the maximum height h attained by the rocket is given by the following expression

(a) $v^2R^2/(2GR - Mv)$
(b) $v^2R^2/(2GM + v^2R)$
(c) $v^2R^2/(2GM - v^2R)$
(d) $v^2R^2/(2Gv + RM)$

15. An iceberg is floating in water. The density of ice in the iceberg is 917 kg/m^3 and the density of water is 1024 kg/m^3. What percentage fraction of the iceberg would be visible ?

(a) 5% (b) 10%
(c) 12% (d) 8%

16. Water is flowing smoothly through a pipe as shown in the figure. Rank the four numbered sections of the pipe according to the water pressure p, greatest first



(a) $p_1 > p_3 > p_2 > p_4$
(b) $p_3 > p_4 > p_2 > p_1$
(c) $p_4 > p_3 > p_2 > p_1$
(d) $p_4 > p_1 > p_2 > p_3$

17. A particle is undergoing a one dimensional simple harmonic oscillation of amplitude X_m about the origin on X-axis with time period T and is at $-X_m$ at $t = 0$ s. What is the position of the particle after a time interval $t = 3.15T$?

(a) Between $-X_m$ and O
(b) Between O and $+X_m$
(c) At the origin
(d) At $+X_m$

18. Two springs of the same force constant k are joined in series to a block of mass m. What is the frequency of oscillation of the block ?

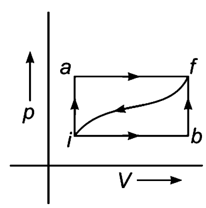
(a) $f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

(c) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (d) $f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$

19. A whistle of frequency 540 Hz is moving in a horizontal circle of radius 60 cm at an angular speed of 15 rad/s. The lowest and highest frequencies heard by a listener a long distance away with respect to the centre of the circle is

(a) 520 Hz and 560 Hz
(b) 500 Hz and 580 Hz
(c) 526 and 555 Hz
(d) 515 Hz and 565 Hz

20. When a system is taken from a state i to f along the path iaf (as shown in the figure). $Q = 50$ cal and $W = 20$ cal; along path ibf, $Q = 36$ cal



(i) What is W along path ibf ?

(ii) If $W = 13$ cal for path fi, what is Q for the path fi ?

(iii) Take $E_{\text{int},\, i} = 10$ cal then what is $E_{\text{int},\, f}$?

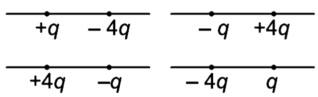
(a) 30, 20 40, cal

(b) 6, – 43, 40 cal

(c) 10, – 20, 30, cal

(d) 15, 35, 25 cal

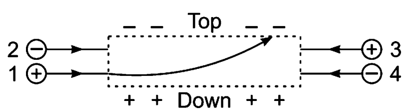
21. The figure shows four situations in which charges as indicated ($q > 0$) are fixed on an axis. In which situation is there a point to the left of the charges where an electron would be in equilibrium ?



(a) 1 and 2 (b) 2 and 4

(c) 3 and 4 (d) 1 and 3

22. The figure shows the path of a positively charged particle 1 through a rectangular region of uniform electric field as shown in the figure. What is the direction of electric field and the direction of deflection of particles 2, 3 and 4 ?



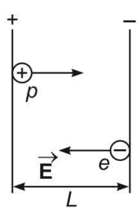
(a) Top, down, top, down

(b) Top, down, down, top

(c) Down, top, top, down

(d) Down, top, down, down

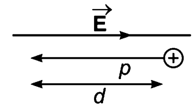
23. Two parallel copper plates as shown in the figure are L distance apart and have a uniform electric field E as shown. An electron is released from the negative plate and at the same time a proton is released from the positive plate. The distance from the + ve plate when they cross each other is



(a) $\dfrac{L}{1+\dfrac{m_e}{m_p}}$ (b) $\dfrac{L}{1-\dfrac{m_e}{m_p}}$

(c) $\dfrac{L}{1-\dfrac{m_p}{m_e}}$ (d) $\dfrac{L}{1+\dfrac{m_e}{m_p}}$

24. In the figure, a proton moves a distance d in a uniform electric field $\vec{\mathbf{E}}$ as shown is the figure. Does the electric field do a positive or negative work on the proton ? Does the electric potential energy of the proton increase or decrease ?



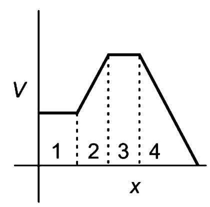
(a) Negative, increase

(b) Positive, decrease

(c) Negative, increase

(d) Positive, increase

25. The figure shows electric potential V as a function of x. Rank the four regions according to the magnitude of x-component of the electric field E within them, greatest first.



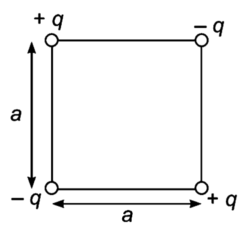
(a) $E_4 > E_2 > E_3 > E_1$

(b) $E_4 > E_2 > E_1 = E_3$

(c) $E_1 > E_2 > E_3 > E_4$

(d) $E_1 > E_3 > E_2 > E_4$

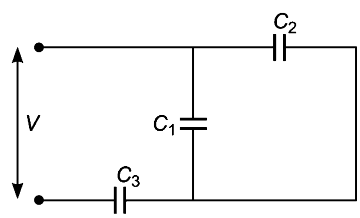
26. Work required to set up the four charge configuration (as shown in the figure) is



(a) $-0.21q^2/\varepsilon_0 a$ (b) $-1.29q^2/\varepsilon_0 a$

(c) $-1.41q^2/\varepsilon_0 a$ (d) $+2.82q^2/\varepsilon_0 a$

27. Three capacitors C_1, C_2 and C_3 are connected as shown in the figure to a battery of V volt. If the capacitor C_3 breaks down electrically the



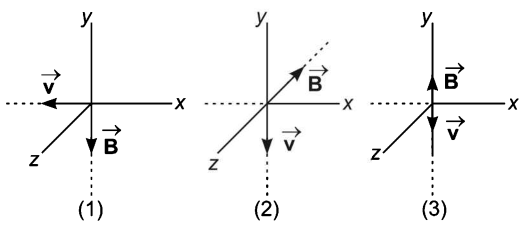
change in total charge on the combination of capacitors is

(a) $(C_1 + C_2) V[1 - C_3/(C_1 + C_2 + C_3)]$
(b) $(C_1 + C_2) V[1 - (C_1 + C_2)/(C_1 + C_2 + C_3)]$
(c) $(C_1 + C_2) V[1 + C_3/(C_1 + C_2 + C_3)]$
(d) $(C_1 + C_2) V[1 + C_2/(C_1 + C_2 + C_3)]$

28. A battery has an emf of 15 V and internal resistance of 1 Ω. Is the terminal to terminal potential difference less than, equal to or greater than 15 V, if the current in the battery is (1) from negative to positive terminal, (2) from positive to negative terminal and (3) zero current ?

(a) Less, greater, equal
(b) Less, less, equal
(c) Greater, greater, equal
(d) Greater, less, equal

29. The figure shows three situations when an electron with velocity $\vec{\mathbf{v}}$ travels through a uniform magnetic field $\vec{\mathbf{B}}$. In each case, what is the direction of magnetic force on the electron ?

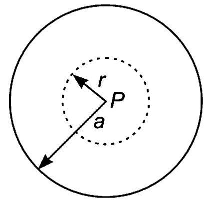


(a) + ve z-axis, – ve x-axis, + ve y-axis
(b) – ve z-axis, – ve x-axis and zero
(c) + ve z-axis, + ve y-axis and zero
(d) – ve z-axis, + ve x-axis and zero

30. A charged particle of mass m and charge q enters a region of uniform magnetic field $\vec{\mathbf{B}}$ perpendicular to its velocity $\vec{\mathbf{v}}$. The particle initially at rest was accelerated by a potential difference V (volt) before it entered the region of magnetic field. What is the diameter of the circular path followed by the charged particle in the region of magnetic field ?

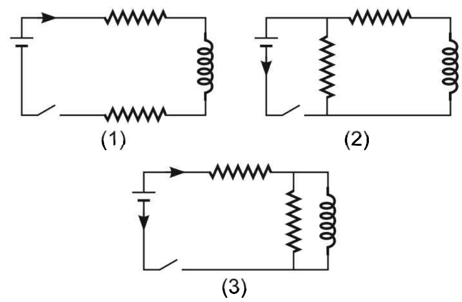
(a) $\frac{2}{B}\sqrt{\frac{mV}{q}}$ (b) $\frac{2}{B}\sqrt{\frac{2mV}{q}}$
(c) $B\sqrt{\frac{2mV}{q}}$ (d) $\frac{B}{q}\sqrt{\frac{2mV}{B}}$

31. The figure shows the cross-section of a long cylindrical conductor of radius a carrying a uniformly distributed current I. The magnetic field due to current at P is



(a) $\frac{\mu_0 Ir}{(2\pi a^2)}$ (b) $\frac{\mu_0 Ir^2}{(2\pi a)}$
(c) $\frac{\mu_0 Ia}{(2\pi r^2)}$ (d) $\frac{\mu_0 Ia^2}{(\pi r^2)}$

32. The figure shows three circuits with identical batteries, inductors and resistances. Rank the currents according to the currents through the battery just after the switch is closed, greatest first



(a) $i_2 > i_3 > i_1$ (b) $i_2 > i_1 > i_3$
(c) $i_1 > i_2 > i_3$ (d) $i_1 > i_3 > i_2$

33. An inductance L and a resistance R are connected in series with a battery of emf ε. The maximum rate at which the energy is stored in the magnetic field is

(a) $\frac{\varepsilon^2}{4R}$ (b) $\frac{\varepsilon^2}{2R}$
(c) $\frac{2R}{\varepsilon}$ (d) $\frac{4R}{\varepsilon}$

34. What direct current will produce the same amount of thermal energy in a resistance $R = 2\Omega$ as an alternating current that a peak value of 4.24 A and frequency 50 Hz ?

(a) 3 A (b) 2 A
(c) 5 A (d) 4 A

35. A ray of light is incident on the surface of a glass plate of thickness t. If the angle of incidence θ is small, the emerging ray would be displaced sideways by an amount
(Take μ = refractive index of glass)

(a) $\frac{t\,\theta\mu}{(\mu+1)}$ (b) $\frac{t\,\theta(\mu-1)}{\mu}$

(c) $\frac{t\,\theta\mu}{(\mu-1)}$ (d) $\frac{t\,\theta(\mu+1)}{\mu}$

36. A parallel beam of light is incident on a solid transparent sphere of a material of refractive index μ. If a point image is produced at the back of the sphere, the refractive index of the material of sphere is

(a) 2.5 (b) 1.5
(c) 1.25 (d) 2.0

37. The wavelength of a certain colour in air is 600 nm. What is the wavelength and speed of this colour in glass of refractive index 1.5 ?

(a) 500 nm and 2×10^{10} cm/s
(b) 400 nm and 2×10^{8} m/s
(c) 300 nm and 3×10^{9} cm/s
(d) 700 nm and 1.5×10^{8} m/s

38. An electron and a neutron can have same (1) kinetic energy, (2) momentum and (3) speed. Which particle has the shorter de-Broglie wavelength ?

(a) Neutron, same, neutron
(b) Neutron, electron, same
(c) Electron, same, neutron
(d) Electron, neutron, electron

39. What is the maximum wavelength of light emitted in Lyman series by hydrogen atom ?

(a) 691 nm (b) 550 nm
(c) 380 nm (d) 122 nm

40. In a *p-n* junction diode acting as a half-wave rectifier, which of the following statements is not true ?

(a) The average output voltage over a cycle is non-zero
(b) The drift current depends on biasing
(c) The depletion zone decreases in forward biasing
(d) The diffusion current increases due to forward biasing

Chemistry

41. A solution is prepared by dissolving 24.5 g of sodium hydroxide in distilled water to give 1 L solution. The molarity of NaOH in the solution is
(Given, molar mass of NaOH = 40.0 g mol^{-1})

(a) 0.2450 M (b) 0.6125 M
(c) 0.9800 M (d) 1.6326 M

42. An electron with values 4, 3, –2 and $+\frac{1}{2}$ for the set of four quantum numbers n, l, m_l and m_s, respectively, belongs to

(a) 4*s*-orbital (b) 4*p*-orbital
(c) 4*d*-orbital (d) 4*f*-orbital

43. The electronic configuration

$$(\sigma 1s)^2(\overset{*}{\sigma} 1s)^2(\sigma 2s)^2(\overset{*}{\sigma} 2s)^2(\sigma 2p_z)^2$$
$$(\pi 2p_x)^2(\pi 2p_y)^2(\overset{*}{\pi} 2p_x)^2(\overset{*}{\pi} 2p_y)^1$$

can be assigned to

(a) O_2 (b) O_2^+
(c) O_2^- (d) O_2^{2-}

44. What is the number of tetrahedral voids per atom in a crystal?

(a) 1 (b) 2
(c) 6 (d) 8

45. In an osmotic pressure measurement experiment, a 5% solution of compound '*X*' is found to be isotonic with a 2% acetic acid solution. The gram molecular mass of '*X*' is

(a) 24 (b) 60
(c) 150 (d) 300

46. On passing 0.5 faraday of electricity through molten sodium chloride,sodium deposited at cathode will be

(a) 29.25 g (b) 11.50 g
(c) 58.50 g (d) 0.00 g

47. The rate constant of a reaction is 3.00×10^{3} L mol^{-1} s^{-1}. The order of this reaction will be

(a) zero (b) first
(c) second (d) third

48. The value of ΔE for combustion of 16 g of CH_4 is –885389 J at 298 K. The ΔH combustion for CH_4 in J mol^{-1} at this temperature will be
(Given that R = 8.314 $JK^{-1}mol^{-1}$)
(a) – 55337 (b) – 880430
(c) – 885389 (d) – 890344

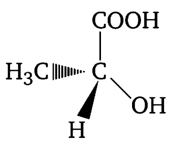
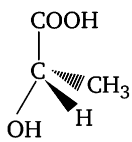
49. When two moles of hydrogen expands isothermally against a constant pressure of 1 atm, at 25°C from 15 L to 50 L, the work done (in L atm) will be
(a) 17.5 (b) 35
(c) 51.5 (d) 70

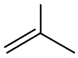
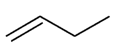
50. The enthalpy change for the transition of liquid water to steam is 40.8 kJ per mol at 100°C. The entropy change for the process will be
(a) 0.408 JK^{-1} mol^{-1}
(b) 408 JK^{-1} mol^{-1}
(c) 109.4 JK^{-1} mol^{-1}
(d) 0.1094 JK^{-1} mol^{-1}

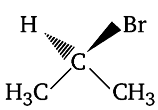
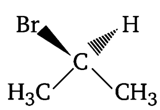
51. The entropy change for the reaction
$H_2(g) + Cl_2(g) \longrightarrow 2HCl(g)$, will be
[Given, $S°(HCl) = 187$ JK^{-1} mol^{-1},
$S°(H_2) = 131$ JK^{-1} mol^{-1}, and
$S°(Cl_2) = 223$ JK^{-1} mol^{-1}]
(a) 20 JK^{-1} mol^{-1} (b) –20 JK^{-1} mol^{-1}
(c) 167 JK^{-1} mol^{-1} (d) –167 JK^{-1} mol^{-1}

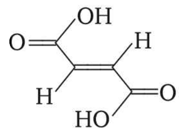
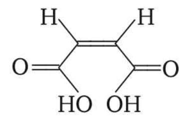
52. If the temperature of 500 mL of air increases from 27°C to 42°C under constant pressure, then the increase in volume shall be
(a) 15 mL (b) 20 mL
(c) 25 mL (d) 30 mL

53. Which of the following pair of structures is an example of diastereomers?

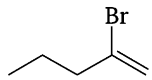
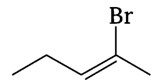
(a)  and 

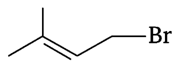
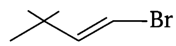
(b)  and 

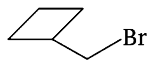
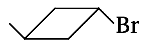
(c)  and 

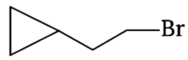
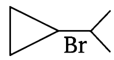
(d)  and 

54. Which of the following pair of compounds is not an example of structural isomers?

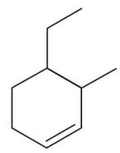
(a)  and 

(b)  and 

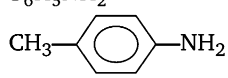
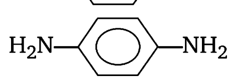
(c)  and 

(d)  and 

55. The systematic (IUPAC) name of the compound with the following structural formula will be



(a) 1-ethyl-2-methyl cyclohexene
(b) 2-methyl-1-ethyl cyclohexene
(c) 3-ethyl-2-methyl cyclohexene
(d) 4-ethyl-3-methyl cyclohexene

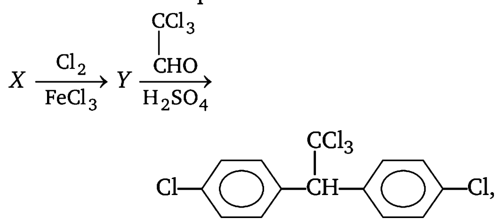
56. Which of the following compounds will form alcohol on treatment with $NaNO_2$, HCl/H_2O at 0°C ?
(a) $(CH_3)_2CHNH_2$
(b) $C_6H_5NH_2$
(c) 
(d) 

57. In the nucleophilic substitution reactions (S_N2 or S_N1), the reactivity of alkyl halides follows the sequence
(a) R—I > R—Br > R—Cl > R—F
(b) R—Cl > R—F > R—Br > R—I
(c) R—F > R—Cl > R—Br > R—I
(d) R—I > R—F > R—Cl > R—Br

58. Compound 'A' reacts with alcoholic KOH to yield compound 'B', which on ozonolysis followed by reaction with Zn/H_2O gives methanal and propanal. Compound '*A*' is

(a) 1-propanol (b) 1-butanol
(c) 1-chlorobutane (d) 1-chloropentane

59. In the reaction sequence



compound '*X*' is

(a) chlorobenzene
(b) benzene
(c) toluene
(d) biphenyl methane

60. In the sequence of reactions,

$$C_2H_4 \xrightarrow{HBr} X \xrightarrow{AgCN} Y \xrightarrow[H_2/Ni]{[H]} Z,$$

compound '*Z*' is

(a) N-methyl ethanamine
(b) N-propylamine
(c) N,N-dimethylamine
(d) ethyl cyanide

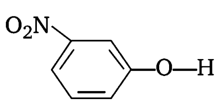
61. In the reaction sequence,

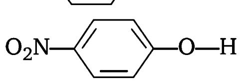
$$A \xrightarrow{CH_3CH_2MgBr} B \xrightarrow{H_3O^{\oplus}} C_5H_{12}O$$

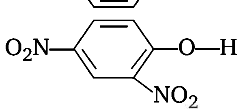
compound '*A*' is

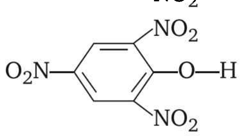
(a) 1-propanol (b) propanal
(c) ethanal (d) 2-propanol

62. Which of the following compounds is weakest acid?

(a) 

(b) 

(c) 

(d) 

63. Acetophenone can be converted to ethylbenzene by reaction with

(a) $LiAlH_4$
(b) H_2NOH
(c) $Pd/BaSO_4—H_2$
(d) $Zn-Hg/HCl$

64. Which of the following hydroxides is amphoteric in nature?

(a) $Be(OH)_2$ (b) $Mg(OH)_2$
(c) $Ca(OH)_2$ (d) $Ba(OH)_2$

65. Number of electrons in 3*d*-orbital of V^{2+}, Cr^{2+}, Mn^{2+} and Fe^{2+} are 3, 4, 5 and 6, respectively. Which of the following ions will have largest value of magnetic moment (μ)?

(a) V^{2+} (b) Cr^{2+}
(c) Mn^{2+} (d) Fe^{2+}

66. The coordination compounds $[Co(NH_3)_6]^{3+}[Cr(CN)_6]^{3-}$ and $[Cr(NH_3)_6]^{3+}[Co(CN)_6]^{3-}$ are the examples of

(a) linkage isomerism
(b) coordination isomerism
(c) ionisation isomerism
(d) geometrical isomerism

67. The spontaneous disintegration of $^{238}_{92}U$ by loss of eight α-particles and six β-particles terminates at

(a) $^{210}_{82}Ra$ (b) $^{210}_{84}Po$
(c) $^{228}_{90}Th$ (d) $^{206}_{82}Pb$

68. The relative Lewis acid character of boron trihalides follows the sequence

(a) $BF_3 > BCl_3 > BBr_3 > BI_3$
(b) $BI_3 > BBr_3 > BCl_3 > BF_3$
(c) $BCl_3 > BF_3 > BI_3 > BBr_3$
(d) $BI_3 > BBr_3 > BF_3 > BCl_3$

69. The tendency for catenation in group 14 elements varies in the order

(a) $C >> Si > Ge = Sn > Pb$
(b) $C << Si < Ge = Sn < Pb$
(c) $C >> Si < Ge < Sn < Pb$
(d) $C >> Si = Ge = Sn > Pb$

70. The thermal stability of CF_4 is

(a) less than SiF_4
(b) more than SiF_4
(c) less than CCl_4
(d) less than $SiCl_4$

71. Which of the following enzymes hydrolyses starch to glucose?
(a) Amylase (b) Invertase
(c) Lactase (d) Maltase

72. Which of the following reactions is an example of heterogeneous catalysis?
(a) $O_3 + O \xrightarrow{Cl} 2O_2$(gas phase)
(b) $2CO(g) + O_2(g) \xrightarrow{NO} 2CO_2(g)$
(c) $CH_3—\overset{\overset{\displaystyle O}{\|}}{C}—OC_2H_5(l) + H_2O(l) \xrightarrow{H_2SO_4} CH_3—\overset{\overset{\displaystyle O}{\|}}{C}—OH(l) + C_2H_5OH(l)$
(d) $CO(g) + 2H_2(g) \xrightarrow{Cu, ZnO—Cr_2O_3} CH_3OH(l)$

73. If one strand of DNA has the sequence TATGACTG, the sequence in the complimentary strand would be
(a) A T A C A C T C
(b) A C G T T G A C
(c) A T A C T G A C
(d) A T A C T G C A

74. Which of the following hormones is secreted from adrenal cortex?
(a) Cortisone (b) Estrogen
(c) Progesterone (d) Testosterone

75. Which of the following compounds is an azo dye?
(a) Martius yellow
(b) Malachite green
(c) Methyl orange
(d) Mercurochrome

76. Which of the following is a thermosetting polymer?
(a) Bakelite (b) Polystyrene
(c) Polyethene (d) Terylene

77. The carbohydrate that will yield glucose and fructose on homogeneous catalytic hydrolysis in presence of dilute sulphuric acid, is
(a) cellulose (b) maltose
(c) starch (d) sucrose

78. The gas that is not considered as a "green house gas" is
(a) CO_2 (b) CH_4
(c) O_2 (d) O_3

79. The enzyme that is used to dissolve blood clot is
(a) trypsin (b) renin
(c) streptokinase (d) tyrosinase

80. Which of the following compounds is known as the antisterility factor?
(a) α-tocopherol (b) Retinol
(c) Calciferol (d) Pyridoxine

Mathematics

81. If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where, a, b, c are non-zero numbers, then a, b, c are in
(a) AP (b) GP
(c) HP (d) None of these

82. It is given that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \text{ to } \infty = \frac{\pi^4}{90}$, then $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots \text{to } \infty$ is equal to
(a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^4}{45}$
(c) $\frac{89}{90}\pi^4$ (d) None of these

83. The polynomial $(ax^2 + bx + c)(ax^2 - bx - c)$, $ac \neq 0$, has
(a) four real roots
(b) atleast two real roots
(c) atmost two real roots
(d) no real roots

84. If $(3 + i)z = (3 - i)\bar{z}$, then the complex number z is
(a) $a(3 - i), a \in R$ (b) $\frac{a}{(3 + i)}, a \in R$
(c) $a(3 + i), a \in R$ (d) $a(-3 + i), a \in R$

85. If $e^{i\theta} = \cos\theta + i\sin\theta$, then for the ΔABC $e^{iA}\,e^{iB}\,e^{iC}$ is
(a) i (b) 1
(c) -1 (d) None of these

86. Numbers lying between 999 and 10000 that can be formed from the digits 0, 2, 3, 6, 7, 8 (repetition of digits not allowed) are
(a) 100 (b) 200
(c) 300 (d) 400

87. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can vote be 126, then the number of contestants is

(a) 4 (b) 5
(c) 6 (d) 7

88. $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\dots$ is equal to

(a) $\frac{2^{n-4}}{n!}$ for even values of n only
(b) $\frac{2^{n-4}+1}{n!}-1$ for odd values of n only
(c) $\frac{2^{n-1}}{n!}$ for all values of n
(d) None of the above

89. $\frac{a-b}{a}+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+\dots$ to ∞ is equal to

(a) $\log ab$ (b) $\log \frac{a}{b}$
(c) $\log \frac{b}{a}$ (d) None of these

90. The sum of infinite terms of the series $\frac{1}{(1+a)(2+a)}+\frac{1}{(2+a)(3+a)}+\frac{1}{(3+a)(4+a)}+\dots+$ to ∞, where a is a constant, is

(a) $\frac{1}{1+a}$ (b) $\frac{2}{1+a}$
(c) ∞ (d) None of these

91. The value of $\log_2 \log_3 \dots \log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$ is equal to

(a) 0 (b) 1
(c) 2 (d) 100!

92. The value of $\tan 20° + 2\tan 50° - \tan 70°$ is

(a) 1 (b) 0
(c) $\tan 50°$ (d) None of these

93. A circular ring of radius 3 cm is suspended horizontally from a point 4 cm vertically above the centre by 4 cm string attached at equal intervals to its circumference. If the angles between two consecutive strings be θ, then $\cos\theta$ is

(a) $\frac{4}{5}$ (b) $\frac{4}{25}$
(c) $\frac{16}{25}$ (d) None of these

94. The number of positive integral solutions of the equation $\tan^{-1} x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$ is

(a) one (b) two
(c) zero (d) None of these

95. If α is a repeated root of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$ is

(a) 0 (b) a
(c) b (d) c

96. $\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$ is equal to

(a) 0 (b) 1
(c) 3 (d) None of these

97. The domain of the function

$$f(x)=\log_e(x-[x]) \text{ is}$$

(a) R (b) $R-I$
(c) $(0,\infty)$ (d) I

98. If $f(x+y, x-y)=xy$, then the arithmetic mean of $f(x,y)$ and $f(y,x)$ is

(a) x (b) y
(c) 0 (d) None of these

99. The equations of the three sides of a triangle are $x=2$, $y+1=0$ and $x+2y=4$. The coordinates of the circumcentre of the triangle are

(a) (4, 0) (b) (2, −1)
(c) (0, 4) (d) (−1, 2)

100. If the point (a, a) falls between the lines $|x+y|=4$, then

(a) $|a|=2$ (b) $|a|=3$
(c) $|a|<2$ (d) $|a|<3$

101. The equation of the image of the pair of rays $y=|x|$ by the line $y=1$ is

(a) $y=|x|+2$ (b) $y=|x|-2$
(c) $y=|x|+1$ (d) $y=|x|-1$

102. Let $P=(1,1)$ and $Q=(3,2)$. The point R on the x-axis such that $PR+RQ$ is minimum, is

(a) $\left(\frac{5}{3}, 0\right)$ (b) $\left(\frac{1}{3}, 0\right)$
(c) (3, 0) (d) (5, 0)

103. L is a variable line such that the algebraic sum of the distances of the points (1, 1), (2, 0) and (0, 2) from the line is equal to zero. The line L will always pass through

(a) (1, 1) (b) (2, 1)
(c) (1, 2) (d) (2, 2)

104. C_1 is a circle of radius 2 touching the x-axis and the y-axis. C_2 is another circle of radius > 2 and touching the axes as well as the circle C_1. The radius of C_2 is

(a) $6-4\sqrt{2}$ (b) $6+4\sqrt{2}$
(c) $6-4\sqrt{3}$ (d) $6+4\sqrt{3}$

105. The locus of the centre of the circle for which one end of a diameter is (1, 1) while the other end is on the line $x+y=3$, is

(a) $x+y=1$ (b) $2(x-y)=5$
(c) $2x+2y=5$ (d) None of these

106. The locus of the middle points of chords of a parabola which subtend a right angle at the vertex of the parabola, is

(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola

107. The point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ is

(a) (0, 0, 0) (b) (1, 1, 1)
(c) (–1, –1, –1) (d) (1, 2, 3)

108. The equation of the plane which meets the axes in A, B, C such that the centroid of the triangle ABC is $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$, is given by

(a) $x+y+z=1$ (b) $x+y+z=2$
(c) $\frac{x}{3}+\frac{y}{3}+\frac{z}{3}=3$ (d) $x+y+z=\frac{1}{3}$

109. If O is the origin and A is the point (a, b, c), then the equation of the plane through A and at right angles to OA is

(a) $a(x-a)-b(y-b)-c(z-c)=0$
(b) $a(x+a)+b(y+b)+c(z+c)=0$
(c) $a(x-a)+b(y-b)+c(z-c)=0$
(d) None of the above

110. The image of the point (5, 4, 6) in the plane $x+y+2z-15=0$ is

(a) (3, 2, 2) (b) (2, 3, 2)
(c) (2, 2, 3) (d) (–5, –4, –6)

111. The radius of the circle $x+2y+2z=15$, which cuts by the sphere $x^2+y^2+z^2-2y-4z=11$ is

(a) 2 (b) $\sqrt{7}$
(c) 3 (d) $\sqrt{5}$

112. A straight line which makes angle of 60° with each of y and z axes, is inclined with x-axis at an angle of

(a) 30° (b) 45°
(c) 60° (d) 75°

113. The value of $\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$ is

(a) 0 (b) 30^x
(c) 30^{-x} (d) 1

114. The rank of the matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$ is

(a) 4 (b) 3
(c) 2 (d) 1

115. The values of a for which the system of equations $ax+y+z=0$, $x-ay+z=0$, $x+y+z=0$ possesses non-zero solutions, are given by

(a) 1, 2 (b) 1, –1
(c) 0 (d) None of these

116. If A is a skew symmetric matrix of order n and C is a column matrix of order $n\times 1$, then C^TAC is

(a) an identity matrix of order n
(b) an identity matrix of order 1
(c) a zero matrix of order 1
(d) None of the above

117. If $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ x-y & z-x & x+y \end{vmatrix}=kxyz$, then the value of k is

(a) 2 (b) 4
(c) 6 (d) 8

118. Let $f(x)$ be a polynomial function of the second degree. If $f(1)=f(-1)$ and a_1, a_2, a_3 are in AP, then $f'(a_1), f'(a_2), f'(a_3)$ are in

(a) AP (b) GP
(c) HP (d) None of these

119. The curve given by $x+y=e^{xy}$ has a tangent parallel to the y-axis at the point

(a) (0, 1) (b) (1, 0)
(c) (1, 1) (d) (–1, –1)

120. Let $f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$. Then $f(x)$ has

(a) more than one minimum
(b) exactly one minimum
(c) atleast one maximum
(d) None of the above

121. The interval in which the function $f(x) = \dfrac{4x^2+1}{x}$ is decreasing, is

(a) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
(c) $(-1, 1)$ (d) $[-1, 1]$

122. A right circular cylinder which is open at the top and has a given surface area, will have the greatest volume if its height h and radius r are related by

(a) $2h = r$ (b) $h = 4r$
(c) $h = 2r$ (d) $h = r$

123. If $f(x) = x^2 - 2x + 4$ on $[1, 5]$, then the value of a constant c such that $\dfrac{f(5) - f(1)}{5 - 1} = f'(c)$, is

(a) 0 (b) 1
(c) 2 (d) 3

124. The value of k for which the function $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, is

(a) $k = 0$ (b) $k = 1$
(c) $k = -1$ (d) None of these

125. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then $f'\left(\dfrac{\pi}{4}\right)$ is

(a) $\sqrt{2}$ (b) $\dfrac{1}{\sqrt{2}}$
(c) 0 (d) $\dfrac{\sqrt{3}}{2}$

126. Let $f(x) = \displaystyle\int \frac{x^2\,dx}{(1 + x^2)(1 + \sqrt{1 + x^2})}$ and $f(0) = 0$. Then, $f(1)$ is

(a) $\log(1 + \sqrt{2})$
(b) $\log(1 + \sqrt{2}) - \dfrac{\pi}{4}$
(c) $\log(1 + \sqrt{2}) + \dfrac{\pi}{4}$
(d) None of the above

127. The value of $\displaystyle\int_1^2 [f\{g(x)\}]^{-1} f'\{g(x)\}\, g'(x)\, dx$, where $g(1) = g(2)$, is equal to

(a) 1 (b) 2
(c) 0 (d) None of these

128. If $\displaystyle\int_0^x f(t)\,dt = x + \int_x^1 t\, f(t)\,dt$, then the value of $f(1)$ is

(a) $\dfrac{1}{2}$ (b) 0
(c) 1 (d) $-\dfrac{1}{2}$

129. If $f(x) = f(a + x)$ and $\displaystyle\int_0^a f(x)\,dx = k$, then $\displaystyle\int_0^{na} f(x)\,dx$ is equal to

(a) nk (b) $(n-1)k$
(c) $(n+1)k$ (d) 0

130. If $\displaystyle\int_a^b x^3\,dx = 0$ and if $\displaystyle\int_a^b x^2\,dx = \frac{2}{3}$, then the values of a and b are respectively

(a) 1, 1 (b) −1, −1
(c) 1, −1 (d) −1, 1

131. A vector has components $2a$ and 1 with respect to a rectangular cartesian system. The axes are rotated through an angle θ about the origin in the anticlockwise direction. If the vector has components $a + 1$ and 1 with respect to the new system, then the values of a are

(a) $1, -\dfrac{1}{3}$ (b) 0
(c) $-1, \dfrac{1}{3}$ (d) 1, −1

132. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{c}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$. If $\vec{\mathbf{b}}$ is a vector satisfying $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$, then $\vec{\mathbf{b}}$ is

(a) $\dfrac{1}{3}(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(b) $\dfrac{1}{3}(5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
(c) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$
(d) None of the above

133. Let $\overrightarrow{\mathbf{OA}} = \vec{\mathbf{a}}$, $\overrightarrow{\mathbf{OB}} = 10\vec{\mathbf{a}} + 2\vec{\mathbf{b}}$ and $\overrightarrow{\mathbf{OC}} = \vec{\mathbf{b}}$, where O, A and C are non-collinear points. Let p denotes the area of the quadrilateral $OABC$ and q denotes the area of the parallelogram with OA and OC as adjacent sides. Then, $\frac{p}{q}$ is equal to

(a) 4 (b) 6

(c) $\frac{|\vec{\mathbf{a}} - \vec{\mathbf{b}}|}{2|\vec{\mathbf{a}}|}$ (d) $\frac{|\vec{\mathbf{a}} + \vec{\mathbf{b}}|}{2|\vec{\mathbf{a}}|}$

134. The value of x so that the four points $A = (0, 2, 0)$, $B = (1, x, 0)$, $C = (1, 2, 0)$ and $D = (1, 2, 1)$ are coplanar, is

(a) 0 (b) 1

(c) 2 (d) 3

135. Constant forces $P_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $P_2 = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $P_3 = \hat{\mathbf{j}} - \hat{\mathbf{k}}$ act on a particle at a point A. The work done when the particle is displaced from the points A to B where $A = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $B = 6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ is

(a) 3 (b) 9

(c) 20 (d) None of these

136. The solution of the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$, when $x = \frac{\pi}{2}$, is

(a) $y = \sin x - \cos x$

(b) $y = \cos x$

(c) $y = \sin x$

(d) $y = \sin x + \cos x$

137. From a point on the ground at a distance 70 ft from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 30 ft on the other side of the wall. The height of the wall is

(a) 20 ft (b) 21 ft

(c) 10 ft (d) 105 ft

138. Three coplanar forces acting on a particle are in equilibrium. The angle between the first and the second is 60° and that between the second and the third is 150°. The ratio of the magnitude of the forces are

(a) $1 : 1 : \sqrt{3}$ (b) $1 : \sqrt{3} : 1$

(c) $\sqrt{3} : 1 : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$

139. A particle having simultaneous velocities 3 m/s, 5 m/s and 7 m/s, is at rest. The angle between the first two velocities is

(a) 30° (b) 45°

(c) 60° (d) 90°

140. A cyclist is beginning to move with an acceleration of 1 m/s^2 and a boy, who is $40\frac{1}{2}$ m behind the cyclist, starts running at 9 m/s to meet him. The boy will be able to meet the cyclist after

(a) 6 s (b) 8 s

(c) 9 s (d) 10 s

141. Two bodies slide from rest down two smooth inclined planes commencing at the same point and terminating in the same horizontal plane. The ratio of the velocities attained if inclinations to the horizontal of the planes are 30° and 60° respectively, is

(a) $\sqrt{3} : 1$ (b) $2 : \sqrt{3}$

(c) $1 : 1$ (d) $1 : 2$

142. A die is thrown $2n + 1$ times. The probability that faces with even numbers show odd number of times, is

(a) $\frac{2n+1}{4n+3}$ (b) $\frac{1}{2}$

(c) $\frac{n+1}{2n+1}$ (d) None of these

143. The probability that exactly one of the independent events A and B occurs, is equal to

(a) $P(A) + P(B) + 2P(A \cap B)$

(b) $P(A) + P(B) - P(A \cap B)$

(c) $P(A') + P(B') - 2P(A' \cap B')$

(d) None of the above

144. A bag contains 30 tickets, numbered from 1 to 30. Five tickets are drawn at random and arranged in the ascending order. The probability that the third number is 20, is

(a) $\frac{{}^{20}C_2 \times {}^{10}C_2}{{}^{30}C_5}$ (b) $\frac{{}^{19}C_2 \times {}^{10}C_2}{{}^{30}C_5}$

(c) $\frac{{}^{19}C_2 \times {}^{11}C_2}{{}^{30}C_5}$ (d) None of these

145. The probability that atleast one of the events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.4

146. The relation of "congruence modulo" is

(a) reflexive only

(b) symmetric only

(c) transitive only

(d) an equivalence relation

147. If flow values of switches x_1, x_2 and x_3 are respectively 0, 0 and 1, then the flow value of the circuit

$s = (x'_1 \cdot x'_2 \cdot x_3) + (x_1 \cdot x'_2 \cdot x'_3) + (x'_1 \cdot x_2 \cdot x'_3)$ is

(a) 0 (b) 1
(c) 2 (d) None of these

148. In a boolean algebra $a \vee (a' \wedge b)$ is equal to

(a) $a \vee b$ (b) $a \wedge b$
(c) a' (d) b'

149. The range of the function

$$f(x) = \sqrt{(x-1)(3-x)} \text{ is}$$

(a) [−1, 1] (b) (−1, 1)
(c) (−3, 3) (d) (−3, 1)

150. The coefficients of x in the quadratic equation $x^2 + bx + c = 0$ was taken as 17 in place of 13, its roots were found to be −2 and −15. The correct roots of the original equation are

(a) −10, − 3 (b) −9, − 4
(c) −8, − 5 (d) −7, − 6

Answers

PHYSICS

1. (b)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (a)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (b)
31. (a)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (b)	**36.** (d)	**37.** (b)	**38.** (a)	**39.** (d)	**40.** (d)

CHEMISTRY

41. (b)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (b)	**50.** (c)
51. (a)	**52.** (c)	**53.** (d)	**54.** (b)	**55.** (d)	**56.** (a)	**57.** (a)	**58.** (c)	**59.** (b)	**60.** (a)
61. (b)	**62.** (a)	**63.** (d)	**64.** (a)	**65.** (c)	**66.** (b)	**67.** (d)	**68.** (b)	**69.** (a)	**70.** (b)
71. (a)	**72.** (d)	**73.** (c)	**74.** (a)	**75.** (c)	**76.** (a)	**77.** (d)	**78.** (c)	**79.** (c)	**80.** (a)

MATHEMATICS

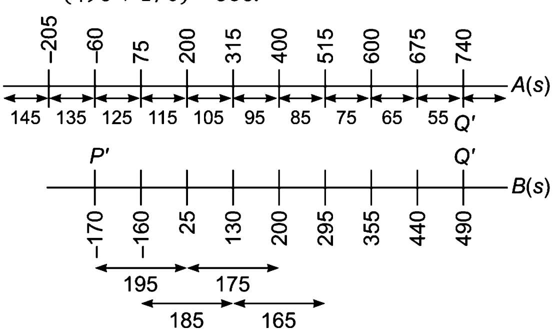
81. (c)	**82.** (a)	**83.** (b)	**84.** (a)	**85.** (c)	**86.** (c)	**87.** (d)	**88.** (c)	**89.** (b)	**90.** (a)
91. (b)	**92.** (b)	**93.** (c)	**94.** (a)	**95.** (b)	**96.** (b)	**97.** (b)	**98.** (c)	**99.** (a)	**100.** (a)
101. (a)	**102.** (a)	**103.** (a)	**104.** (b)	**105.** (c)	**106.** (c)	**107.** (c)	**108.** (a)	**109.** (c)	**110.** (a)
111. (b)	**112.** (b)	**113.** (a)	**114.** (c)	**115.** (b)	**116.** (c)	**117.** (d)	**118.** (a)	**119.** (b)	**120.** (b)
121. (a)	**122.** (d)	**123.** (b)	**124.** (b)	**125.** (c)	**126.** (b)	**127.** (c)	**128.** (a)	**129.** (a)	**130.** (d)
131. (a)	**132.** (a)	**133.** (b)	**134.** (c)	**135.** (b)	**136.** (c)	**137.** (b)	**138.** (a)	**139.** (c)	**140.** (c)
141. (c)	**142.** (b)	**143.** (c)	**144.** (b)	**145.** (c)	**146.** (d)	**147.** (b)	**148.** (a)	**149.** (a)	**150.** (a)

Hints & Explanations

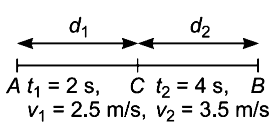
Physics

1. **Key Idea** *Total degrees of latitude on earth is 360°.*
   Degrees of latitudes completed in 24 h by earth is 360°.
   ∴ Degrees of latitudes completed in 1 h
   $$= \frac{360°}{24} = 15°$$
   ∴ The degrees of latitudes completed in 2 h
   $$= 2 \times 15° = 30°$$
   Hence, to reset a watch by 2 h a person must travel 30° of latitude.

2. From the clock A it becomes clear that in order to have a difference of 800, two events should occur between P and Q. Thus, these two events in clock A will appear in clock B at respective points P' and Q' having separation $(490 + 170) = 660$.

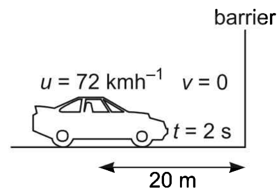


3. **Key Idea** *Average speed* $= \dfrac{\textit{total distance}}{\textit{total time}}$.
   Let d_1 be distance covered in 2 s and d_2 in 4 s then



$$d_1 = 2 \times 2.5 = 5 \text{ m}$$
$$d_2 = 4 \times 3.5 = 14 \text{ m}$$
∴ Total distance covered $= (5 + 14) = 19$ m
Total time taken $= 2 + 4 = 6$ s
∴ Average speed $= \dfrac{19}{6} = 3.16$ m/s

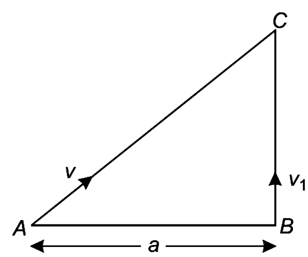
4. **Key Idea** *Final velocity is zero.*
   From equation of motion
   $$v = u + at$$



We have, $v = 0$ (final velocity),
$$u = 72 \text{ km/h}$$
$$= 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$
$$t = 2 \text{s}$$
∴ $$a = -\frac{20}{2} = -10 \text{ ms}^{-2}$$
⇒ $$a = -10 \text{ ms}^{-2}$$

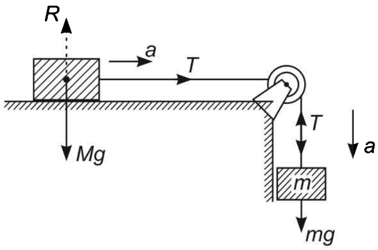
NOTE Negative sign indicates that acceleration is retarding or it is deceleration which decreases the speed of the car.

5. Let two boys meet at C after time t from the starting.



Then, $AC = vt$, $BC = v_1 t$
$$(AC)^2 = (AB)^2 + (CB)^2$$
⇒ $$v^2 t^2 = a^2 + v_1^2 t^2$$
By solving, we get,
$$t = \sqrt{\frac{a^2}{v^2 - v_1^2}}$$

6. The various forces acting on the system are as shown. Let T be the tension in the string, and a the downward acceleration, then
   $$T = Ma \quad \text{...(i)}$$



For block m

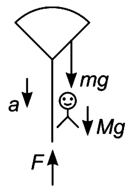
$$ma = mg - T \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$ma = mg - Ma$$

$$\Rightarrow \quad a = \frac{mg}{m + M}$$

7. Key Idea *Apply Newton's second law of motion.*



Let upward force acting be F, and $M + m = 85$ kg be the total mass.

From Newton's second law of motion, we have

$$F = (m + M)g - (m + M)a$$

$$\Rightarrow \quad F = (m + M)(g - a)$$

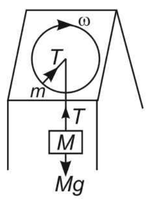
Given, $g = 9.8\ \text{m/s}^2$, $a = 2.8\ \text{ms}^{-2}$,

$$M + m = 85\ \text{kg}$$

$$\therefore \quad F = 85(9.8 - 2.8)$$

$$= 85 \times 7 = 595\ \text{N}$$

8. Key Idea *Centripetal force is provided by weight of body.*



In order to keep the mass M steady, let T be the tension in the string joining the two. Then, for particle m,

$$T = \frac{mv^2}{R} \quad \text{...(i)}$$

For mass M $\quad T = Mg \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{mv^2}{R} = Mg$$

$$\Rightarrow \quad v = \sqrt{\frac{MgR}{m}}$$

9. Key Idea *Force is equal to rate of change of momentum.*

Let v be the final velocity of the ball, from Newton's second law

We have, $\quad F = -\dfrac{dp}{dt}$

= rate of change of momentum

Given, $\quad F = -1200$ N,

$$t = 25\ \text{m-s} = 25 \times 10^{-3}\ \text{s}$$

$$-1200 = \frac{mv - mu}{t}$$

$$-\frac{1200t}{m} = -u + v$$

$$\Rightarrow \quad -\frac{1200\,t}{m} + u = v$$

$$\Rightarrow \quad v = \frac{-1200 \times 25 \times 10^{-3}}{0.5} + 14$$

$$= -46\ \text{ms}^{-1}$$

NOTE Negative sign shows that now the ball will move in the direction of force.

10. Key Idea *Rate of change of momentum is equal to force acting on the body.*

From Newton's second law of motion, we have

$$F = -\frac{dp}{dt} = -\left(\frac{mv - mu}{t}\right)$$

Given, $\quad m = 1.5$ kg, $\quad u = 25\ \text{ms}^{-1}$,

$$v = 15\ \text{ms}^{-1}, \quad t = 0.03\ \text{s}.$$

$$\therefore \quad F = \frac{1.5 \times 15 - (1.5) \times (-25)}{0.03}$$

$$\Rightarrow \quad F = 2000\ \text{N}$$

11. Key Idea *Angular momentum is conserved.*

Since, no external torque acts on the system, its angular momentum remains conserved.

$$J = I\omega = \text{constant}$$

where I is moment of inertia, ω the angular velocity.

Moment of inertia I of a circular disc of radius R is $\frac{1}{2}MR^2$.

$$\therefore \quad \left(\frac{1}{2}MR^2\right)\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right)\omega'$$

$$\Rightarrow \quad \left(\frac{1}{2}MR^2\right)\omega = \frac{5}{8}MR^2\,\omega'$$

$$\Rightarrow \quad \omega' = \frac{4}{5}\omega$$

12. Key Idea $\quad \tau = \dfrac{dJ}{dt}$

Torque is defined as rate of change of angular momentum

$$\tau = \frac{dJ}{dt}$$

Given, $J = 4\sqrt{t}$

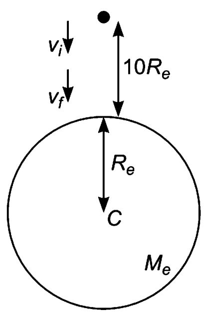
Using $\frac{d}{dt} t^n = nt^{n-1}$, we have

$$\tau = \frac{d}{dt}(4\sqrt{t})$$

$$= 4\frac{d\sqrt{t}}{dt} = 4\left(\frac{1}{2}t^{1/2-1}\right)$$

$$\Rightarrow \quad \tau = \frac{4}{2}.t^{-1/2} \text{ N-m} = \frac{2}{\sqrt{t}} \text{ N-m}$$

13. Key Idea *Apply law of conservation of energy.*



Let KE_i, U_i and KE_f, U_f be the initial and final kinetic and potential energy, then

$$KE_i + U_i = KE_f + U_f \quad \ldots\text{(i)}$$

Also, $\quad KE_i = \frac{1}{2}mv_i^2$

and $\quad U_i = -\frac{GM_em}{10R_e}$

$$KE_f = \frac{1}{2}mv_f^2$$

and $\quad U_f = -\frac{GM_em}{R_e}$

Putting these values in Eq. (i), we get

$$\frac{1}{2}mv_i^2 - \frac{GM_em}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_em}{R_e}$$

$$\Rightarrow \quad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + \frac{GM_em}{R_e} - \frac{GM_em}{10R_e}$$

$$\Rightarrow \quad v_f^2 = v_i^2 + \frac{2GM_e}{R_e} - \frac{2GM_e}{10R_e}$$

$$\Rightarrow \quad v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

14. Key Idea *Change in gravitational potential energy is equal to kinetic energy.*

Increase in gravitational potential energy of rocket at a height h from earth's surface is

$$\Delta U = \frac{GMmh}{(R+h)R} \quad \ldots\text{(i)}$$

If v is escape velocity of rocket, then

$$\Delta U = \frac{1}{2}mv^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}mv^2 = \frac{GMmh}{(R+h)R}$$

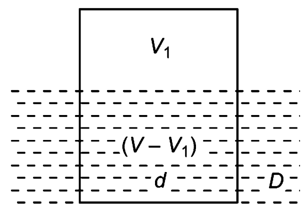
$$\Rightarrow \quad mv^2R^2 + mv^2Rh = 2GMmh$$

$$\Rightarrow \quad v^2R^2 = (2GM - v^2R)h$$

$$\Rightarrow \quad h = \frac{v^2R^2}{2GM - v^2R}$$

15. Key Idea *At equilibrium,*
weight of iceberg = weight of displaced water.

Let V be total volume of iceberg, and V_1 be volume of visible part.



From law of floation, we have

$$Vdg = (V - V_1)Dg$$

$$\Rightarrow \quad \frac{V - V_1}{V} = \frac{d}{D}$$

$$\Rightarrow \quad 1 - \frac{V_1}{V} = \frac{d}{D}$$

$$\Rightarrow \quad \frac{V_1}{V} = 1 - \frac{d}{D}$$

Percentage fraction of visible iceberg is

$$\frac{V_1}{V} \times 100\% = \left(1 - \frac{d}{D}\right) \times 100\%$$

Given, $\quad d = 917 \text{ kg m}^{-3}, \quad D = 1024 \text{ kg m}^{-3}$

$$\therefore \quad \frac{V_1}{V} \times 100\% = \left(1 - \frac{917}{1024}\right) \times 100$$

$$= \frac{10700}{1024} \approx 10\%$$

16. From the principle of continuity, we have

$$Av = \text{constant}$$

where A is area, v the velocity.

Also, from Bernoulli's principle total energy per unit volume is a constant.

Hence, lesser the breadth of pipe, the more is the pressure of water. Thus

$$p_1 > p_3 > p_2 > p_4$$

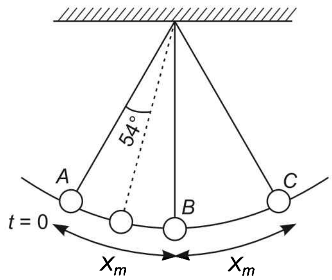
NOTE Change option (a) to $p_1 > p_3 > p_2 > p_4$.

17. Key Idea *General equation of SHM is*

$$y = a \sin \omega t.$$

Let a be the amplitude of oscillation, then

$$y = a \sin \omega t$$



Also, $\omega = \frac{2\pi}{T}$

$\therefore$ $y = a \sin \frac{2\pi t}{T}$

From the figure $a = X_m$, thus at $t = 3.15\,T$

$$y = X_m \sin \frac{2\pi}{T}(3.15\,T)$$

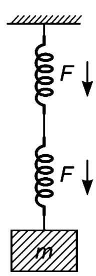
$\Rightarrow$ $y = X_m \sin (6\pi + 54°)$

$\Rightarrow$ $y = X_m \sin 54°$

$\Rightarrow$ $y = X_m \sin \frac{(\sqrt{5} - 1)}{4}.$

Since, measurement starts from position A, after 3.15 T particle will be between O and X_m.

18. Key Idea *The spring mass system executes SHM.*



When a force F is applied in the downward direction, then

Force $F \propto$ displacement (x)

Hence, motion SHM, the frequency of vibration of which is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k'}{m'}}$$

where k' is equivalent spring constant, given by

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$$

Given, $k_1 = k_2 = k$

$\therefore$ $k' = \frac{k^2}{2k} = \frac{k}{2}$

$\therefore$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

19. Key Idea $v = r\omega$

From Doppler's effect, the perceived frequency is given by

$\therefore$ $\nu' = \nu \cdot \frac{v}{v - v_s}$

When distance between source and listener is less.

Given, $\nu = 540$ Hz, $v = 330$ m/s,

also $v_s = r\,\omega = 0.60 \times 15 = 9$ m/s

$\therefore$ $\nu' = 540 \times \frac{330}{330 - 9}$

$= 540 \times \frac{330}{321} = 555$ Hz

When source is moving away from listener, then sound heard will be lowest

$$\nu'' = \frac{\nu' v}{v + v_s}$$

$$\nu'' = \frac{540 \times 330}{330 + 9} = 526 \text{ Hz}$$

20. Key Idea *First law of thermodynamics is a form of law of conservation of energy.*

From first law of thermodynamics, we have

$$\Delta U = Q - W$$

Along path *iaf* $Q = 50$ cal, $W = 20$ cal

$\therefore$ $\Delta U = 50 - 20 = 30$ cal

Along path *ibf*

$\Delta U = 30$ cal

$\therefore$ $W = Q - \Delta U = 36 - 30 = 6$ cal

Along path *fi* volume is decreasing

Hence,

$W = -13$ cal, $\Delta U = -30$ cal

$\therefore$ $Q = W + \Delta U$

$= -13 + (-30) = -43$ cal

$E_{\text{int},i} = 10$ cal

$E_{\text{int},f} = E_{\text{int},i} + \Delta U$

$= 10 + 30 = 40$ cal

21. Key Idea *When no net force acts electron is in equilibrium.*

From Coulomb's law, the force between charges placed at a distance r apart is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Taking each case individually, we have

Case I Let distance between $+q$ and $-4q$ be d, between $-e$ and $+q$ be x, and that between $-e$ and $-4q$ be $(x + d)$.

$$\therefore \quad F_1 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qe}{x^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qe}{(x+d)^2}$$

For equilibrium $\Sigma F = 0$

$$\therefore \quad x = d \quad \Rightarrow \quad F_1 = -F_2$$

Case II Force (F) between $-q$ and $-e$ is

$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qe}{x^2}$$

between $-e$ and $+4q$

$$F_2 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{4qe}{(x+d)^2}$$

Solving $\quad x = d$

$$F_1 = -F_2$$

$\therefore$ Net force on $-e$ is zero.

Case III Force (F) between $-e$ and $4q$

$$F_1 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{4qe}{x^2}$$

between $-q$ and $-e$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qe}{(x+d)^2}$$

Since, electron is closer to $+4q$ and $-q$

So, $\quad F_1 > F_2$.

In this case electron will not remain at rest and starts moving towards the system.

Case IV In case force between $-e$ and $-4q$

$$F_1 = +\frac{1}{4\pi\varepsilon_0} \cdot \frac{4qe}{x^2}$$

between $-e$ and $+q$

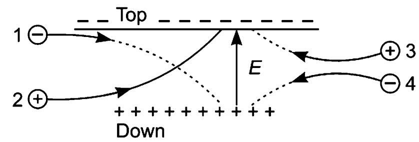
$$F_2 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{qe}{(x+d)^2}$$

Since, electron is closer to $-4q$ than $+q$ then $F_1 > F_2$.

Thus, electron will move away from the system. It means equilibrium stage cannot be obtained.

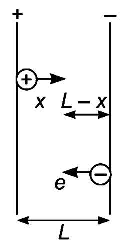
22. Key Idea *Electric lines of force originate from positive charge and end on negative charge.*

We know that like charges repel each other and opposites attract. Also direction of electric field is from positive to negative charge. In the given case since positive charged particle moves as a parabolic path in an electric field. It means 1 the direction of electric field E is upward. The direction of deflection of particle 2 which is negative is downward.



Similarly direction of deflection of particle 3 is upward and of 4 is downward.

23. Key Idea *Both electron and proton are accelerated by electric field.*



Let both particle cross each other at time t, with acceleration a_e and a_p respectively, at distances x and L-x from the plates from equation of motion

$$s = ut + \frac{1}{2}at^2$$

we have, $\quad u = 0, \quad s_1 = x,$

$$s_2 = L - x, \quad a_1 = a_p, a_2 = a_e$$

$$x = \frac{1}{2}a_p t^2 \quad \ldots\text{(i)}$$

$$L - x = \frac{1}{2}a_e t^2 \quad \ldots\text{(ii)}$$

Since, time taken is same, hence

$$\frac{2(L-x)}{a_e} = \frac{2x}{a_p} \quad \Rightarrow \quad x = \frac{L}{1 + \frac{a_e}{a_p}}$$

Force (F) due to electric field (E) on a particle with charge q is

$$F = qE \quad \ldots\text{(iii)}$$

Also, from Newton's law

$$F = ma \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

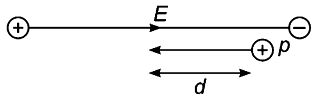
$$a = \frac{qE}{m}$$

$$\therefore \quad a_e = -\frac{eE}{m_e}, \quad a_p = \frac{eE}{m_p}$$

$$\therefore \quad x = \frac{L}{1 - \frac{m_p}{m_e}}$$

24. Key Idea *Electric field is directed from positive to negative charge.*

We know that electric lines of force originate from positive charge and end on negative charge, also proton is a positively charged particle.



Hence, proton is moving against the direction of electric field, hence work is done by the proton against electric field. It implies that electric field does negative work on the proton. Also proton is moving in an electric field from low potential to high potential region hence, its potential energy increases.

25. **Key Idea** *Rate of change of electric potential gives electric field.*

In the given graph,

For region I Electric potential V is constant

Since, $E_1 = -\frac{dV_1}{dx}$

$dV_1 = 0, \quad \therefore \quad E_1 = 0$

For region II V_2 is positive (increases)

$\therefore \qquad E_2 = -\frac{dV_2}{dx} = -\text{ ve}$

For region III V_3 is constant

$\therefore \qquad \frac{dV_3}{dx} = 0$

$E_3 = 0$

For region IV $V_4 = -f(x)$

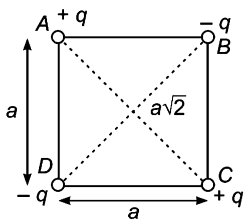
$E_4 = -\frac{dV_4}{dx} = +\text{ ve}$

Hence, $E_4 > E_2 > E_1 = E_3$

26. **Key Idea** *Work required to set up the four charge configration is equal to potential energy.*

The potential energy (U) due to charges q_1q_2 separated at a distance r is

$$U = \Sigma \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r}$$



which is equal to work done.

Therefore,

$$W = \frac{1}{4\pi\varepsilon_0}\left[\frac{(+q)(-q)}{AB} + \frac{(-q)(+q)}{BC} + \frac{(+q)(-q)}{CD} + \frac{(-q)(+q)}{DA} + \frac{(+q)(+q)}{AC} + \frac{(-q)(-q)}{BD}\right]$$

$$W = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{a\sqrt{2}} + \frac{q^2}{a\sqrt{2}}\right]$$

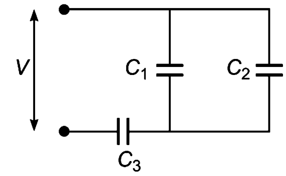
$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}[-4 + \sqrt{2}]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}[-4 + 1.414]$$

$$\Rightarrow \quad W = -0.21\frac{q^2}{\varepsilon_0 a}$$

27. **Key Idea** *The circuit consists of two capacitors in parallel joined to one in series.*

The given circuit can be redrawn as follows



The equivalent capacitance of combination is

$$C' = C_1 + C_2$$

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_1 + C_2}$$

$$\Rightarrow \quad C'' = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}$$

The voltage of battery is V, hence charge q of system is

$$q = CV$$

$$\Rightarrow \quad q = \frac{(C_1 + C_2)C_3V}{C_1 + C_2 + C_3}$$

When capacitor C_3 breaks down, then total equivalent capacitance is

$$C' = C_1 + C_2$$

New charge stored

$$q' = C'V = (C_1 + C_2)V$$

Change in total charge

$$\Delta q = q' - q$$

$$\therefore \quad \Delta q = (C_1 + C_2)V - \frac{(C_1 + C_2)C_3V}{C_1 + C_2 + C_3}$$

$$\Delta q = (C_1 + C_2)V\left[1 - \frac{C_3}{C_1 + C_2 + C_3}\right]$$

28. **Key Idea** *When current is drawn from a battery, potential difference across its is reduced.*

If E is emf of battery, r the internal resistance, I the current drawn, V the potential difference, then

$$V = E - Ir$$

Case I During discharging $I = -$ ve

$\therefore \quad V > E$

Case II During charging $I = +$ ve

$V < E$

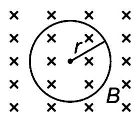
Case III When $I = 0$, $V = E$

29. **Key Idea** *Use Fleming's left hand rule.*

From Fleming's left hand rule, magentic force in figure (1) and (2) on electron will be directed in $-$ve z-axis and $-$ve x-axis respectively. In figure (3) velocity of electron and direction of magentic field are antiparallel so, no force will act on the electron.

30. **Key Idea** *Force due to magnetic field provides the required centripetal force.*

When a particle with charge q, moves with velocity v in a magnetic field B, then force F due to magnetic field is



$$F = qvB \sin \theta$$

This force provides the required centripetal force.

Given, $\theta = 90°$

$$\therefore \quad qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB} \quad \ldots(i)$$

Also, kinetic energy $= \frac{1}{2} mv^2 = qV$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$r = \frac{1}{B} \cdot \sqrt{\frac{2mV}{q}}$$

Diameter $D = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}$

31. **Key Idea** *Use Ampere's circuital law.*

From ampere's circuital law

$$\oint \vec{\mathbf{B}} \cdot dl = \mu_0 I$$

where B is magnetic field, I is current.

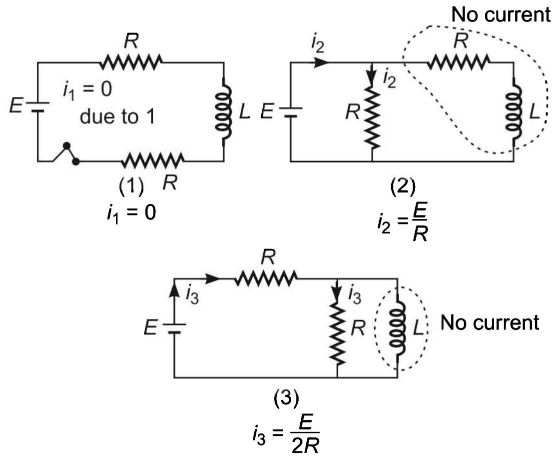
Current enclosed within the circle is

$$\frac{I}{\pi a^2} \cdot \pi r^2 = \frac{I}{a^2} \cdot r^2$$

$$\therefore \quad B \cdot 2\pi r = \frac{\mu_0 I r^2}{a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

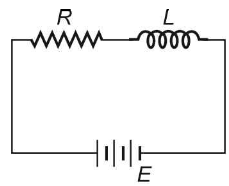
32. Just after closing the switch



So, $\quad i_2 > i_3 > i_1 \quad (i_1 = 0)$

33. **Key Idea** *Rate of energy* (U) *stored is maximum when*

$$\frac{dU}{dt} = 0$$



The energy (U) stored in a magnetic field at time t is

$$U = \frac{1}{2} LI^2$$

In an L-R circuit, current I is given by

$$I = I_0 (1 - e^{-t/\tau})$$

where, τ is time constant of circuit $\left(\tau = \frac{L}{R}\right)$

$$\therefore \quad U = \frac{1}{2} \cdot LI_0^2 (1 - e^{-t/\tau})^2$$

Rate of energy stored is

$$\frac{dU}{dt} = LI_0^2 (1 - e^{-t/\tau})(- e^{-t/\tau})\left(-\frac{1}{\tau}\right)$$

$$\frac{dU}{dt} = \frac{LI_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}) \quad \ldots(i)$$

This rate will be maximum, when

$$\frac{dU}{dt^2} = 0$$

$$\therefore \quad -\frac{1}{\tau} e^{-t/\tau} + \frac{2}{\tau} e^{-2t/\tau} = 0$$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{2} \quad \ldots(ii)$$

$$P = \frac{dU}{dt} = \frac{LI_0^2}{\tau}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\Rightarrow \quad \frac{dU}{dt} = \frac{L\,\varepsilon^2}{4R^2\left(\frac{L}{R}\right)} = \frac{\varepsilon^2}{4R}$$

34. From Joule's law, the amount of heat (H) produced is

$$H = I^2 R\,t = I_{\text{rms}}^2 Rt$$

where i is current, R the resistance, t the time.

Also, $\quad I_d^2 = I_{\text{rms}}^2$

$$I_d = \sqrt{I_{\text{rms}}^2}$$

But $\quad I_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}$

Given, $\quad I_0 = 4.24\text{ A}$

$$\therefore \quad I_d = \sqrt{\left(\frac{4.24}{\sqrt{2}}\right)^2}$$

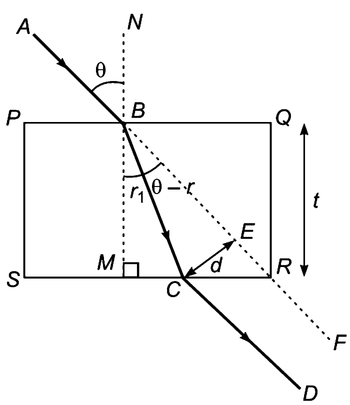
$$\Rightarrow \quad I_d = \frac{4.24}{\sqrt{2}} = \frac{4.24}{2}\sqrt{2}$$

$$= 2.12 \times 1.4 = 2.96\text{ A}$$

$$\Rightarrow \quad I_d = 3\text{ A (approx)}$$

35. Key Idea *For small angle* $\sin\theta \simeq \theta$.

Let AB be the incident ray at an angle θ with the normal.



From ΔBCE

$$\sin(\theta - r) = \frac{CE}{BC}$$

$$\Rightarrow \quad CE = BC\sin(\theta - r)$$

$$\therefore \quad d = BC\sin(\theta - r) \qquad \ldots\text{(i)}$$

In ΔBMC

$$\cos r = \frac{BM}{BC}$$

$$\Rightarrow \quad BC = \frac{BM}{\cos r} = \frac{t}{\cos r} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$d = \frac{t}{\cos r}\sin(\theta - r)$$

$$d = \frac{t}{\cos r}(\sin\theta\cos r - \cos\theta\sin r)$$

using $\quad \frac{\sin r}{\cos r} = \tan r$

$$d = t\,(\sin\theta - \cos\theta\tan r)$$

From Snell's law, refractive index (μ) is given by

$$\mu = \frac{\sin\theta}{\sin r}$$

For small angle $\mu \approx \frac{\theta}{r}$

$$\Rightarrow \quad r = \frac{\theta}{\mu}$$

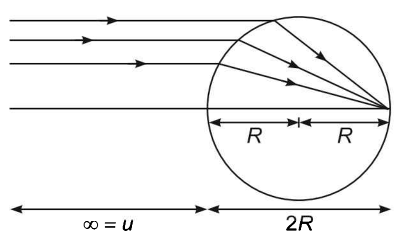
and $\quad d = t\left(\theta - \frac{\theta}{\mu}\right) = t\,\theta.\left(1 - \frac{1}{\mu}\right)$

$$\therefore \quad d = \frac{t\,\theta(\mu - 1)}{\mu}$$

36. Key Idea *For parallel beam of light object distance is infinity.*

Let R be the radius of the sphere, then $u = \infty$,

$v = 2R$



$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

Putting the values, we have

$$\frac{\mu}{2R} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\frac{\mu}{2R} = \frac{\mu - 1}{R}$$

$$\Rightarrow \quad \mu = 2\mu - 2$$

$$\Rightarrow \quad \mu = 2$$

37. The refractive index of a material is the factor by which the phase velocity of electromagnetic radiation is slowed in that material, relative to its velocity in vacuum (air),

$$\mu = \frac{v_0}{v_m} = \frac{\lambda_0}{\lambda_m}$$

where, λ_0 and λ_m is wavelength in vacuum (air) and medium respectively.

Given, $\mu = 1.5, \lambda_0 = 600$ nm

$$\therefore \quad \lambda_m = \frac{600}{1.5} = 400 \text{ nm}$$

Also, $v_0 = 3 \times 10^8$ m/s, $\mu = 1.5$

$$\therefore \quad v_m = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

NOTE Phase velocity decreases in the medium.

38. Key Idea *Kinetic energy (KE) and momentum (p) are related as* $p = \sqrt{2mKE}$.

From de-Broglie's wavelength (λ), we have

$$\lambda = \frac{h}{p}$$

where, h is Planck's constant, p the momentum.

For electron, $\lambda_e = \dfrac{h}{p_e}$,

For neutron, $\lambda_n = \dfrac{h}{p_n}$

$$\therefore \quad \frac{\lambda_e}{\lambda_n} = \frac{p_n}{p_e} \quad \ldots\text{(i)}$$

Case I $(\text{KE})_{\text{electron}} = (\text{KE})_{\text{neutron}}$

$$\Rightarrow \quad \frac{p_e^2}{2m_e} = \frac{p_n^2}{2m_n}$$

$$\Rightarrow \quad \frac{p_n}{p_e} = \sqrt{\frac{m_n}{m_e}} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda_e}{\lambda_n} = \sqrt{\frac{m_n}{m_e}}$$

but $m_n > m_e \quad \therefore \quad \dfrac{m_n}{m_e} > 1$

$$\Rightarrow \quad \frac{\lambda_e}{\lambda_n} >> 1$$

$$\Rightarrow \quad \lambda_e >> \lambda_n$$

Case II When momentum are equal, then

$$p_e = p_n$$

From Eq. (i) $\dfrac{\lambda_e}{\lambda_n} = 1$

Case III When speeds are same

$$v_e = v_n$$

$$\therefore \quad \frac{\lambda_e}{\lambda_n} = \frac{p_n}{p_e} = \frac{m_n v_n}{m_e v_e} = \frac{m_n}{m_e}$$

Since, $m_n >> m_e$, $\dfrac{m_n}{m_e} >> 1$, $\dfrac{\lambda_e}{\lambda_n} >> 1$

39. Key Idea *When an atom comes from some higher energy level to the first energy level Lyman series is obtained.*

The wavelength (λ) of Lyman series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad \text{where } n = 2, 3, 4, \ldots$$

The longest wavelength is for $n = 2$

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.097 \times 10^7 \times \frac{3}{4}$$

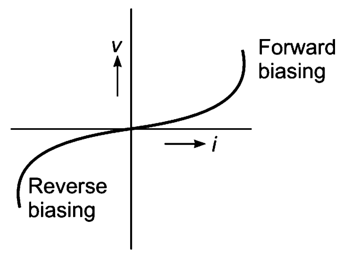
$$\Rightarrow \quad \lambda_m = \frac{4}{3.291 \times 10^7} = 1216 \text{ Å}$$

$$\Rightarrow \quad \lambda_m = 122 \text{ nm}$$

NOTE Shortest wavelength is obtained for $n = \infty$, given by approximately 912 Å.

40. (a) In half wave rectifier diode conducts only for positive half cycle and not for negative half. Hence, average output voltage has some finite value.

(b) The drift current that is current flowing due to majority carriers (holes in p-region and electrons in n-region) depends upon biasing of junction. The current increases for forward biasing and reduces to zero for reverse biasing.



(c) In forward biasing, more electrons in n-region cross the barrier to reach the p-region and more holes in p-region cross the barrier to reach the n-region. In this way, the depletion width decreases.

(d) The diffusion current in p-n junction is due to flow of minority carriers (that is electrons in p-region and holes in n-region). During forward biasing it supports majority carriers that is electrons in n-side and holes in p-side, while it opposes minority carriers. So, diffusion current decreases during forward biasing.

41. Key Idea

$$Molarity = \frac{moles\ of\ solute}{volume\ of\ solution\ in\ litre}$$

Given, mass of solute (NaOH) = 24.5 g

Volume of solution = 1 L

$$\text{Moles of solute} = \frac{\text{mass}}{\text{molar mass}} = \frac{24.5}{40} = 0.6125\text{ g}$$

$$\text{Molarity} = \frac{0.6125}{1} = 0.6125\text{ M}$$

42. Key Idea *The value of n tells the number of orbit and the value of l tells about subshell ($l = 0$ is s, 1 is p, 2 is d, 3 is f)*

Given, $n = 4,\ l = 3,\ m = -2,\ s = +\frac{1}{2}$

$\therefore$ Electron belongs to $4f$-orbital.

43. Key Idea *From the given molecular orbital configuration, find the total electrons and then, find the ion.*

Total electrons in

$\sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2p_y^2,$

$\overset{*}{\pi} 2p_x^2, \overset{*}{\pi} 2p_y^1$, are 17.

O_2 has 16 electrons.

O_2^+ has 15 electrons.

O_2^- has 17 electrons.

O_2^{2-} has 18 electrons.

$\therefore$ The electronic configuration is of O_2^-.

44. Key Idea *In close packing of 'n' atoms the number of tetrahedral voids are 2n.*

$\therefore$ The number of tetrahedral voids per atom is 2.

45. Key Idea *Osmotic pressures of isotonic solutions are equal.*

Given,

2% mean weight of acetic acid in solution = 2g

5% means weight of unknown solute (X) in solution = 5 g

Molarity of CH_3COOH solution

$$= \frac{\text{moles of } CH_3COOH}{\text{volume of solution in litre}}$$

$$= \frac{\text{mass/ mol. wt.}}{\text{volume of solution in litre}}$$

$$= \frac{\frac{2}{100 \times 60}}{\frac{1}{1000}} = 0.33\text{ M}$$

$\because$ CH_3COOH solution is isotonic with solution of 'X'.

$\therefore$ Molarity of CH_3COOH

= molarity of solution of 'X'

$$\therefore \quad 0.33 = \frac{5/100 \times \text{mol. wt. of } X}{(1/1000)}$$

$$\text{or} \quad 0.33 = \frac{0.05 \times 1000}{\text{mol. wt. of } X}$$

$\therefore$ Molecular weight of $X = 150$

46. Key Idea *One faraday charge liberates one gram equivalent of a metal.*

One gram equivalent of sodium = 23

$$Na^+ + e^- \longrightarrow Na$$

$\because$ One faraday liberates = 23 g of Na

$\therefore$ 0.5 faraday liberates = 23×0.5

= 11.50 g of Na

47. Key Idea *The units of rate constant are* $(\text{conc.})^{1-n}\ time^{-1}$, *where n is the order of reaction.*

Given, units of reaction are L $mol^{-1}\ s^{-1}$.

We get these units by substituting $n = 2$

$\therefore$ It is a second order reaction.

48. Key Idea 16 g CH_4 = 1 *mole of* CH_4

$$\Delta H = \Delta E + \Delta n_g RT$$

Given, $R = 8.314\ JK^{-1}\ mol^{-1}$

$\Delta E = -885389$ J

$T = 298$ K

$$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$$

$$\Delta n_g = n_P - n_R - n_R$$

$$= 1 - 3 = -2$$

$\therefore \Delta H = -885389 + [(-2) \times (8.314) \times (298)]$

$= -885389 - 4955.1440 = -890344$ J

49. Key Idea $W = -p\Delta V$

Given, $p = 1$ atm,

$\Delta V = (50\text{L} - 15)\text{ L} = 35\text{ L}$

$\therefore \quad W = -1 \times 35 = -35$ L atm

50. Key Idea $\Delta S = \dfrac{\Delta H_V}{T}$

Given, $\Delta H_V = 40.8$ kJ

$= 40.8 \times 10^3$ J $= 40800$ J

$T = 100°C$

$= 273 + 100 = 373$ K

$\therefore \quad \Delta S = \dfrac{40800}{373}$

$= 109.4$ JK^{-1}mol^{-1}

51. Key Idea $\Delta S° = \Sigma S_P° - \Sigma S_R°$

Given, $S°(HCl) = 187$ JK^{-1}mol^{-1}

$S°(H_2) = 131$ JK^{-1}mol^{-1}

$S°(Cl_2) = 223$ JK^{-1} mol^{-1}

$$H_2(g) + Cl_2(g) \longrightarrow 2HCl(g)$$

$\Delta S° = 2S°\,HCl(g) - [S°\,H_2(g) + S°\,Cl_2(g)]$

$= 2 \times 187 - (131 + 223)$

$= 374 - 354$

$= 20$ JK^{-1}mol^{-1}

52. Key Idea *Use Charles' law,*

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad V_2 = \frac{V_1 T_2}{T_1}$$

and then increase in volume $= V_2 - V_1$

Given, $V_1 = 500$ mL

$V_2 = ?$

$T_1 = 27°C = 27 + 273 = 300$ K

$T_2 = 42°C = 42 + 273 = 315$ K

$\therefore \quad V_2 = \dfrac{500 \times 315}{300} = 525$ mL

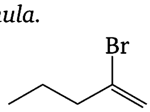
$\therefore$ Increase in volume $= V_2 - V_1$

$= 525 - 500 = 25$ mL

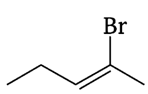
53. Key Idea *Diastereomers are the pair of optical isomers which are non-superimposable and also are not the mirror images of each other.*

Only optical isomers drawn in choice (d) are not the mirror images of each other and also non-superimposable.

54. Key Idea *The structural isomers have same molecular formula but different structural formula.*

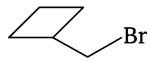
(a) 

2-bromo-1-pentene

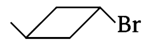
and 

2-bromo-2-pentene

Both have same molecular formula C_5H_9Br but different structures. So, they are structural isomers.

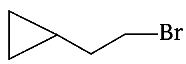
(c) 

1-bromomethyl cyclobutane

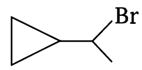
and 

1-methyl-3-bromo-cyclobutane

Molecular formula is C_5H_9Br for the both structures but structural formula is different. So, they are structural isomers.

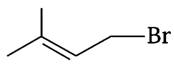
(d) 

2-bromomethyl cyclopropane

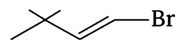
and 

1-bromomethyl cyclopropane

They are also structural isomers.

(b) 

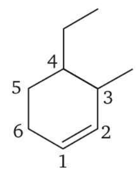
C_5H_9Br
1-bromo-3-methyl-2-butene

and 

$C_6H_{11}Br$
1-bromo-3,3-dimethyl-1-butene

$\because$ They have different molecular formulae.

$\therefore$ They are not structural isomers.

55. 

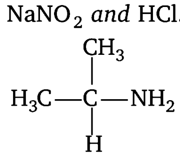
4-ethyl-3-methyl cyclohexene

56. Key Idea

(i) *Only aliphatic* 1° *amine on reaction with* $NaNO_2$ *and* HCl *form alcohol.*

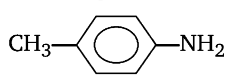
(ii) *Aromatic amines are diazotised by same reagent.*

(iii) 2° *aliphatic and* 2° *aromatic amines form yellow oily liquid on reaction with* $NaNO_2$ *and* HCl.

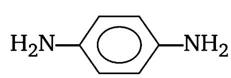


(a)
1° aliphatic amine

$C_6H_5NH_2$
(b)
aromatic amine

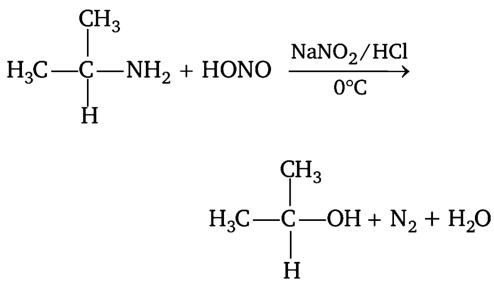


(c)
aromatic amine



(d)
aromatic amine

$\therefore$ Only compound given in option (a) will form alcohol.



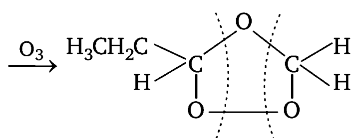
57. Key Idea *Weaker the base, better it will be the leaving group.*

I^- is best leaving group.

$\therefore$ The order of reactivity is

$$RI > RBr > RCl > RF$$

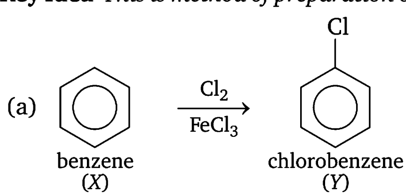
58. $\underset{\substack{\text{1-chlorobutane} \\ (A)}}{CH_3CH_2CH_2CH_2Cl} \xrightarrow{\text{Alc. KOH}} \underset{\substack{\text{1-butene} \\ (B)}}{CH_3CH_2CH{=}CH_2}$

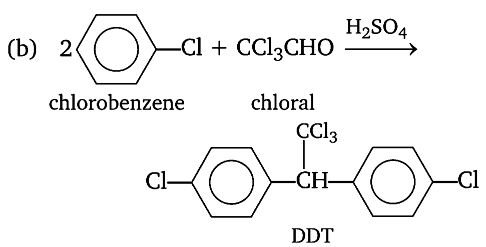


$\xrightarrow{Zn/H_2O} \underset{\text{propanal}}{CH_3CH_2CHO} + \underset{\text{methanal}}{HCHO}$

$\therefore$ The compound (A) is 1-chlorobutane.

59. Key Idea *This is method of preparation of DDT.*



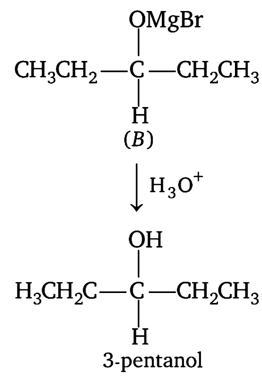


60. Key Idea *Alkenes give addition product with hydrogen halide which undergoes substitution by nucleophile (like NC^-). The substituted product on reduction with N/H_2 gives 2° amines.*

$\underset{\text{ethene}}{CH_2{=}CH_2} \xrightarrow{HBr} \underset{(X)}{CH_3CH_2Br} \xrightarrow{AgCN}$

$\underset{(Y)}{CH_3CH_2NC} \xrightarrow{H_2/Ni} \underset{\substack{(Z) \\ \text{N-methylethanamine}}}{CH_3CH_2NHCH_3}$

61. $\underset{\substack{(A) \\ \text{propanal}}}{CH_3CH_2CH{=}O} + CH_3CH_2MgBr \longrightarrow$



Molecular formula $C_5H_{12}O$.

$\therefore$ Compound (A) is propanal.

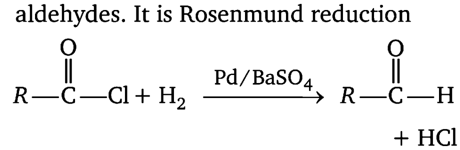
62. Key Idea

(i) *Proton donors are acids. The electron withdrawing groups (eg, $-NO_2$) increase the acidity of phenols by stabilizing the phenoxide ion. More the number of electron withdrawing groups, more will be acidity.*

(ii) *The effect of any group is minimum at m-position due to lack of increased delocalisation of electrons in it.*

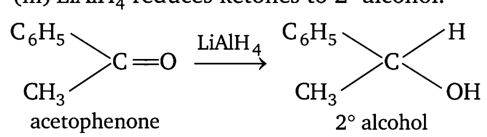
$\therefore$ Among given choices *m*-nitrophenol is weakest acid.

63. (i) $Pb/BaSO_4$ converts acid chlorides to aldehydes. It is Rosenmund reduction

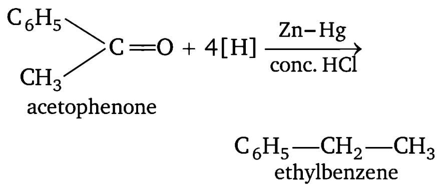


(ii) NH_2OH converts carbonyl compounds to oxime.

(iii) $LiAlH_4$ reduces ketones to 2° alcohol.



(iv) Reduction by Zn–Hg and HCl is Clemmensen reduction. The carbonyl compounds are reduced to hydrocarbons by this reaction.



64. Key Idea *Basicity of hydroxides of group* II *increases down the group due to decrease in ionisation energy.*

$\therefore$ $Be(OH)_2$ is amphoteric in nature and rest are basic.

$Be(OH)_2$ reacts with acids and bases both due to its amphoteric nature.

$$Be(OH)_2 + 2HCl \longrightarrow BeCl_2 + 2H_2O$$
$$Be(OH)_2 + 2NaOH \longrightarrow Na_2BeO_2 + 2H_2O$$

65. Key Idea *Greater the number of unpaired electrons, larger the value of magnetic moment.*

Magnetic moment

$$(\mu) = \sqrt{n(n+2)} \text{ BM}$$

where n = Number of unpaired electrons.

(a) V^{2+} Number of unpaired electrons = 3 (given)

$$\therefore \quad \mu = \sqrt{3(3+2)} = \sqrt{3 \times 5} = 3.87 \text{ BM}$$

(b) Cr^{2+} Number of unpaired electrons = 4 (given)

$$\therefore \quad \mu = \sqrt{n(n+2)} \text{ BM} = \sqrt{4(4+2)} = \sqrt{24} = 4.89 \text{ BM}$$

(c) Mn^{2+} Number of unpaired electrons = 5 (given)

$$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92 \text{ BM}$$

(d) Fe^{2+} Number of unpaired electrons = 4 (given)

$$\therefore \quad \mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 \text{ BM}$$

$\therefore$ Mn^{2+} has highest magnetic moment (due to presence of maximum unpaired electrons).

66. Key Idea *When both the positive and negative ions are complex and ligands are exchanged between the central metal atoms, coordination isomers are formed.*

$\therefore$ $[Co(NH_3)_6][Cr(CN)_6]$ and $[Cr(NH_3)_6][Co(CN)_6]$ represent coordination isomerism.

(i) Linkage isomerism is shown by complexes which have ambidentate ligands.

(ii) Ionisation isomerism is shown by complexes in which ligands are exchanged with ions outside coordination sphere.

(iii) Geometrical isomerism is due to different arrangement of atoms.

67. Key Idea

(i) *Loss of one α-particle results in loss of* 4 *units in atomic mass and loss of* 2 *units in atomic number.*

(ii) *Loss of one β-particle results in increase of atomic number by* 1 *and no change in atomic mass.*

$$^{238}_{92}U \xrightarrow{-8\alpha \text{ particle}} {}^{206}_{76}Os \xrightarrow{-6\beta} {}^{206}_{82}Pb$$

$\therefore$ The series will terminate at $^{206}_{82}Pb$.

68. Key Idea *Electron deficient compounds are Lewis acid and* $p\pi - p\pi$ *back bonding compensate the electron deficiency. The tendency to form* $p\pi$*-* $p\pi$ *back bond decreases with increase in size of halogen.*

(i) Boron trihalides are acidic in nature because they are electron deficient compounds.

(ii) The electron deficiency of BF_3 is maximum compensated due to capacity to do $p\pi$- $p\pi$ back bonding.

$\therefore$ Lewis acid character decreases from BI_3 to BF_3.

$\therefore$ Correct order of acidity among halides of boron is

$$BI_3 > BBr_3 > BCl_3 > BF_3$$

69. Key Idea *The tendency of ccatenation decreases while we move down in group* 14.

Catenation is capability of forming bond with its own atom or it is called property of self linkage.

The correct order is

$$C >> Si > Ge = Sn > Pb.$$

70. Key Idea *As the size of ions, forming molecule, decreases, thermal stability increases.*

$\because$ CF_4 has more ionic character as compared to CCl_4, SiF_4 and $SiCl_4$ due to small size of ions.

$\therefore$ CF_4 has more lattice energy and more thermal stability than CCl_4, SiF_4 and $SiCl_4$.

71. (i) Starch $\xrightarrow{\text{Amylase}}$ glucose

(ii) Sucrose $\xrightarrow{\text{Invertase}}$ glucose + fructose

(iii) Lactose $\xrightarrow{\text{Lactase}}$ glucose + galactose

(iv) Maltose $\xrightarrow{\text{Maltase}}$ glucose + glucose

72. **Key Idea** *A reaction in which catalyst and reactants are in different physical states is an example of heterogeneous catalysis.*

(a) Reactants and catalyst all in gaseous state.
(b) Reactants and catalyst all in gaseous state.
(c) Reactants and catalyst all in liquid state.
$\therefore$ (a), (b) and (c) choices are example of homogeneous catalysis.

(d) $CO(g) + 2H_2(g) \xrightarrow{Cu, ZnO-Cr_2O_3} CH_3OH(l)$

In this reaction reactants are in gaseous state and catalysts are in solid state. So, it is an example of heterogeneous catalysis.

73. **Key Idea** *In DNA strands complementary base pairing takes place. Thymine (T) always joins adenine (A) by 2-hydrogen bonds and cytosine (C) always joins guanine (G) by 3-hydrogen bonds.*

DNA strand T A T G A C T G
Complementary strand A T A C T G A C

74. (i) Adrenal cortex secretes corticoids, cortisones, corticosterone and aldosterone.
(ii) Ovary secretes estrogens.
(iii) Corpus luteum secretes progesteron.
(iv) Testes secretes testosterone.

75. **Key Idea** *The dyes having azo group* (—N=N—) *are azo dyes.*

(i) Methyl orange is a azo dye.
(ii) Martius yellow is a nitro dye.
(iii) Malachite green is a triphenylmethane dye.
(iv) Mercurochrome is a derivative of fluorescein dye.

76. (i) Thermosetting polymers have cross links and they cannot be remelted or reworked, *eg,* bakelite.
(ii) Thermoplastics are linear polymers which can be remolded, *eg,* terylene, polystyrene, polyethene.

77. **Key Idea** *Write hydrolysis reaction for all the given carbohydrates.*

(a) Cellulose $\xrightarrow[H_2SO_4 \text{ or } H^+]{\text{Hydrolysis}}$ β-D-glucose

(b) Maltose $\xrightarrow[H_2SO_4 \text{ or } H^+]{\text{Hydrolysis}}$ glucose

(c) Starch $\xrightarrow[H_2SO_4 \text{ or } H^+]{\text{Hydrolysis}}$ α-D-glucose

(d) Sucrose $\xrightarrow[H_2SO_4 \text{ or } H^+]{\text{Hydrolysis}}$ glucose + fructose

78. **Key Idea** *Green house gases cause green house effect. They absorb infra-red radiations of high wavelengths reflected by earth* CO_2, CH_4 *and* O_3 *are green house gases as they absorb infra-red radiations.*

$\because$ O_2 does not absorb infra-red radiations.
$\therefore$ O_2 is not a green house gas.

79. (a) Trypsin converts proteins into amino acids.
(b) Renin is found in kidney and participate in renin-angiotensin system.
(c) Streptokinase converts plasminogen into plasmin and is used to dissolve blood clots.
(d) Tyrosinase catalyses the oxidation of phenols.

80. (i) α-tocopherol is vitamin E. Its deficiency causes sterility. It is called antisterility factor.
(ii) Retinol is vitamin A.
(iii) Calciferol is vitamin-D.
(iv) Pyridoxine is vitamin B-6.

Mathematics

81. **Key Idea** *Three numbers a, b, c are in HP, if* $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ *are in AP.*

Given, $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$

$\Rightarrow 8a^2 + 18b^2 + 32c^2 = 12ab + 24bc + 16ca$

$= 0$

$\Rightarrow (4a^2 - 12ab + 9b^2) + (9b^2 - 24bc + 16c^2) + (16c^2 - 16ca + 4a^2) = 0$

$\Rightarrow (2a - 3b)^2 + (3b - 4c)^2 + (4c - 2a)^2 = 0$

$\Rightarrow 2a = 3b, 3b = 4c, 4c = 2a$ $[\because x^2 \geq 0]$

Let $2a = 3b = 4c = A$

$\therefore$ $a = \frac{A}{2}, b = \frac{A}{3}, c = \frac{A}{4}$

$\therefore$ $\frac{1}{a} = \frac{2}{A}, \frac{1}{b} = \frac{3}{A}, \frac{1}{c} = \frac{4}{A}$

$\Rightarrow \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

$\Rightarrow \quad a, b, c$ are in HP.

NOTE If a, b, c are in HP, then $a^n + c^n > 2b^n$.

82. Given, $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots \infty = \frac{\pi^4}{90}$

Let $\quad S = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots \infty \quad \ldots(\text{i})$

$$\Rightarrow \quad S = \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\right) + \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \ldots\right)$$

$$\Rightarrow \quad S = \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\right) + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\right)$$

$$\Rightarrow \quad S = \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\right) + \frac{1}{2^4} S \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad S = \left(\frac{1}{1^4} + \frac{1}{3^4} + \ldots \infty\right) + \frac{1}{2^4} S$$

$$\Rightarrow \quad S\left(1 - \frac{1}{2^4}\right) = \frac{1}{1^4} + \frac{1}{3^4} + \ldots \infty$$

Given, $\quad S = \frac{\pi^4}{90}$

$$\Rightarrow \quad \frac{\pi^4}{90}\left(\frac{15}{2^4}\right) = \frac{1}{1^4} + \frac{1}{3^4} + \ldots \infty$$

$$\Rightarrow \quad \frac{1}{6}\left(\frac{\pi}{2}\right)^4 = \frac{1}{1^4} + \frac{1}{3^4} + \ldots \infty$$

$\therefore$ Required value is $\frac{\pi^4}{96}$.

83. Let $f(x) = (ax^2 + bx + c)(ax^2 - bx - c) = 0$

ie,

$f(x) = -(ax^2 + bx + c)(-ax^2 + bx + c) = 0$

Let D_1 be the discriminant of $ax^2 + bx + c = 0$

and D_2 be the discriminant of

$$-ax^2 + bx + c = 0$$

$\therefore \quad D_1 = b^2 - 4ac$

and $\quad D_2 = b^2 + 4ac$

Now, $\quad ac \neq 0$

either $\quad ac > 0 \quad$ or $\quad ac < 0$

Now, if $\quad ac > 0 \quad D_2 > 0$

If $\quad ac < 0 \quad D_1 > 0$

$\therefore$ In either case atleast two real roots of given polynomial will exist.

84. Let $\quad z = a + ib$

Given, $\quad (3 + i) z = (3 - i) \bar{z}$

$\Rightarrow \quad (3 + i)(a + ib) = (3 - i)(a - ib)$

$\Rightarrow \quad 3a - b + (3b + a)i = (3a - b) + (-3b - a)$

$\Rightarrow \quad 3a - b = 3a - b$

and $\quad 3b + a = -3b - a$

$\Rightarrow \quad 6b = -2a$

$\Rightarrow \quad 3b = -a$

$\therefore \quad z = a - \frac{1}{3} ai = 3a(3 - i)$

$\quad a \in R$

$\Rightarrow \quad 3a = k \in R$

$\therefore \quad z = k(3 - i)$ for some $k \in R$

Alternative

Given, $\quad (3 + i) z = (3 - i) \bar{z}$

$$\Rightarrow \quad \frac{z}{\bar{z}} = \frac{3 - i}{3 + i}$$

Let $\quad z = x + iy$

$$\Rightarrow \quad \frac{x + iy}{x - iy} = \frac{3 - i}{3 + i}$$

$\Rightarrow \quad x = 3a, \; y = -a$

$\Rightarrow \quad z = a(3 - i), \; a \in R$

NOTE If z_1 is any complex number, then $z_1 \bar{z}_1 = \{\text{Re } z_1\}^2 + \{\text{Im}(z_1)\}^2$

85. Given, $\quad e^{i\theta} = \cos\theta + i \sin\theta$

$\therefore \quad e^{iA} = \cos A + i \sin A$

$\quad e^{iB} = \cos B + i \sin B$

and $\quad e^{iC} = \cos C + i \sin C$

$$\therefore \quad e^{iA} e^{iB} e^{iC} = (\cos A + i \sin A) \times (\cos B + i \sin B)(\cos C + i \sin C)$$

$$= \cos(A + B + C) + i \sin(A + B + C)$$

$$\left[\because \prod_{r=1}^{n} \cos r\theta + i \sin r\theta = \cos\left(\sum_{r=1}^{n} r\right)\theta + i \sin \sum_{r=1}^{n} r\theta\right]$$

$$\Rightarrow \quad e^{iA} \cdot e^{iB} \cdot e^{iC} = \cos\pi + i \sin\pi \quad (\because A + B + C = \pi)$$

$$\Rightarrow \quad e^{iA} \cdot e^{iB} \cdot e^{iC} = -1$$

Alternative

$e^{iA} \cdot e^{iB} \cdot e^{iC} = e^{i(A+B+C)} = e^{i\pi}$

$[\because A + B + C = \pi \text{ in } \Delta ABC]$

$e^{iA} \cdot e^{iB} \cdot e^{iC} = \cos \pi + i \sin \pi = -1$

NOTE If $z_i = r_i\, e^{i\theta_i}$

Then $\prod_{i=1}^{n} z_i = \prod_{i=1}^{n} r_i\, e^{i(\theta_1 + \theta_2 + \dots \theta_n)}$

86. Number lying between 999 and 10000 is a four digit number.

$\therefore$ Four digit numbers have to be formed using 0, 2, 3, 6, 7, 8 (No repetition)

$\therefore$ Ist digit from right side can be occupied in 5 ways ...(i)

($\because$ Zero is excluded since 0 at first place make it three digit number)

IInd digit from right side can be occupied in 5 ways ...(ii)

IIIrd digit from right side can be occupied in 4 ways ...(iii)

IVth digit from right side can be occupied in 3 ways ...(iv)

From statements (i), (ii), (iii), (iv)

Number of numbers that can be formed are

$5 \times 5 \times 4 \times 3 = 300$

Alternate Solution

The numbers lying between 999 and 10000 are all four digit numbers.

All four digits numbers that can be formed using 0, 2, 3, 6, 7, 8 without repetition are $3 \times 4 \times 5 \times 6 = 360$.

Number of those numbers in which 0 is the first digit $= 5 \times 4 \times 3 = 60$

$\therefore$ Required number of numbers $= 360 - 60$

$= 300$

87. Key Idea

${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} + {}^nC_n = 2^n$

Let the total number of contestants $= n$

$\therefore$ A voter can vote to $n - 1$ candidates.

$\therefore \quad {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} = 126$ (given)

$\Rightarrow \quad 2^n - 2 = 126$

$\Rightarrow \quad 2^n = 128 \Rightarrow 2^n = 2^7$

$\therefore \quad n = 7$

88. Let $S = \dfrac{1}{1!(n-1)!} + \dfrac{1}{3!(n-3)!} + \dots$

$\Rightarrow \quad S = \dfrac{1}{n!}\left[\dfrac{n!}{1!(n-1)!} + \dfrac{n!}{3!(n-3)!} + \dots\right]$

$\Rightarrow \quad S = \dfrac{1}{n!}[{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots]$

$\Rightarrow \quad S = \dfrac{1}{n!} 2^{n-1} = \dfrac{2^{n-1}}{n!} \; \forall\, n$

89. Key Idea $-\log(1-x) = x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \dots$

Let $A = \dfrac{a-b}{a} + \dfrac{1}{2}\left(\dfrac{a-b}{a}\right)^2 + \dfrac{1}{3}\left(\dfrac{a-b}{a}\right)^3 + \dots \text{ to } \infty$

Let $x = \dfrac{a-b}{a}$ ...(i)

$\therefore \quad A = x + \dfrac{1}{2}x^2 + \dfrac{x^3}{3} + \dots$

$\therefore \quad A = -\log(1-x)$

$\Rightarrow \quad A = -\log\left[1 - \left(\dfrac{a-b}{a}\right)\right]$ [from Eq. (i)]

$\Rightarrow \quad A = -\log\left[\dfrac{a-a+b}{a}\right]$

$\Rightarrow \quad A = \log \dfrac{a}{b}$

90. Let $S = \dfrac{1}{(1+a)(2+a)} + \dfrac{1}{(2+a)(3+a)} + \dots \infty$

Let T_n be nth term of series S.

$\therefore \quad T_n = \dfrac{1}{(n+a)(n+1+a)}$

$= \dfrac{1}{n+a} - \dfrac{1}{(n+1)+a}$

$\therefore \quad T_1 = \dfrac{1}{1+a} - \dfrac{1}{2+a}$ ...(i)

$T_2 = \dfrac{1}{2+a} - \dfrac{1}{3+a}$ ...(ii)

... ...

... ...

$T_n = \dfrac{1}{n+a} - \dfrac{1}{(n+1)a}$ ...(n)

$\therefore \quad S_n = T_1 + T_2 + \dots T_n$

$= \dfrac{1}{1+a} - \dfrac{1}{(n+1)+a}$

$\therefore \quad S_\infty = \lim_{n\to\infty} \dfrac{1}{1+a} - \lim_{n\to\infty} \dfrac{1}{(n+1)+a}$

$= \dfrac{1}{1+a} - 0$

$\Rightarrow \quad S_\infty = \dfrac{1}{1+a}$

91. Key Idea $\log_a a = 1$

Given, $\log_2 \log_3 \ldots \log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}}$

$= \log_2 \log_3 \ldots \log_{99} 99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}} (\log_{100} 100)$

$= \log_2 \log_3 \ldots \log_{98} 98^{97^{\cdot^{\cdot^{\cdot^{2^1}}}}} \log_{99} 99 \cdot 1$

$= \log_2 \log_3 \ldots \log_{97} 97^{96^{\cdot^{\cdot^{\cdot^{2^1}}}}} \cdot 1$

Proceeding like wise, we get

$= \log_2 2^1 = 1$

92. Key Idea $\tan(A - B) = \dfrac{\tan A - \tan B}{\tan A \tan B + 1}$

Now, $\tan(70° - 20°) = \dfrac{\tan 70° - \tan 20°}{1 + \tan 70° \tan 20°}$

$\Rightarrow \tan 50° (1 + \tan 70° \tan 20°)$
$= \tan 70° - \tan 20° \quad \ldots(\text{i})$

Now, $\tan(70° + 20°) = \dfrac{\tan 70° + \tan 20°}{1 - \tan 70° \tan 20°}$

$\Rightarrow \infty = \dfrac{\tan 70° + \tan 20°}{1 - \tan 20° \tan 70°}$

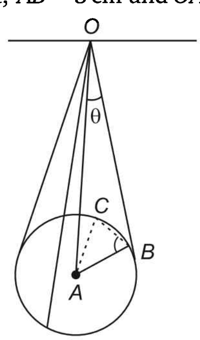
$\Rightarrow 1 = \tan 20° \tan 70° \quad \ldots(\text{ii})$

From Eqs. (ii) and (i), we get

$\tan 50° (1 + 1) = \tan 70° - \tan 20°$

$\Rightarrow \tan 70° - 2\tan 50° - \tan 20° = 0$

93. Given that, $AB = 3$ cm and $OA = 4$ cm



$\therefore$ In ΔAOB,

$OB^2 = OA^2 + AB^2$

$= 4^2 + 3^2 = 16 + 9 = 5^2$

$\therefore \quad OB = 5$ cm

Since, 4 strings attached at equal intervals to its circumference, then ΔABC is a right angled triangle.

So, $BC^2 = AB^2 + AC^2 = 3^2 + 3^2 = 18$

($\because AB = AC =$ radius of circle)

$\Rightarrow \quad BC = 3\sqrt{2}$

Now, in ΔOBC,

$$\cos\theta = \frac{OB^2 + OC^2 - BC^2}{2 \cdot OB \cdot OC}$$

$$= \frac{25 + 25 - 18}{2 \times 5 \times 5} = \frac{32}{50} = \frac{16}{25}$$

94. Key Idea $\tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1 + x^2}}$

and $\tan^{-1} \dfrac{1}{x} = \cos^{-1} \dfrac{x}{\sqrt{1 + x^2}}$

Given, $\tan^{-1} x + \cos^{-1} \dfrac{y}{\sqrt{1 + y^2}} = \sin^{-1} \dfrac{3}{\sqrt{10}}$

$\Rightarrow \quad \tan^{-1} x + \tan^{-1} \dfrac{1}{y} = \sin^{-1} \dfrac{3}{\sqrt{10}}$

$\Rightarrow \quad \tan^{-1} \left(\dfrac{x + \dfrac{1}{y}}{1 - \dfrac{x}{y}} \right) = \tan^{-1} 3$

$\Rightarrow \quad x + \dfrac{1}{y} = 3\left(1 - \dfrac{x}{y}\right)$

$\Rightarrow \quad x = 1, y = 2$

Hence, One solution exists.

95. Key Idea *If* α *is repeated root of* $ax^2 + bx + c = 0$*, then* $ax^2 + bx + c = a(x - \alpha)^2$

Now, $\lim\limits_{x \to \alpha} \dfrac{\sin(ax^2 + bx + c)}{(x - \alpha)^2}$

$= \lim\limits_{x \to \alpha} \dfrac{\sin a(x - \alpha)^2}{(x - \alpha)^2}$

$= \lim\limits_{x \to \alpha} \dfrac{\sin a(x - \alpha)^2}{a(x - \alpha)^2} \cdot a$

$= \lim\limits_{x \to \infty} \dfrac{\sin a(x - \alpha)^2}{a(x - \alpha)^2} \lim\limits_{x \to \alpha} a = a$

NOTE If α, β are the two roots of $ax^2 + bx + c = 0$, then $ax^2 + bx + c = 0 = a(x - \alpha)(x - \beta)$

96. Key Idea $\lim\limits_{x \to 0} \dfrac{\log(1 + x)}{x} = 1$

$$\lim_{x \to 0} \frac{\log(1 + x^3)}{\sin^3 x} = \lim_{x \to 0} \frac{\log(1 + x^3)}{x^3} \cdot \frac{x^3}{\sin^3 x}$$

$$= \lim_{x \to 0} \frac{\log(1 + x^3)}{x^3} \lim_{x \to 0} \left(\frac{x}{\sin x}\right)^3$$

$$= 1 \cdot 1 = 1$$

Alternative

$$\lim_{x \to 0} \frac{\log(1 + x^3)}{\sin^3 x}$$

$$= \lim_{x \to 0} \frac{\left(\frac{x^3}{1} - \frac{(x^3)^2}{2} + \frac{(x^3)^3}{3} - \dots \infty\right)}{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \infty\right)}$$

$$= \lim_{x \to 0} \frac{\left(1 + \frac{x^3}{2} + \frac{(x^3)^2}{3} + \dots \infty\right)}{\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \infty\right)^3}$$

$$= 1$$

97. Key Idea $x - [x] = \{x\}$

Given, that $\log_e (x - [x]) = \log_e \{x\}$

When x is integer, $\{x\} = 0$

$\therefore$ $\log_e \{x\}$ is not defined when $x \in I$

$\therefore$ Required domain $= R - I$

98. Let $x + y = u$ and $x - y = v$

$\Rightarrow$ $x = \frac{(u + v)}{2}, y = \frac{u - v}{2}$

$\therefore$ $F(u, v) = \frac{u + v}{2} \cdot \frac{u - v}{2}$

Arithmetic mean of $f(u, v)$ and $f(v, u)$ is

$$\frac{f(u, v) + f(v, u)}{2}$$

$$= \frac{\frac{u + v}{2} \cdot \left(\frac{u - v}{2}\right) + \left(\frac{u + v}{2}\right)\left(\frac{v - u}{2}\right)}{2}$$

$$= \frac{u^2 - v^2 + v^2 - u^2}{8} = 0$$

$\therefore$ Arithmetic mean of $f(x, y)$ and $f(y, x)$ is 0.

99. Key Idea *Circumcentre of a right angle triangle is mid point of its hypotenuse.*

Given, equation of sides of a triangle are

$$x = 2, \quad y + 1 = 0, \quad x + 2y = 4$$

On solving these equations, we get the coordinates of the vertices of the triangle which are $A(2, -1)$, $B(6, -1)$ and $C(2, 1)$

$\therefore$ Δ is right triangle whose hypotenuse is BC.

$\therefore$ Circumcentre is mid point of BC

ie, $M\left(\frac{6 + 2}{2}, \frac{-1 + 1}{2}\right)$ *ie*, $(4, 0)$

NOTE If a Δ is right angle, orthocentre lies on that vertex at which Δ is right angle.

100. Given that, $|x + y| = 4$

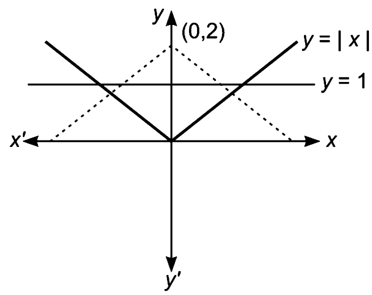
If point (a, a) lies between the lines, then

$$|a + a| = 4$$

$\Rightarrow$ $|a| = 2$

101. Given that $y = |x|$ and $y = 1$

It is clear from the figure that image of rays $y = |x|$ about line $y = 1$ is $y = |x| + 2$



102. Given $P = (1, 1)$, $Q = (3, 2)$, let the point R on x-axis is $(x, 0)$.

$\therefore$ $PR = \sqrt{(x - 1)^2 + 1}$,

$RQ = \sqrt{(x - 3)^2 + 2^2}$

$\therefore$ Let $L = PR + RQ$

$\Rightarrow$ $L = \sqrt{(x - 1)^2 + 1} + \sqrt{(x - 3)^2 + 2^2}$ ...(i)

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{dL}{dx} = \frac{1}{2\sqrt{(x - 1)^2 + 1}} 2(x - 1) + \frac{2(x - 3)}{2\sqrt{(x - 3)^2 + 2^2}}$$

For extremum $\frac{dL}{dx} = 0$

$\Rightarrow$ $4(x - 1)\sqrt{(x - 3)^2 + 4} + 4(x - 3)\sqrt{(x - 1)^2 + 1} = 0$

$\Rightarrow$ $(x - 1)^2 \{(x - 3)^2 + 4\} = (x - 3)^2 \{(x - 1)^2 + 1\}$

$\Rightarrow$ $x = \frac{5}{3}$ and $\frac{d^2L}{dx^2} > 0$ at $x = \frac{5}{3}$

$\therefore$ Required point is $\left(\frac{5}{3}, 0\right)$.

103. Key Idea *If the algebraic sum of the perpendicular distance from the vertices $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ of the triangle ABC to a variable line is zero, then line passes through the centroid of ΔABC.*

Given, sum of distances of the points (1, 1), (2, 0) and (0, 2) from a line is zero.

$\therefore$ Line will pass through

$$\left(\frac{1+2+0}{3}, \frac{1+0+2}{3}\right) = (1, 1)$$

$\therefore$ Required point is (1, 1).

Alternative

Let the equation of line be

$$ax + by + c = 0 \quad \dots(i)$$

The perpendicular distances from (1, 1), (2, 0), (0, 2) to the line $ax + by + c = 0$ are

$$p_1 = \frac{a+b+c}{\sqrt{a^2+b^2}}, p_2 = \frac{2a+c}{\sqrt{a^2+b^2}}, p_3 = \frac{2b+c}{\sqrt{a^2+b^2}}$$

Given, $p_1 + p_2 + p_3 = 0$

$\Rightarrow a + b + c + 2a + c + 2b + c = 0$

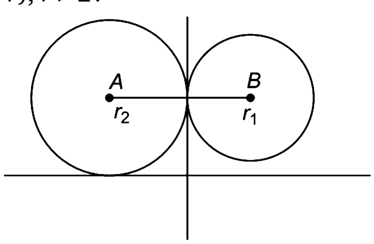
$\Rightarrow 3(a + b + c) = 0$

$\Rightarrow a + b + c = 0 \quad \dots(ii)$

From Eq. (ii) it is clear that line (i) will pass through (1, 1).

104. Key Idea *Two circles touch each other externally, iff $|r_1 + r_2| = AB$*

Since, a radius of circle C_1 is 2 and this circle touches both axes. So, its centre is (2, 2). Let radius of another circle be r and its centre be (r, r), $r > 2$.



Since, both circles touch each other.

$\therefore \sqrt{(r-2)^2 + (r-2)^2} = 2 + r$

$\Rightarrow 2(r-2)^2 = (r+2)^2$

$\Rightarrow r^2 - 12r + 4 = 0$

$\Rightarrow r = \frac{12 \pm \sqrt{128}}{2} = 6 \pm 4\sqrt{2}$

Since, $r > 2$

$\therefore r = 6 + 4\sqrt{2}$

NOTE If two circles touches internally, then $|AB| = |r_1 - r_2|$

105. Let the centre of circle be (g, f). If one end of a diameter is (1, 1), then the other end of diameter is $(2g - 1, 2f - 1)$.

Since, other end lie on $x + y = 3$

$\Rightarrow 2g - 1 + 2f - 1 = 3$

$\Rightarrow 2g + 2f = 5$

$\therefore$ Locus of centre is $2x + 2y = 5$

106. Key Idea *Locus of the mid points of the chords of the parabola $y^2 = 4ax$ which are right angle at the vertex of the parabola is $y^2 = 2ax - 8a^2$, which is also a parabola.*

Alternate Solution

Let the equation of the parabola be

$$y^2 = 4ax \quad \dots(i)$$

Equation of a chord in terms of mid point (x_1, y_1) is $T = S_1$

ie, $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

$\Rightarrow -2ax + yy_1 - 2ax_1 - y_1^2 + 4ax_1 = 0$

$\Rightarrow 2ax - yy_1 + y_1^2 - 2ax_1 = 0$

$\Rightarrow \frac{2ax - yy_1}{2ax_1 - y_1^2} = 1 \quad \dots(ii)$

On combining Eqs. (i) and (ii), we get

$$y^2 = 4ax\left(\frac{2ax - y_1 y}{2ax_1 - y_1^2}\right)$$

$\Rightarrow 8a^2x^2 + (y_1^2 - 2ax_1)y^2 - 4axyy_1 = 0$

Since, lines are perpendicular

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow 8a^2 + y_1^2 - 2ax_1 = 0$

$\Rightarrow y_1^2 = 2ax_1 - 8a^2$

$\therefore$ Locus of (x_1, y_1) is a parabola.

NOTE Locus of mid points of the chords of the parabola $y^2 = 4ax$ those passes through focus of the parabola is given by $y^2 = 2ax - a^2$.

107. Let L_1 be the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r_1 \quad \text{(say)}$$

Any point on the line L_1 is $(2r_1 + 1, 3r_1 + 2, 4r_1 + 3)$

And let L_2 be the another line $\frac{x-4}{5}$

$$= \frac{y-1}{2} = z = r_2 \quad \text{(say)}$$

Any point on the line L_2 is $(5r_2 + 4, 2r_2 + 1, r_2)$

Since, lines intersect each other.

$\therefore \quad 2r_1 + 1 = 5r_2 + 4 \quad \ldots(i)$

$3r_1 + 2 = 2r_2 + 1 \quad \ldots(ii)$

and $\quad 4r_1 + 3 = r_2 \quad \ldots(iii)$

$\Rightarrow \quad 2r_1 + 1 = 5(4r_1 + 3) + 4$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad 2r_1 + 1 = 20r_1 + 15 + 4$

$\Rightarrow \quad -18r_1 = 18$

$\Rightarrow \quad r_1 = -1$

$\therefore \quad r_2 = 4(-1) + 3 = -1$

$\therefore$ Point of intersection is $(-1, -1, -1)$.

108. Key Idea *A plane meets the coordinate axes in A, B, C such that the centroid of ΔABC is point (p, q, r), then equation of plane is*

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$$

Given, centroid of $\Delta ABC = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$\therefore \quad p = \frac{1}{3}, \quad q = \frac{1}{3}, \quad r = \frac{1}{3}$$

$\therefore$ Required equation of plane

$$\frac{x}{1/3} + \frac{y}{1/3} + \frac{z}{1/3} = 3$$

$\Rightarrow \quad x + y + z = 1$ is the equation of required plane.

Alternative

Let the equation of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \ldots(i)$$

The coordinate of A, B and C are $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.

The centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ but it is given $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

$\therefore \quad a = 1, b = 1, c = 1 \quad \ldots(ii)$

Required equation of plane is

$x + y + z = 1$ [from Eqs. (i) and (ii)]

109. Equation of any plane passing through (a, b, c) is

$a'(x - a) + b'(y - b) + c'(z - c) = 0 \quad \ldots(i)$

Where a', b', c' are the direction ratios of a plane.

Direction ratio's of $OA = (a - 0, b - 0, c - 0)$

$= (a, b, c)$

Since, plane (i) is perpendicular to the line OA.

$\therefore$ Its Direction ratio's is proportional to (a, b, c).

$\therefore$ Required equation of plane is

$a(x - a) + b(y - b) + c(z - c) = 0$

110. Key Idea *Image of a point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is given by*

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2|ax_1 + by_1 + cz_1 - d|}{a^2 + b^2 + c^2}$$

Given plane is $x + y + 2z - 15 = 0$ and point $(5, 4, 6)$.

$\therefore$ Image of $(5, 4, 6)$ in given plane is given by

$$\frac{x - 5}{1} = \frac{y - 4}{1} = \frac{z - 6}{2} = \frac{-2|5 + 4 + 12 - 15|}{1 + 1 + 4}$$

$$\Rightarrow \quad x - 5 = y - 4 = \frac{z - 6}{2} = -\frac{12}{6} = -2$$

$\therefore \quad x = 3, \quad y = 2, \quad z = 2$

$\therefore$ Required image is $(3, 2, 2)$.

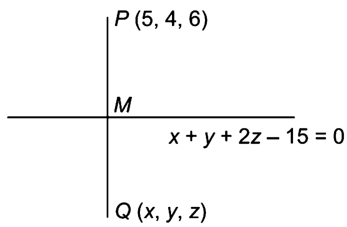
Alternative

Let the image of $P(5, 4, 6)$ be $Q(x_1, y_1, z_1)$

The equation of straight line PQ which is perpendicular to the plane $x + y + 2z - 15 = 0$ is

$$\frac{x - 5}{1} = \frac{y - 4}{1} = \frac{z - 6}{2} = k \quad \text{(say)}$$

$\therefore$ Any point on the line is $M(k + 5, k + 4, 2k + 6)$.



This point lies on the given plane.

$\therefore \quad k + 5 + k + 4 + 2(2k + 6) - 15 = 0$

$\Rightarrow \quad 6k + 6 = 0$

$\Rightarrow \quad k = -1$

$\therefore$ Coordinate of M is $(4, 3, 4)$.

Since, M is the mid point of PQ, then coordinate of Q are $(3, 2, 2)$.

111. Key Idea *Radius of circle whose plane $ax + by + cz + d_2 = 0$ cut by the sphere*

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d_1 = 0$$

is $\sqrt{R^2 - P^2}$

where R is radius of sphere and P is length of perpendicular from centre of sphere to the plane.

Given sphere is

$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$$

$\therefore$ Centre (0, 1, 2)

and radius $= \sqrt{0^2 + 1^2 + 2^2 + 11} = 4$

$$\Rightarrow \quad R = 4 \quad \ldots(i)$$

Length of perpendicular from (0, 1, 2) to given plane $x + 2y + 2z - 15 = 0$ is

$$P = \frac{|0 + 2 + 4 - 15|}{\sqrt{1 + 4 + 4}} = \frac{9}{3} = 3 \quad \ldots(ii)$$

$\therefore$ Centre of circle $= \sqrt{R^2 - P^2}$

$$= \sqrt{16 - 9} = \sqrt{7}$$

[using Eqs. (i) and (ii)]

112. Let a line makes an angle α to the x-axis, we know that

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2\alpha + \cos^2 60° + \cos^2 60° = 1$$

$$\Rightarrow \cos^2\alpha = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$$

$$\therefore \cos\alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \alpha = 45°$$

113. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ (2^x + 2^{-x})^2 & (3^x + 3^{-x})^2 & (5^x + 5^{-x})^2 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$[\because (a + b)^2 - (a - b)^2 = 4ab]$

$$\Rightarrow \Delta = 4\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$$\Rightarrow \Delta = 0 \quad (\because R_1 = R_2)$$

114. Key Idea *A number r is said to be the rank of an $m \times n$ matrix A, if*

(1) *every square submatrix of order $(r + 1)$ or more is singular and*

(2) *there exist atleast one square submatrix of order r which is non-singular.*

Let $A = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 6 & 0 & 2 & 0 \end{bmatrix}$

Now, we take submatrix of order 3×3

Let $B = \begin{bmatrix} 4 & 1 & 0 \\ 3 & 0 & 1 \\ 6 & 0 & 2 \end{bmatrix}$

$\therefore \quad |B| = 0$

Now, we take submatrix of order 2×2

Let $C = \begin{bmatrix} 4 & 1 \\ 3 & 0 \end{bmatrix}$

$\therefore \quad |C| \neq 0$

$\therefore \quad$ Rank $A = 2$

115. Key Idea *If $AX = O$ be a homogeneous system of equations, then it has non-zero solution, if $|A| = 0$*

Given, system of equations are

$$ax + y + z = 0,$$
$$x - ay + z = 0$$

and $\quad x + y + z = 0$

For non-zero solution

$$\begin{vmatrix} a & 1 & 1 \\ 1 & -a & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a(-a - 1) - 1(1 - 1) + 1(1 + a) = 0$$

$$\Rightarrow -a^2 - a + 1 + a = 0$$

$$\Rightarrow a^2 = 1$$

$$\Rightarrow a = \pm 1$$

NOTE
1. For trivial solution *ie*, $x = y = z = 0 \; |A| \neq 0$
2. Homogeneous system of equations is never inconsistent.

116. Key Idea *For skew symmetric matrix*

$$A^T = -A.$$

Here, C, A and C^T are matrix of order $n \times 1$, $n \times n$, $1 \times n$ respectively.

Let $\quad C^T AC = k$

$$\Rightarrow (C^T AC)^T = C^T A^T (C^T)^T$$

$$= C^T A^T C$$

$$= C^T(-A)C$$
$$= -C^TAC = -k$$
$\therefore$ $k = -k$

$\Rightarrow$ $2k = 0$

$k = 0$

$\therefore C^TAC$ is a null matrix of order 1.

117. Let $A = \begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ x-y & z-x & x+y \end{vmatrix} = k\,xyz$ ...(i)

Consider $A = \begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ x-y & z-x & x+y \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2x & x-z & x-y \\ 2y & z+x & y-x \\ 2z & z-x & x+y \end{vmatrix}$$

Applying $R_2 \to R_2 + R_1, R_3 \to R_3 + R_1$

$$= \begin{vmatrix} 2x & x-z & x-y \\ 2(y+x) & 2x & 0 \\ 2(z+x) & 0 & 2x \end{vmatrix}$$

$$= 4\begin{vmatrix} 2x & x-z & x-y \\ y+x & x & 0 \\ z+x & 0 & x \end{vmatrix}$$

$$= 4[2x(x^2) - (x-z)(yx + x^2) + (x-y)(-xz - x^2)]$$

$$= 4[2x^3 - x^3 - yx^2 + zxy + zx^2 - x^2z - x^3 + xyz + x^2y]$$

$$= 4(2xyz) = 8xyz$$

$\therefore$ From Eq. (i),

$$8xyz = kxyz$$

$\Rightarrow$ $k = 8$

118. Let $f(x) = ax^2 + bx + c$

Given, $f(1) = f(-1)$

$\Rightarrow$ $a + b + c = a - b + c$

$\Rightarrow$ $b = 0$

$\therefore$ $f(x) = ax^2 + c$

$$f'(x) = 2ax$$
$$f'(a_1) = 2aa_1, \quad f'(a_2) = 2aa_2$$
$$f'(a_3) = 2aa_3$$

Since, a_1, a_2, a_3 are in AP

$\Rightarrow$ $2aa_1, 2aa_2, 2aa_3$ are in AP

$\Rightarrow$ $f'(a_1), f'(a_2), f'(a_3)$ are in AP.

119. Given, $x + y = e^{xy}$ ...(i)

On differentiating Eq. (i) w.r.t. x, we get,

$$1 + \frac{dy}{dx} = e^{xy}\left(y + x\frac{dy}{dx}\right)$$

$\Rightarrow$ $\frac{dy}{dx}(1 - xe^{xy}) = ye^{xy} - 1$

$\Rightarrow$ $\frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}}$

Since, tangent is parallel to y-axis, *ie*, $\frac{dy}{dx} = \infty$

$\Rightarrow$ $1 - xe^{xy} = 0$

$\Rightarrow$ $x = 1, y = 0$

$\therefore$ Coordinates of required point is (1, 0).

120. Given,

$f(x) = 1 + 2x^2 + 2^2x^4 + \ldots + 2^{10}x^{20}$ ...(i)

On differentiating Eq. (i) w.r.t. x, we get

$f'(x) = 4x + 4\cdot 2^4x^3 + \ldots 2^{10}\cdot 20x^{19}$

For extremum $f'(x) = 0$

$\Rightarrow$ $4x + 4\cdot 2^4x^3 + \ldots + 2^{10}\cdot 20\cdot x^{19} = 0$

$\Rightarrow$ $x = 0$

Also, $f''(x) > 0$ for $x = 0$

$\therefore$ $x = 0$ is the only point of minima.

121. Key Idea *$f(x)$ is decreasing in the given interval, if $f'(x) < 0$ in the given interval*

Given, $f(x) = \frac{4x^2 + 1}{x}$ ...(i)

On differentiating Eq.(i) w.r.t. x, we get

$$f'(x) = \frac{x(8x) - (4x^2 + 1)1}{x^2}$$

$\Rightarrow$ $f'(x) = \frac{4x^2 - 1}{x^2}$

For decreasing function, $f'(x) < 0$

$\Rightarrow$ $4x^2 - 1 < 0$

$\Rightarrow$ $4x^2 < 1$

$\Rightarrow$ $-\frac{1}{2} < x < \frac{1}{2}$.

$\therefore$ $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

NOTE $f(x)$ is increasing in the interval, if $f'(x) > 0$ in the interval.

122. Let V = volume of cylinder = $\pi r^2 h$ ...(i)

Total surface area, if one top is open

ie, $S = 2\pi rh + \pi r^2$ ...(ii)

From Eq. (ii),

$$h = \frac{S - \pi r^2}{2\pi r} \quad \ldots\text{(iii)}$$

$$\therefore \quad V = \pi r^2 \left(\frac{S - \pi r^2}{2\pi r}\right)$$

$$\Rightarrow \quad V = \frac{r}{2}(S - \pi r^2) \quad \ldots\text{(iv)}$$

On differentiating Eq. (iv) w.r.t. r, we get

$$\frac{dV}{dr} = \frac{r}{2}(-2\pi r) + \frac{1}{2}(S - \pi r^2)$$

$$= -\pi r^2 + \frac{1}{2}S - \frac{1}{2}\pi r^2$$

$$= -\frac{3}{2}\pi r^2 + \frac{1}{2}S$$

For greatest volume,

$$\frac{dV}{dr} = 0$$

$$\Rightarrow \quad -3\pi r^2 + S = 0$$

$$\Rightarrow \quad S = 3\pi r^2$$

On putting $S = 3\pi r^2$ in Eq. (iii), we get

$$h = \frac{3\pi r^2 - \pi r^2}{2\pi r} = r$$

$$\Rightarrow \quad h = r$$

123. Given, $f(x) = x^2 - 2x + 4$

$$f'(x) = 2x - 2$$

By applying Lagrange's mean value theorem

$$f'(c) = 2c - 2 = 0$$

$$\Rightarrow \quad c = 1$$

124. Given, $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$

$$\Rightarrow \quad k = \lim_{x \to 0} f(x)$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{1 - \cos 4x}{8x^2} = k$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{2\sin^2 2x}{2(2x)^2} = k$$

$$\Rightarrow \quad \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^2 = k$$

$$\Rightarrow \quad 1 = k$$

$\Rightarrow$ Required value of $k = 1$

NOTE A function $f(x)$ is continuous at $x = a$, if

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$$

125. Given

$$f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x \quad \ldots\text{(i)}$$

$$\therefore \quad f\left(\frac{\pi}{4}\right) = 0 \quad \ldots\text{(ii)}$$

$$\left(\because \cos\frac{\pi}{2} = 0\right)$$

Now, taking log on both sides of Eq. (i) we get

$$\log f(x) = \log \cos x + \log \cos 2x + \log \cos 4x + \log \cos 8x + \log \cos 16x$$

On differentiating both sides w.r.t. x, we get

$$\frac{1}{f(x)} f'(x) = \frac{d}{dx}[\log \cos x + \log \cos 2x + \ldots + \log \cos 16x]$$

$$\Rightarrow \quad f'(x) = f(x)\frac{d}{dx}[\log \cos x + \ldots + \log \cos 16x]$$

$$\Rightarrow \quad f'\left(\frac{\pi}{4}\right) = 0$$

$$\therefore \quad f\left(\frac{\pi}{4}\right) = 0$$

126. Given, $f(x) = \int \dfrac{x^2\,dx}{(1 + x^2)(1 + \sqrt{1 + x^2})}$

Given, $f(0) = 0$

Put, $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore \quad f(x) = \int \frac{\tan^2\theta \sec^2\theta}{\sec^2\theta(1 + \sec\theta)}\, d\theta$$

$$\Rightarrow \quad f(x) = \int \frac{\tan^2\theta}{1 + \sec\theta}\, d\theta$$

$$\Rightarrow \quad f(x) = \int \frac{\sec^2\theta - 1}{\sec\theta + 1}\, d\theta$$

$$\Rightarrow \quad f(x) = \int (\sec\theta - 1)\, d\theta$$

$$\Rightarrow \quad f(x) = \log(\sec\theta + \tan\theta) - \theta + c$$

$$\Rightarrow \quad f(x) = \log(\sqrt{1 + x^2} + x) - \tan^{-1} x + c$$

At $x = 0$,

$$f(0) = \log(1 + 0) - 0 + c$$

$$\Rightarrow \quad c = 0$$

$$\therefore \quad f(x) = \log(\sqrt{1 + x^2} + x) - \tan^{-1} x$$

$\therefore$ At $x = 1$,

$$f(1) = \log(\sqrt{2} + 1) - \frac{\pi}{4}$$

127. Let $I = \int_1^2 [f\{g(x)\}]^{-1} f'(g(x))\, g'(x)\, dx$

and $g(1) = g(2)$

Let $f(g(x)) = t$

$\Rightarrow f'(g(x))\, g'(x)\, dx = dt$

$\therefore I = \int_1^2 t^{-1}\, dt = [\log t]_1^2$

$= \log 2$

$\Rightarrow I = [\log f(g(x))]_1^2$

$= \log f(g(2)) - \log f(g(1))$

$= \log f(g(1)) - \log f(g(1))$

$[\because g(2) = g(1)]$

$\therefore I = 0$

128. **Key Idea** $\frac{d}{dx}\int_{\phi(x)}^{\Psi(x)} f(x)\, dx$

$= f(\Psi(x))\, \Psi'(x) - f(\phi(x))\, \phi'(x)$

Given that,

$\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$

On differentiating w.r.t. x, we get

$f(x) = 1 + (-x\, f(x))$

$\Rightarrow f(x)(1 + x) = 1$

$\Rightarrow f(x) = \frac{1}{1 + x}$

$\therefore f(1) = \frac{1}{2}$

129. Given, $f(x) = f(a + x)$

$\Rightarrow f(x)$ is periodic function with period a.

and $\int_0^a f(x)\, dx = k$

$\Rightarrow \int_0^{na} f(x)\, dx = n\int_0^a f(x)\, dx$

[$\because f(x)$ is periodic]

$= nk$

130. Given that, $\int_a^b x^3\, dx = 0$

and $\int_a^b x^2\, dx = \frac{2}{3}$

If we take $a = -1$ and $b = 1$, then it will satisfy the given integration.

131. **Key Idea** *If $\vec{\mathbf{a}}$ is obtained by vector $\vec{\mathbf{b}}$ through rotation of axes, then $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$*

Let $\vec{\mathbf{b}} = (a + 1)\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is obtained from $\vec{\mathbf{a}}$ by rotating the axes *ie*, $\vec{\mathbf{a}} = 2a\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$

Now, $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$

$\Rightarrow (a + 1)^2 + 1 = (2a)^2 + 1$

$\Rightarrow a^2 + 1 + 2a + 1 = 4a^2 + 1$

$\Rightarrow 3a^2 - 2a - 1 = 0$

$\Rightarrow a = 1, -\frac{1}{3}$

132. Let $\vec{\mathbf{b}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$

Given, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{c}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$.

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$

$\Rightarrow x + y + z = 3$ ...(i)

Also, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}}$

$$\Rightarrow \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$\Rightarrow z - y = 0,\quad z - x = 1,\quad y - x = -1$

$\Rightarrow z = y,\quad z = 1 - x,\quad x = 1 + y$

From Eq. (i)

$x + y + z = 3$

$\Rightarrow 1 + y + y + y = 3$

$\Rightarrow 3y = 2$

$\Rightarrow y = \frac{2}{3},\quad z = \frac{2}{3}$

and $x = 1 + \frac{2}{3} = \frac{5}{3}$

$\therefore$ Required vector is $\frac{5}{3}\hat{\mathbf{i}} + \frac{2}{3}\hat{\mathbf{j}} + \frac{2}{3}\hat{\mathbf{k}}$.

133. **Key Idea** *Area of parallelogram with $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OC}}$ as adjacent sides is $|\overrightarrow{\mathbf{OA}} \times \overrightarrow{\mathbf{OC}}|$.*

Given, $\overrightarrow{\mathbf{OA}} = \vec{\mathbf{a}}$, $\overrightarrow{\mathbf{OB}} = 10\vec{\mathbf{a}} + 2\vec{\mathbf{b}}$ and

$\overrightarrow{\mathbf{OC}} = \vec{\mathbf{b}}$

q = Area of parallelogram with $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OC}}$ as adjacent sides

$= |\overrightarrow{\mathbf{OA}} \times \overrightarrow{\mathbf{OC}}|$

$\therefore q = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$ ...(i)

p = area of quadrilateral $OABC$

$= \frac{1}{2}|\overrightarrow{\mathbf{OA}} \times \overrightarrow{\mathbf{OB}}| + \frac{1}{2}|\overrightarrow{\mathbf{OB}} \times \overrightarrow{\mathbf{OC}}|$

$= \frac{1}{2}|\vec{\mathbf{a}} \times (10\vec{\mathbf{a}} + 2\vec{\mathbf{b}})| + \frac{1}{2}|(10\vec{\mathbf{a}} + 2\vec{\mathbf{b}}) \times \vec{\mathbf{b}}|$

$= |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| + 5|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 6|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| \quad \ldots\text{(ii)}$

$\therefore \quad \dfrac{p}{q} = \dfrac{6|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|} = \dfrac{6}{1}$

[From Eqs. (i) and (ii)]

$\therefore \quad \dfrac{p}{q} = 6$

134. Key Idea $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ *are coplanar iff* $[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = 0$

Given, $A = (0, 2, 0)$, $B = (1, x, 0)$, $C = (1, 2, 0)$ and $D = (1, 2, 1)$ are four points.

Let O be the origin.

$\therefore$ The position vector of

$\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{j}}, \quad \overrightarrow{\mathbf{OB}} = \hat{\mathbf{i}} + x\hat{\mathbf{j}},$

$\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}, \quad \overrightarrow{\mathbf{OD}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AB}} = \hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}}, \quad \overrightarrow{\mathbf{BC}} = (2-x)\hat{\mathbf{j}},$

and $\overrightarrow{\mathbf{CD}} = \hat{\mathbf{k}}$

Since, these are coplanar.

$$\therefore \quad \begin{vmatrix} 1 & x-2 & 0 \\ 0 & 2-x & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(2-x) = 0$

$\Rightarrow \quad x = 2$

135. Key Idea *Work done*

= Total force · Net displacement

Total forces $= (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) + (\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Displacement $= \overrightarrow{\mathbf{AB}}$

$= (6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Word done = force · displacement

$= (2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= 8 + 1 = 9$ unit

136. Given equation is

$$x\frac{dy}{dx} + y = x\cos x + \sin x$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$

IF $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ Solution is

$$y \cdot x = \int (x\cos x + \sin x)\,dx$$

$\Rightarrow \quad y \cdot x = x\sin x + c$

At $\quad x = \dfrac{\pi}{2}, \quad y = 1$

$\Rightarrow \quad \dfrac{\pi}{2} = \dfrac{\pi}{2} + c$

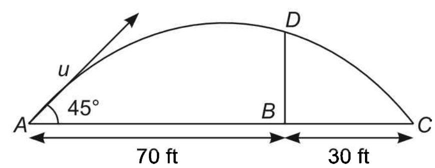
$\Rightarrow \quad c = 0$

$\therefore \quad xy = x\sin x$

$\Rightarrow$ $y = \sin x$ is the required solution.

137. Key Idea *If a particle is projected with velocity u and angle of projection* α*, then the horizontal range R is given by* $R = \dfrac{u^2}{g}\sin 2\alpha$ *and is maximum at* $\alpha = \dfrac{\pi}{4}$

$$R_{\max} = \frac{u^2}{g}$$



Let $x = 70$ ft, $\quad BD = y$ ft

Since, $\quad R_{\max} = 100$ ft

$\therefore \quad \dfrac{u^2}{g} = 100$

$\Rightarrow \quad u^2 = 100\,g \quad \ldots\text{(i)}$

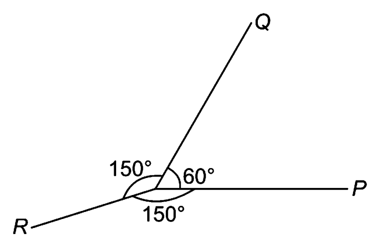
Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\Rightarrow \quad y = 70\tan 45° - \frac{g\,(70)^2}{2g\,100 \cdot \cos^2 45°}$$

$\Rightarrow \quad y = 21$ ft

138. Let three forces be P, Q, R are acting on a particle.



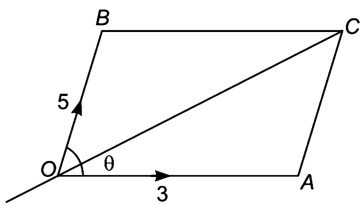
Using Lami's theorem

$$\frac{P}{\sin 150^\circ} = \frac{Q}{\sin 150^\circ} = \frac{R}{\sin 60^\circ}$$

$$\Rightarrow \quad \frac{P}{\frac{1}{2}} = \frac{Q}{\frac{1}{2}} = \frac{R}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \quad P : Q : R = 1 : 1 : \sqrt{3}$$

139. Let the three velocities act along the direction $\overrightarrow{\mathbf{OA}}$, $\overrightarrow{\mathbf{OB}}$ and $\overrightarrow{\mathbf{OC}}$. Let θ be the angle between $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OB}}$.



$\Rightarrow$ Resultant act along the direction $\overrightarrow{\mathbf{DC}}$

$$\Rightarrow \quad 7^2 = 3^2 + 5^2 + 2 \cdot 3 \cdot 5 \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

140. Let the boy meet the cyclist after t second.

$\therefore$ Distance travelled by cyclist in t second

$$= \frac{1}{2} t^2$$

Distance travelled by boy in t seconds $= 9t$

Initial distance between them $= \frac{81}{2}$

$$\Rightarrow \quad \frac{1}{2} t^2 + \frac{81}{2} = 9t$$

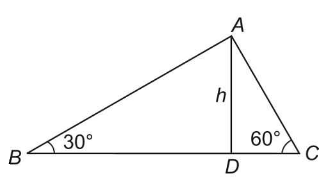
$$\Rightarrow \quad t^2 - 18t + 81 = 0$$

$$\Rightarrow \quad (t - 9)^2 = 0$$

$$\Rightarrow \quad t = 9 \text{ s}$$

$\Rightarrow$ They both meet after 9 s.

141. Let two plane be AB and AC. They inclined at angles of 30° and 60° to the horizontal and connected at A, $AD = h$.



In ΔABD, $\quad AB = h \operatorname{cosec} 30^\circ = 2h$

Acceleration of the bodies to slides on the two planes are $f_1 = g \sin 30^\circ = \frac{g}{2}$

and $\quad f_2 = g \sin 60^\circ = \sqrt{3}\,\frac{g}{2}$

Let v_1 and v_2 be the velocities at B and C.

Then, $$\frac{v_1^2}{v_2^2} = \frac{0 + 2f_1\, AB}{0 + 2f_2\, AC}$$

$$= \frac{g \cdot 2h}{\sqrt{3}\, g \left(\frac{2h}{\sqrt{3}}\right)} = 1$$

$$\therefore \quad v_1 : v_2 = 1 : 1$$

142. In a throw of dice probability of getting an even number $= \frac{3}{6} = \frac{1}{2} = p$

$$\Rightarrow \quad q = 1 - \frac{1}{2} = \frac{1}{2}$$

$\Rightarrow$ Probability of getting even number in odd number of trials.

$$= P(X = 1) + P(X = 3) + P(X = 5) + \ldots + P(X = 2n + 1)$$

$$= {}^{2n+1}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{2n} + {}^{2n+1}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{2n-2} + \ldots + {}^{2n+1}C_{2n+1} \left(\frac{1}{2}\right)^{2n+1} \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^{2n+1}} \left({}^{2n+1}C_1 + {}^{2n+1}C_3 + \ldots + {}^{2n+1}C_{2n+1}\right)$$

$$= \frac{1}{2^{2n+1}}\, 2^{2n+1-1} = \frac{1}{2}$$

$$[\because {}^nC_1 + {}^nC_3 + \ldots + {}^nC_n = 2^{n-1}]$$

143. P (exactly one of the event occur)

$$= P(A' \cap B) + P(A \cap B')$$

$$= P(A') - P(A' \cap B') + P(B') - P(A' \cap B')$$

$$= P(A') + P(B') - 2P(A' \cap B')$$

144. In out of five ticket, third ticket number be 20 as per given, therefore, we have to select 2 ticket before the number 20 and rest 2 ticket selected after numbered 20.

$\therefore$ Required probability $= \frac{{}^{19}C_2 \times {}^{10}C_2}{{}^{30}C_5}$

145. Given that $\quad P(A \cup B) = 0.6$

and $\quad P(A \cap B) = 0.2$

We know

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1 - P(\overline{A}) + 1 - P(\overline{B}) - P(A \cap B)$

$\Rightarrow \quad P(\overline{A}) + P(\overline{B}) = 2 - P(A \cap B) - P(A \cup B)$

$= 2 - 0.2 - 0.6$

$= 1.2$

146. Let relation $\equiv$ (mod n)

Reflexive Let a be any number, then $a \equiv a \pmod n$

$\Rightarrow \quad a - a = 0$ which is divisible by n

$\therefore \quad$ It is reflexive.

Symmetric Let a and b be any two numbers, then $a \equiv b \pmod n$

$\Rightarrow \quad a - b$ is divisible by n

$= (b - a)$ is divisible by n

$\therefore \quad a\,R\,b = b\,R\,a$

Transitive Let a, b and c be any three numbers, then

$a \equiv b \pmod n$ and $b \equiv c \pmod n$

$\Rightarrow a - b$ is divisible by n and $b - c$ is divisible by n.

$\Rightarrow a - b + b - c$ is divisible by n.

$\therefore$ It is transitive.

Hence, it is an equivalence relation.

147. The boolean function corresponding to given circuit is

$f(x_1, x_2, x_3) = (x'_1 \cdot x'_2 \cdot x_3) + (x_1 \cdot x'_2 \cdot x'_3) + (x'_1 \cdot x_2 \cdot x'_3)$

$\therefore \quad f(0, 0, 1) = (0' \cdot 0' \cdot 1) + (0 \cdot 0' \cdot 1') + (0' \cdot 0 \cdot 1')$

$= (1 \cdot 1 \cdot 1) + (0 \cdot 1 \cdot 0) + (1 \cdot 0 \cdot 0)$

$= 1 + 0 + 0 = 1$

148.

a	b	a'	$a' \wedge b$	$a \vee (a' \wedge b)$	$a \vee b$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

$\Rightarrow \quad a \vee (a' \wedge b) = a \vee b$

149. Given, $\quad y = \sqrt{(x-1)(3-x)}$

$\Rightarrow \quad y^2 = 3x - x^2 - 3 + x$

$\Rightarrow \quad x^2 - 4x + 3 + y^2 = 0$

This is a quadratic in x, we get

$$x = \frac{4 \pm \sqrt{16 - 4(3 + y^2)}}{2(1)}$$

$$\Rightarrow \quad x = \frac{4 \pm 2\sqrt{1 - y^2}}{2(1)}$$

Since, x is real.

$\therefore \quad 1 - y^2 \ge 0$

$\Rightarrow \quad y^2 \le 1$

$\Rightarrow \quad -1 \le y \le 1$

$\Rightarrow \quad y \in [-1, 1]$

150. Given that, $x^2 + bx + c = 0$

and $\quad b = 17$

Now, $\quad (x + 2)(x + 15) = x^2 + 17x + c$

On comparing both sides, we get

$c = 30$

$\therefore$ Correct equation is

$x^2 + 13x + 30 = 0$

$\Rightarrow \quad (x + 10)(x + 3) = 0$

$\Rightarrow \quad x = -10, -3$

$\therefore \quad$ Required correct roots are $-10, -3$.