1.2 THE FUNDAMENTAL EQUATION OF DYNAMICS

1.59 Let R be the constant upward thurst on the aerostat of mass m, coming down with a constant acceleration w. Applying Newton's second law of motion for the aerostat in projection form

$$F_y = mw_y$$

$$mg - R = mw \qquad (1)$$

Now, if Δm be the mass, to be dumped, then using the Eq. $F_y = mw_y$

$$R - (m - \Delta m)\, g = (m - \Delta m)\, w, \qquad (2)$$

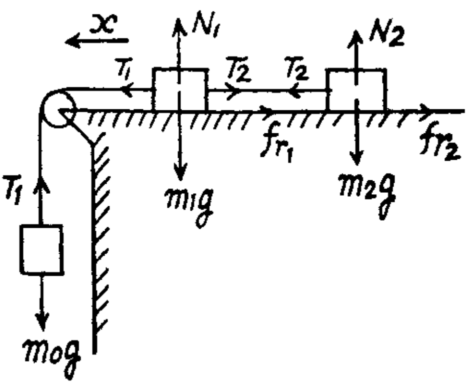
From Eqs. (1) and (2), we get, $\Delta m = \dfrac{2\, mw}{g + w}$

1.60 Let us write the fundamental equation of dynamics for all the three blocks in terms of projections, having taken the positive direction of x and y axes as shown in Fig; and using the fact that kinematical relation between the accelerations is such that the blocks move with same value of acceleration (say w)

$$m_0\, g - T_1 = m_0\, w \qquad (1)$$

$$T_1 - T_2 - km_1\, g = m_1\, w \qquad (2)$$

and $$T_2 - km_2\, g = m_2\, w \qquad (3)$$



The simultaneous solution of Eqs. (1), (2) and (3) yields,

$$w = g\, \frac{[\, m_0 - k\,(m_1 + m_2)\,]}{m_0 + m_1 + m_2}$$

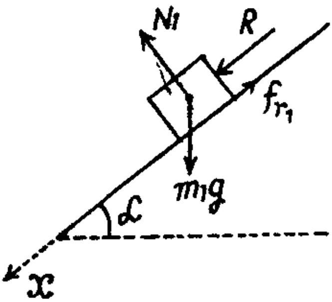
and $$T_2 = \frac{(1 + k)\, m_0}{m_0 + m_1 + m_2}\, m_2\, g$$

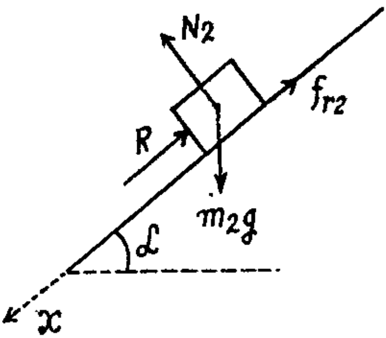
As the block m_0 moves down with acceleration w, so in vector form

$$\vec{w} = \frac{[m_0 - k\,(m_1 + m_2)\,]\,\vec{g}}{m_0 + m_1 + m_2}$$

1.61 Let us indicate the positive direction of x-axis along the incline (Fig.). Figures show the force diagram for the blocks.

Let, R be the force of interaction between the bars and they are obviously sliding down with the same constant acceleration w.





Newton's second law of motion in projection form along x-axis for the blocks gives :

$$m_1 g \sin\alpha - k_1 m_1 g \cos\alpha + R = m_1 w \quad (1)$$

$$m_2 g \sin\alpha - R - k_2 m_2 g \cos\alpha = m_2 w \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we get

$$w = g \sin\alpha - g \cos\alpha \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2} \text{ and}$$

$$R = \frac{m_1 m_2 (k_1 - k_2) g \cos\alpha}{m_1 + m_2} \quad (3)$$

(b) when the blocks just slide down the plane, $w = 0$, so from Eqn. (3)

$$g \sin\alpha - g \cos\alpha \frac{k_1 m_1 + k_2 m_2}{m_1 + m_2} = 0$$

$$\text{or,} \quad (m_1 + m_2) \sin\alpha = (k_1 m_1 + k_2 m_2) \cos\alpha$$

Hence $$\tan\alpha = \frac{(k_1 m_1 + k_2 m_2)}{m_1 + m_2}$$

1.62 Case 1. When the body is launched up :

Let k be the coefficeint of friction, u the velocity of projection and l the distance traversed along the incline. Retarding force on the block $= mg \sin\alpha + k\, mg \cos\alpha$ and hence the retardation $= g \sin\alpha + k\, g \cos\alpha$.

Using the equation of particle kinematics along the incline,

$$0 = u^2 - 2(g \sin\alpha + k\, g \cos\alpha)\, l$$

or, $$l = \frac{u^2}{2(g \sin\alpha + k\, g \cos\alpha)} \quad (1)$$

and $$0 = u - (g \sin\alpha + k\, g \cos\alpha)\, t$$

or, $$u = (g \sin\alpha + k\, g \cos\alpha)\, t \quad (2)$$

Using (2) in (1) $l = \frac{1}{2}(g \sin\alpha + k\, g \cos\alpha)\, t^2$ (3)

Case (2). When the block comes downward, the net force on the body $= mg \sin\alpha - km\, g \cos\alpha$ and hence its acceleration $= g \sin\alpha - k\, g \cos\alpha$

Let, t be the time required then,

$$l = \frac{1}{2}(g \sin\alpha - k\, g \cos\alpha)\, t'^2 \quad (4)$$

From Eqs. (3) and (4)

$$\frac{t^2}{t'^2} = \frac{\sin\alpha - k \cos\alpha}{\sin\alpha + k \cos\alpha}$$

But $\frac{t}{t'} = \frac{1}{\eta}$ (according to the question),

Hence on solving we get

$$k = \frac{(\eta^2 - 1)}{(\eta^2 + 1)} \tan\alpha = 0{\cdot}16$$

1.63 At the initial moment, obviously the tension in the thread connecting m_1 and m_2 equals the weight of m_2.

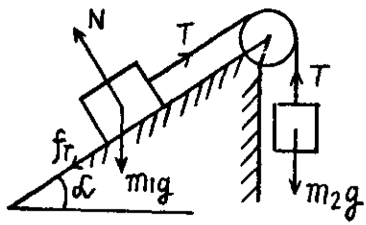
(a) For the block m_2 to come down or the block m_1 to go up, the conditions is

$$m_2 g - T \geq 0 \quad \text{and} \quad T - m_1 g \sin\alpha - fr \geq 0$$

where T is tension and f_r is friction which in the limiting case equals $km_1 g\cos\alpha$. Then

or $\quad m_2 g - m_1 \sin\alpha > k m_1 g \cos\alpha$

or $\quad \frac{m_2}{m_1} > (k\cos\alpha + \sin\alpha)$



(b) Similarly in the case

$$m_1 g \sin\alpha - m_2 g > fr_{\text{lim}}$$

or, $m_1 g \sin\alpha - m_2 g > km_1 g\cos\alpha$

or, $\quad \frac{m_2}{m_1} < (\sin\alpha - k\cos\alpha)$

(c) For this case, neither kind of motion is possible, and fr need not be limiting.

Hence, $\quad (k\cos\alpha + \sin\alpha) > \frac{m_2}{m_1} > (\sin\alpha - k\cos\alpha)$

1.64 From the conditions, obtained in the previous problem, first we will check whether the mass m_2 goes up or down.

Here, $m_2/m_1 = \eta > \sin\alpha + k\cos\alpha$, (substituting the values). Hence the mass m_2 will come down with an acceleration (say w). From the free body diagram of previous problem,

$$m_2 - g - T = m_2 w \tag{1}$$

and $\quad T - m_1 g \sin\alpha - k m_1 g \cos\alpha = m_1 w \quad (2)$

Adding (1) and (2), we get,

$$m_2 g - m_1 g \sin\alpha - k m_1 g \cos\alpha = (m_1 + m_2) w$$

$$w = \frac{(m_2/m_1 - \sin\alpha - k\cos\alpha) g}{(1 + m_2/m_1)} = \frac{(\eta - \sin\alpha - k\cos\alpha) g}{1 + \eta}$$

Substituting all the values, $w = 0{\cdot}048\,g = 0{\cdot}05\,g$

As m_2 moves down with acceleration of magnitude $w = 0.05\,g > 0$, thus in vector form acceleration of m_2 :

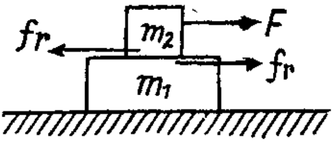
$$\vec{w}_2 = \frac{(\eta - \sin\alpha - k\cos\alpha)\vec{g}}{1 + \eta} = 0.05\,\vec{g}.$$

1.65 Let us write the Newton's second law in projection form along positive x-axis for the plank and the bar

$$fr = m_1 w_1\,,\ fr = m_2 w_2 \tag{1}$$

At the initial moment, fr represents the static friction, and as the force F grows so does the friction force fr, but up to it's limiting value i.e. $fr = fr_{s(max)} = kN = k\,m_2\,g$.

Unless this value is reached, both bodies moves as a single body with equal acceleration. But as soon as the force fr reaches the limit, the bar starts sliding over the plank i.e. $w_2 \geq w_1$.



Substituting here the values of w_1 and w_2 taken from Eq. (1) and taking into account that $f_r = k\,m_2\,g$, we obtain, $(at - km_2\,g)/m_2 \geq \frac{km_2}{m_1}\,g$, were the sign "=" corresponds to the moment $t = t_0$ (say)

Hence, $$t_0 = \frac{k\,g\,m_2\,(m_1 + m_2)}{a\,m_1}$$

If $$t \leq t_0, \text{ then } w_1 = \frac{km_2\,g}{m_1} \text{ (constant). and}$$

$$w_2 = (at - km_2\,g)/m_2$$

On this basis $w_1\,(t)$ and $w_2\,(t)$, plots are as shown in the figure of answersheet.

1.66 Let us designate the x-axis (Fig.) and apply $F_x = m\,w_x$ for body A :

$$mg\sin\alpha - k\,m\,g\cos\alpha = m\,w$$

or, $$w = g\sin\alpha - k\,g\cos\alpha$$

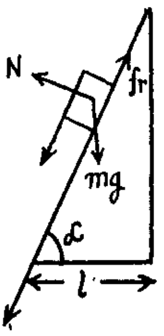
Now, from kinematical equation :

$$l\sec\alpha = 0 + (1/2)\,w\,t^2$$

or, $$t = \sqrt{2\,l\sec\alpha/(\sin\alpha - k\cos\alpha)\,g}$$

$$= \sqrt{2\,l/(\sin 2\alpha/2 - k\cos^2\alpha)\,g}$$

(using Eq. (1)).



for t_{min}, $$\frac{d\left(\frac{\sin 2\alpha}{2} - k\cos^2\alpha\right)}{d\alpha} = 0$$

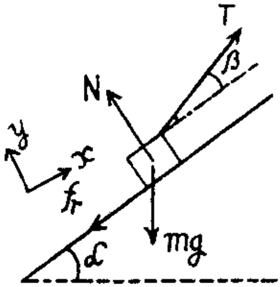
i.e. $$\frac{2\cos 2\alpha}{2} + 2\,k\cos\alpha\sin\alpha = 0$$

or, $$\tan 2\alpha = -\frac{1}{k} \Rightarrow \alpha = 49°$$

and putting the values of α, k and l in Eq. (2) we get $t_{min} = 1$s.

1.67 Let us fix the $x - y$ co-ordinate system to the wedge, taking the x – axis up, along the incline and the y – axis perpendicular to it (Fig.).

Now, we draw the free body diagram for the bar.



Let us apply Newton's second law in projection form along x and y axis for the bar :

$T \cos \beta - m g \sin \alpha - fr = 0$ (1)

$T \sin \beta + N - m g \cos \alpha = 0$

or, $N = m g \cos \alpha - T \sin \beta$ (2)

But $f_r = kN$ and using (2) in (1), we get

$$T = m g \sin \alpha + k\, mg \cos \alpha / (\cos \beta + k \sin \beta) \quad (3)$$

For T_{min} the value of $(\cos \beta + k \sin \beta)$ should be maximum

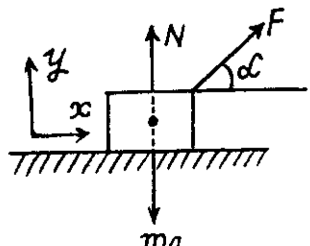
So, $$\frac{d (\cos \beta + k \sin \beta)}{d \beta} = 0 \text{ or } \tan \beta = k$$

Putting this value of β in Eq. (3) we get,

$$T_{min} = \frac{m g (\sin \alpha + k \cos \alpha)}{1/\sqrt{1+k^2} + k^2/\sqrt{1+k^2}} = \frac{m g (\sin \alpha + k \cos \alpha)}{\sqrt{1+k^2}}$$

1.68 First of all let us draw the free body diagram for the small body of mass m and indicate x – axis along the horizontal plane and y – axis, perpendicular to it, as shown in the figure. Let the block breaks off the plane at $t = t_0$ i.e. $N = 0$

So, $N = m g - a t_0 \sin \alpha = 0$



or, $$t_0 = \frac{mg}{a \sin \alpha} \quad (1)$$

From $F_x = m w_x$, for the body under investigation :

$m\, d v_x / dt = a t \cos \alpha$; Integrating within the limits for $v(t)$

$$m \int_0^v dv_x = a \cos \alpha \int_0 t\, dt \quad \text{(using Eq. 1)}$$

So, $$v = \frac{ds}{dt} = \frac{a \cos \alpha}{2m} t^2 \quad (2)$$

Integrating, Eqn. (2) for $s(t)$

$$s = \frac{a \cos \alpha}{2m} \frac{t^3}{3} \quad (3)$$

Using the value of $t = t_0$ from Eq. (1), into Eqs. (2) and (3)

$$v = \frac{m g^2 \cos \alpha}{2 a \sin^2 \alpha} \text{ and } s = \frac{m^2 g^3 \cos \alpha}{6 a^2 \sin^3 \alpha}$$

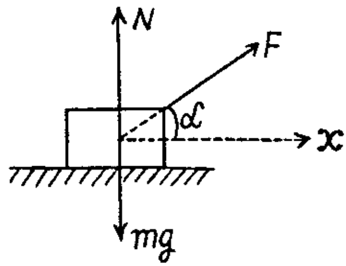
1.69 Newton's second law of motion in projection form, along horizontal or x – axis i.e. $F_x = m\,w_x$ gives.

$$F\cos(as) = m\,v\,\frac{dv}{ds} \quad (as\ \alpha = as)$$

or, $F\cos(as)\,ds = m\,v\,dv$

Integrating, over the limits for $v(s)$

$$\frac{F}{m}\int_0^{\infty}\cos(as)\,ds = \frac{v^2}{2}$$



or $$v = \sqrt{\frac{2F\sin\alpha}{m\,a}}$$

$$= \sqrt{2\,g\sin\alpha/3\,a}\ \text{(using } F = \frac{m\,g}{3}\text{)}$$

which is the sought relationship.

1.70 From the Newton's second law in projection from :

For the bar,

$$T - 2\,kmg = (2m)\,w \qquad (1)$$

For the motor,

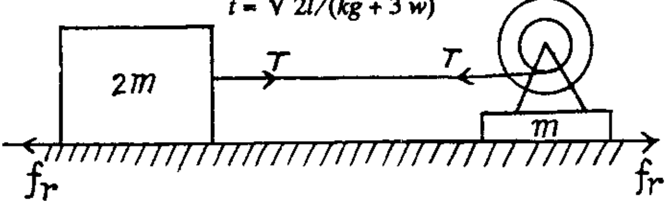
$$T - kmg = m\,w' \qquad (2)$$

Now, from the equation of kinematics in the frame of bar or motor :

$$l = \frac{1}{2}(w + w')\,t^2 \qquad (3)$$

From (1), (2) and (3) we get on eliminating T and w'

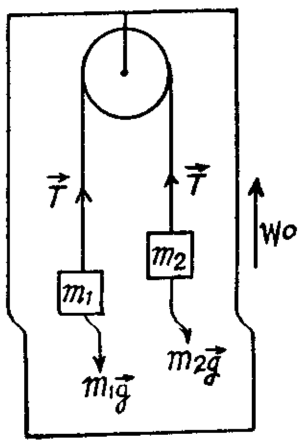
$$t = \sqrt{2l/(kg + 3\,w)}$$



1.71 Let us write Newton's second law in vector from $\vec{F} = m\,\vec{w}$, for both the blocks (in the frame of ground).

$$\vec{T} + m_1\,\vec{g} = m_1\,\vec{w}_1 \qquad (1)$$

$$\vec{T} + m_2\,\vec{g} = m_2\,\vec{w}_2 \qquad (2)$$



These two equations contain three unknown quantities $\vec{w}_1$, $\vec{w}_2$ and $\vec{T}$. The third equation is provided by the kinematic relationship between the accelerations :

$$\vec{w}_1 = \vec{w}_0 + \vec{w}', \quad \vec{w}_2 = \vec{w}_0 - \vec{w}' \qquad (3)$$

where $\vec{w}'$ is th acceleration of the mass m_1 with respect to the pulley or elevator car.

Summing up termwise the left hand and the right-hand sides of these kinematical equations, we get

$$\vec{w}_1 + \vec{w}_2 = 2\,\vec{w}_0 \qquad (4)$$

The simultaneous solution of Eqs. (1), (2) and (4) yields

$$\vec{w}_1 = \frac{(m_1 - m_2)\,\vec{g} + 2\,m_2\,\vec{w}_0}{m_1 + m_2}$$

Using this result in Eq. (3), we get,

$$\vec{w}' = \frac{m_1 - m_2}{m_1 + m_2}(\vec{g} - \vec{w}_0) \quad \text{and} \quad \vec{T} = \frac{2\,m_1\,m_2}{m_1 + m_2}(\vec{w}_0 - \vec{g})$$

Using the results in Eq. (3) we get $\vec{w}' = \dfrac{m_1 - m_2}{m_1 + m_2}(\vec{g} - \vec{w}_0)$

(b) obviously the force exerted by the pulley on the celing of the car

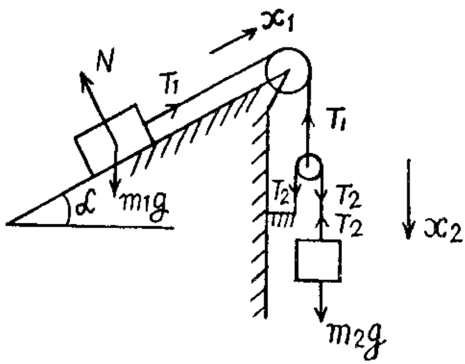
$$\vec{F} = -2\,\vec{T} = \frac{4\,m_1\,m_2}{m_1 + m_2}(\vec{g} - \vec{w}_0)$$

Note : *one could also solve this problem in the frame of elevator car.*

1.72 Let us write Newton's second law for both, bar 1 and body 2 in terms of projection having taken the positive direction of x_1 and x_2 as shown in the figure and assuming that body 2 starts sliding, say, upward along the incline

$$T_1 - m_1\,g\sin\alpha = m_1\,w_1 \qquad (1)$$

$$m_2\,g - T_2 = m_2\,w \qquad (2)$$



For the pulley, moving in vertical direction from the equation $F_x = m\,w_x$

$$2\,T_2 - T_1 = (m_p)\,w_1 = 0$$

(as mass of the pulley $m_p = 0$)

or $$T_1 = 2\,T_2 \qquad (3)$$

As the length of the threads are constant, the kinematical relationship of accelerations becomes

$$w = 2\,w_1 \qquad (4)$$

Simultaneous solutions of all these equations yields :

$$w = \frac{2\,g\left(2\dfrac{m_2}{m_1} - \sin\alpha\right)}{\left(4\dfrac{m_2}{m_1} + 1\right)} = \frac{2\,g\,(2\,\eta - \sin\alpha)}{(4\,\eta + 1)}$$

As $\eta > 1$, w is directed vertically downward, and hence in vector form

$$\vec{w} = \frac{2\,\vec{g}\,(2\,\eta - \sin\alpha)}{4\,\eta + 1}$$

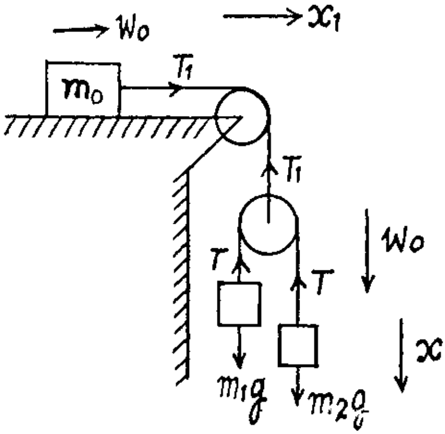
1.73 Let us write Newton's second law for masses m_1 and m_2 and moving pully in vertical direction along positive x – axis (Fig.) :

$$m_1 g - T = m_1 w_{1x} \qquad (1)$$

$$m_2 g - T = m_2 w_{2x} \qquad (2)$$

$$T_1 - 2T = 0 \text{ (as } m = 0)$$

or $$T_1 = 2T \qquad (3)$$



Again using Newton's second law in projection form for mass m_0 along positive x_1 direction (Fig.), we get

$$T_1 = m_0 w_0 \qquad (4)$$

The kinematical relationship between the accelerations of masses gives in terms of projection on the x – axis

$$w_{1x} + w_{2x} = 2 w_0 \qquad (5)$$

Simultaneous solution of the obtained five equations yields :

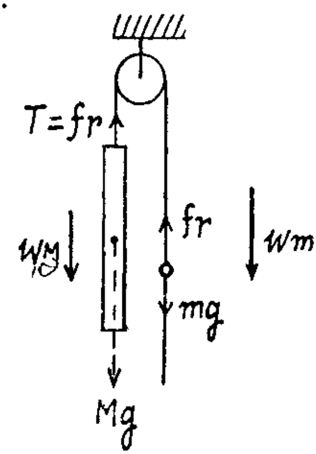
$$w_1 = \frac{[4 m_1 m_2 + m_0 (m_1 - m_2)] g}{4 m_1 m_2 + m_0 (m_1 + m_2)}$$

In vector form

$$\vec{w}_1 = \frac{[4 m_1 m_2 + m_0 (m_1 - m_2)] \vec{g}}{4 m_1 m_2 + m_0 (m_1 + m_2)}$$

1.74 As the thread is not tied with m, so if there were no friction between the thread and the ball m, the tension in the thread would be zero and as a result both bodies will have free fall motion. Obviously in the given problem it is the friction force exerted by the ball on the thread, which becomes the tension in the thread. From the condition or language of the problem $w_M > w_m$ and as both are directed downward so, relative acceleration of $M = w_M - w_m$ and is directed downward. Kinematical equation for the ball in the frame of rod in projection form along upward direction gives :

$$l = \frac{1}{2} (w_M - w_m) t^2 \qquad (1)$$



Newton's second law in projection form along vertically down direction for both, rod and ball gives,

$$Mg - fr = M w_M \qquad (2)$$

$$mg - fr = m w_m \qquad (3)$$

Multiplying Eq. (2) by m and Eq. (3) by M and then subtracting Eq. (3) from (2) and after using Eq. (1) we get

$$fr = \frac{2 l M m}{(M - m) t^2}$$

1.75 Suppose, the ball goes up with accleration w_1 and the rod comes down with the acceleration w_2.

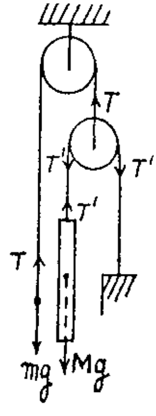
As the length of the thread is constant, $2\,w_1 = w_2$ (1)

From Newton's second law in projection form along vertically upward for the ball and vertically downward for the rod respectively gives,

$$T - m\,g = m\,w_1 \quad (2)$$

and $$M\,g - T' = M\,w_2 \quad (3)$$

but $$T = 2\,T \quad \text{(because pulley is massless)} \quad (4)$$



From Eqs. (1), (2), (3) and (4)

$$w_1 = \frac{(2M - m)\,g}{m + 4M} = \frac{(2 - \eta)\,g}{\eta + 4} \text{ (in upward direction)}$$

and $$w_2 = \frac{2\,(2 - \eta)\,g}{(\eta + 4)} \text{ (downwards)}$$

From kinematical equation in projection form, we get

$$l = \frac{1}{2}(w_1 + w_2)\,t^2$$

as, w_1 and w_2 are in the opposite direction.

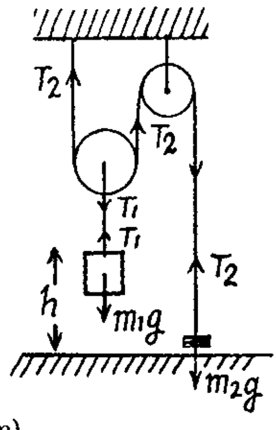
Putting the values of w_1 and w_2, the sought time becomes

$$t = \sqrt{2\,l\,(\eta + 4)\,/\,3\,(2 - \eta)\,g} = 1{\cdot}4\text{ s}$$

1.76 Using Newton's second law in projection form along x – axis for the body 1 and along negative x – axis for the body 2 respectively, we get

$$m_1\,g - T_1 = m_1\,w_1 \quad (1)$$

$$T_2 - m_2\,g = m_2\,w_2 \quad (2)$$



For the pulley lowering in downward direction from Newton's law along x axis,

$$T_1 - 2\,T_2 = 0 \text{ (as pulley is mass less)}$$

or, $$T_1 = 2\,T_2 \quad (3)$$

As the length of the thread is constant so,

$$w_2 = 2\,w_1 \quad (4)$$

The simultaneous solution of above equations yields,

$$w_2 = \frac{2\,(m_1 - 2m_2)\,g}{4\,m_2 + m_1} = \frac{2\,(\eta - 2)}{\eta + 4} \quad \left(\text{as } \frac{m_1}{m_2} = \eta\right) \quad (5)$$

Obviously during the time interval in which the body 1 comes to the horizontal floor covering the distance h, the body 2 moves upward the distance $2h$. At the moment when the body 2 is at the height $2h$ from the floor its velocity is given by the expression :

$$v_2^2 = 2\,w_2\,(2h) = 2\left[\frac{2\,(\eta - 2)\,g}{\eta + 4}\right]2h = \frac{8\,h\,(\eta - 2)\,g}{\eta + 4}$$

After the body m_1 touches the floor the thread becomes slack or the tension in the thread zero, thus as a result body 2 is only under gravity for it's subsequent motion.

Owing to the velocity v_2 at that moment or at the height $2h$ from the floor, the body 2 further goes up under gravity by the distance,

$$h' = \frac{v_2^2}{2g} = \frac{4h(\eta - 2)}{\eta + 4}$$

Thus the sought maximum height attained by the body 2 :

$$H = 2h + h' = 2h + \frac{4h(\eta - 2)}{(\eta + 4)} = \frac{6\eta h}{\eta + 4}$$

1.77 Let us draw free body diagram of each body, i.e. of rod A and of wedge B and also draw the kinemetical diagram for accelerations, after analysing the directions of motion of A and B. Kinematical relationship of accelarations is :

$$\tan\alpha = \frac{w_A}{w_B} \tag{1}$$

Let us write Newton's second law for both bodies in terms of projections having taken positive directions of y and x axes as shown in the figure.

$$m_A g - N\cos\alpha = m_A w_A \tag{2}$$

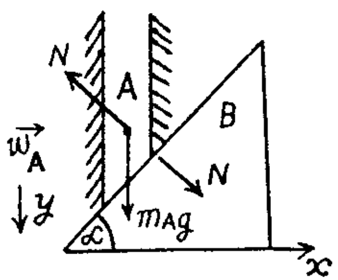
and

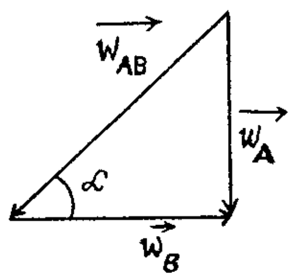
$$N\sin\alpha = m_B w_B \tag{3}$$

Simultaneous solution of (1), (2) and (3) yields :

$$w_A = \frac{m_A g \sin\alpha}{m_A \sin\alpha + m_B \cot\alpha \cos\alpha} = \frac{g}{(1 + \eta\cot^2\alpha)} \text{ and}$$

$$w_B = \frac{w_A}{\tan\alpha} = \frac{g}{(\tan\alpha + \eta\cot\alpha)}$$





Note : *We may also solve this problem using conservation of mechanical energy instead of Newton's second law.*

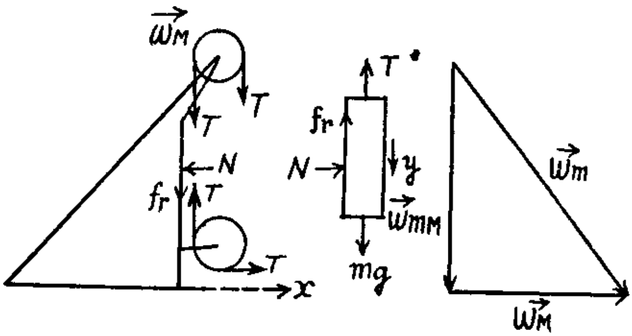
1.78 Let us draw free body diagram of each body and fix the coordinate system, as shown in the figure. After analysing the motion of M and m on the basis of force diagrams, let us draw the kinematical diagram for accelerations (Fig.).

As the length of threads are constant so,

$ds_{mM} = ds_M$ and as $\vec{v}_{mM}$ and $\vec{v}_M$ do not change their directions that why

$$|\vec{w}_{mM}| = |\vec{w}_M| = w \text{ (say) and}$$

$$\vec{w}_{mM} \uparrow\uparrow \vec{v}_M \text{ and } \vec{w}_M \uparrow\uparrow \vec{v}_M$$



As $\vec{w}_m = \vec{w}_{mM} + \vec{w}_M$

so, from the triangle law of vector addition

$$w_m = \sqrt{2}\, w \tag{1}$$

From the Eq. $F_x = m\, w_x$, for the wedge and block :

$$T - N = Mw\,, \tag{2}$$

and

$$N = mw \tag{3}$$

Now, from the Eq. $F_y = mw_y$, for the block

$$mg - T - kN = mw \tag{4}$$

Simultaneous solution of Eqs. (2), (3) and (4) yields :

$$w = \frac{mg}{(km + 2m + M)} = \frac{g}{(k + 2 + M/m)}$$

Hence using Eq. (1)

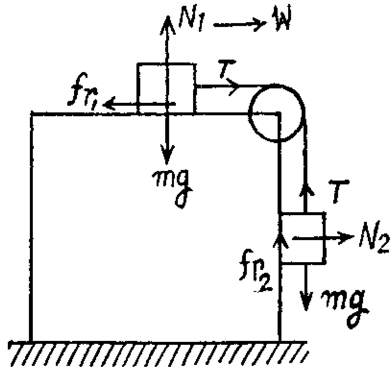
$$w_m = \frac{g\sqrt{2}}{(2 + k + M/m)}$$

1.79 Bodies 1 and 2 will remain at rest with repect to bar A for $w_{min} \le w \le w_{max}$, where w_{min} is the sought minimum acceleration of the bar. Beyond these limits there will be a relative motion between bar and the bodies. For $0 \le w \le w_{min}$, the tendency of body 1 in relation to the bar A is to move towards right and is in the opposite sense for $w \ge w_{max}$. On the basis of above argument the static friction on 2 by A is directed upward and on 1 by A is directed towards left for the purpose of calculating w_{min}.

Let us write Newton's second law for bodies 1 and 2 in terms of projection along positive x – axis (Fig.).

$$T - fr_1 = m\,w \quad \text{or,} \quad fr_1 = T - mw \tag{1}$$

$$N_2 = mw \tag{2}$$



As body 2 has no acceleration in vertical direction, so

$$fr_2 = mg - T \tag{3}$$

From (1) and (3)

$$(fr_1 + fr_2) = m\,(g - w) \tag{4}$$

But $\quad fr_1 + fr_2 \le k\,(N_1 + N_2)$

or $\quad fr_1 + fr_2 \le k\,(mg + mw) \qquad (5)$

From (4) and (5)

$$m(g-w) \le mk(g+w), \text{ or } w \ge \frac{g(1-k)}{(1+k)}$$

Hence

$$w_{\min} = \frac{g(1-k)}{(1+k)}$$

1.80 On the basis of the initial argument of the solution of 1.79, the tendency of bar 2 with respect to 1 will be to move up along the plane.

Let us fix $(x-y)$ coordinate system in the frame of ground as shown in the figure.

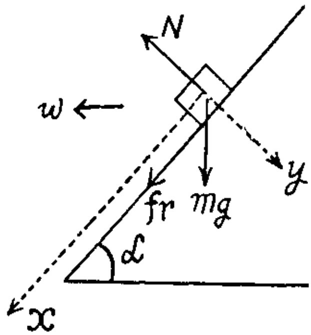
From second law of motion in projection form along y and x axes :

$m\,g\cos\alpha - N = m\,w\sin\alpha$

or, $N = m(g\cos\alpha - w\sin\alpha)$ (1)

$m\,g\sin\alpha + fr = m\,w\cos\alpha$

or, $fr = m(w\cos\alpha - g\sin\alpha)$ (2)



but $fr \le kN$, so from (1) and (2)

$(w\cos\alpha - g\sin\alpha) \le k(g\cos\alpha + w\sin\alpha)$

or, $w(\cos\alpha - k\sin\alpha) \le g(k\cos\alpha + \sin\alpha)$

or, $$w \le g\frac{(\cos\alpha + \sin\alpha)}{\cos\alpha - k\sin\alpha},$$

So, the sought maximum acceleration of the wedge :

$$w_{\max} = \frac{(k\cos\alpha + \sin\alpha)g}{\cos\alpha - k\sin\alpha} = \frac{(k\cot\alpha + 1)g}{\cot\alpha - k} \text{ where } \cot\alpha > k$$

1.81 Let us draw the force diagram of each body, and on this basis we observe that the prism moves towards right say with an acceleration $\vec{w}_1$ and the bar 2 of mass m_2 moves down the plane with respect to 1, say with acceleration $\vec{w}_{21}$, then, $\vec{w}_2 = \vec{w}_{21} + \vec{w}_1$ (Fig.)

Let us write Newton's second law for both bodies in projection form along positive y_2 and x_1 axes as shown in the Fig.

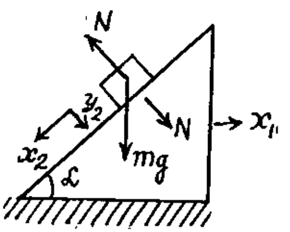
$$m_2 g\cos\alpha - N = m_2 w_{2(y_2)} = m_2\left[w_{21(y_2)} + w_{1(y_2)}\right] = m_2\left[0 + w_1\sin\alpha\right]$$

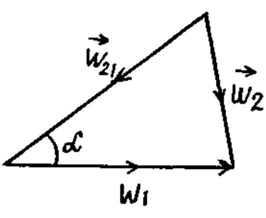
or, $$m_2 g\cos\alpha - N = m_2 w_1 \sin\alpha \quad (1)$$

and $$N\sin\alpha = m_1 w_1 \quad (2)$$

Solving (1) and (2), we get

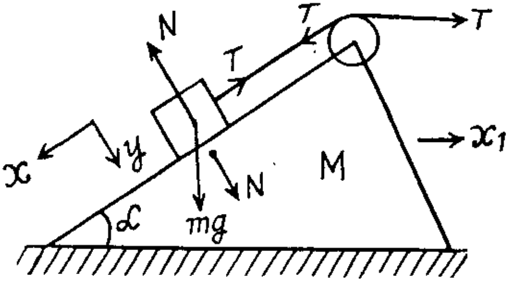
$$w_1 = \frac{m_2 g\sin\alpha\cos\alpha}{m_1 + m_2\sin^2\alpha} = \frac{g\sin\alpha\cos\alpha}{(m_1/m_2) + \sin^2\alpha}$$

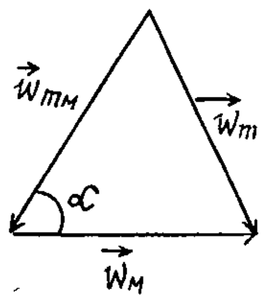




1.82 To analyse the kinematic relations between the bodies, sketch the force diagram of each body as shown in the figure.

On the basis of force diagram, it is obvious that the wedge M will move towards right and the block will move down along the wedge. As the length of the thread is constant, the distance travelled by the block on the wedge must be equal to the distance travelled by the wedge on the floor. Hence $d s_{mM} = d s_M$. As $\vec{v}_{mM}$ and $\vec{v}_M$ do not change their directions and acceleration that's why $\vec{w}_{mM} \uparrow\uparrow \vec{v}_{mM}$ and $\vec{w}_M \uparrow\uparrow \vec{v}_M$ and $w_{mM} = w_M = w$ (say) and accordingly the diagram of kinematical dependence is shown in figure.





As $\vec{w}_m = \vec{w}_{mM} + \vec{w}_M$, so from triangle law of vector addition.

$$w_m = \sqrt{w_M^2 + w_{mM}^2 - 2\, w_{mM}\, w_M \cos\alpha} = w\sqrt{2(1-\cos\alpha)} \qquad (1)$$

From $F_x = m\, w_x$, (for the wedge),

$$T - T\cos\alpha + N\sin\alpha = Mw \qquad (2)$$

For the bar m let us fix $(x - y)$ coordinate system in the frame of ground Newton's law in projection form along x and y axes (Fig.) gives

$$mg\sin\alpha - T = m\, w_{m(x)} = m\left[w_{mM(x)} + w_{M(x)}\right]$$

$$= m\left[w_{mM} + w_M\cos(\pi-\alpha)\right] = m\, w\,(1-\cos\alpha) \qquad (3)$$

$$m\, g\cos\alpha - N = m\, w_{m(y)} = m\left[w_{mM(y)} + w_{M(y)}\right] = m\,[\,0 + w\sin\alpha\,] \qquad (4)$$

Solving the above Eqs. simultaneously, we get

$$w = \frac{m\, g\sin\alpha}{M + 2m\,(1-\cos\alpha)}$$

Note : *We can study the motion of the block m in the frame of wedge also, alternately we may solve this problem using conservation of mechanical energy.*

1.83 Let us sketch the diagram for the motion of the particle of mass m along the circle of radius R and indicate x and y axis, as shown in the figure.

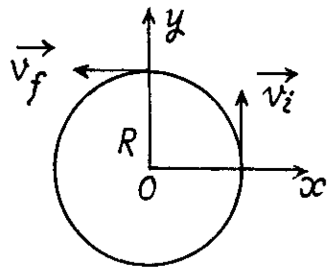
(a) For the particle, change in momentum $\Delta\vec{p} = mv(-\vec{i}) - m\,v(\vec{j})$

so, $\quad |\Delta\vec{p}| = \sqrt{2}\, m\, v$

and time taken in describing quarter of the circle,

$$\Delta t = \frac{\pi R}{2v}$$

Hence, $<\vec{F}> = \frac{|\Delta \vec{p}|}{\Delta t} = \frac{\sqrt{2}\, mv}{\pi R/2v} = \frac{2\sqrt{2}\, m v^2}{\pi R}$



(b) In this case

$\vec{p_i} = 0$ and $\vec{p_f} = m w_t t(-\vec{i})$,

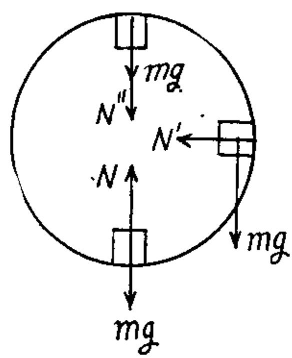
so $|\Delta \vec{p}| = m w_t t$

Hence, $|<\vec{F}>| = \frac{|\Delta \vec{p}|}{t} = m w_t$

1.84 While moving in a loop, normal reaction exerted by the flyer on the loop at different points and uncompensated weight if any contribute to the weight of flyer at those points.

(a) When the aircraft is at the lowermost point, Newton's second law of motion in projection form $F_n = m w_n$ gives

$$N - mg = \frac{m v^2}{R}$$



or, $N = mg + \frac{m v^2}{R} = 2{\cdot}09$ kN

(b) When it is at the upper most point, again from $F_n = m w_n$ we get

$$N'' + mg = \frac{mv^2}{R}$$

$$N'' = \frac{m v^2}{R} - m g = 0{\cdot}7 \text{ kN}$$

(c) When the aircraft is at the middle point of the loop, again from $F_n = m w_n$

$$N' = \frac{m v^2}{R} = 1{\cdot}4 \text{ kN}$$

The uncompensated weight is mg. Thus effective weight $= \sqrt{N'^2 + m^2 g^2} = 1{\cdot}56$ kN acts obliquely.

1.85 Let us depict the forces acting on the small sphere m, (at an arbitrary position when the thread makes an angle θ from the vertical) and write equation $\vec{F} = m \vec{w}$ via projection on the unit vectors $\hat{u}_t$ and $\hat{u}_n$. From $F_t = m w_t$, we have

$$mg \sin \theta = m \frac{dv}{dt}$$

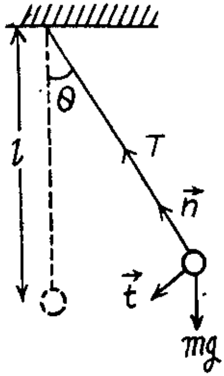
$$= m \frac{v\, dv}{ds} = m \frac{v\, dv}{l(-d\theta)}$$

(as vertical is refrence line of angular position) or $$v\,dv = -gl\sin\theta\, d\theta$$

Integrating both the sides :

$$\int_0^v v\,dv = -g\,l\int_{\pi/2}^{\theta}\sin\theta\, d\theta$$

or, $$\frac{v^2}{2} = g\,l\cos\theta$$



Hence $\frac{v^2}{l} = 2\,g\cos\theta = w_n$ (1)

(Eq. (1) can be easily obtained by the conservation of mechanical energy).

From $F_n = m\,w_n$

$$T - m\,g\cos\theta = \frac{m\,v^2}{l}$$

Using (1) we have

$$T = 3\,mg\cos\theta \quad (2)$$

Again from the Eq. $F_t = m\,w_t$:

$$mg\sin\theta = m\,w_t \text{ or } w_t = g\sin\theta \quad (3)$$

Hence $w = \sqrt{w_t^2 + w_n^2} = \sqrt{(g\sin\theta)^2 + (2g\cos\theta)^2}$ (using 1 and 3)

$$= g\sqrt{1 + 3\cos^2\theta}$$

(b) Vertical component of velocity, $v_y = v\sin\theta$

So, $$v_y^2 = v^2\sin^2\theta = 2g\,l\cos\theta\sin^2\theta \text{ (using 1)}$$

For maximum v_y or v_y^2, $\frac{d(\cos\theta\sin^2\theta)}{d\theta} = 0$

which yields $$\cos\theta = \frac{1}{\sqrt{3}}$$

Therefore from (2) $T = 3mg\frac{1}{\sqrt{3}} = \sqrt{3}\,mg$

(c) We have $\vec{w} = w_t\,\hat{u}_t + w_n\,\hat{u}_n$ thus $w_y = w_{t(y)} + w_{n(y)}$

But in accordance with the problem $w_y = 0$

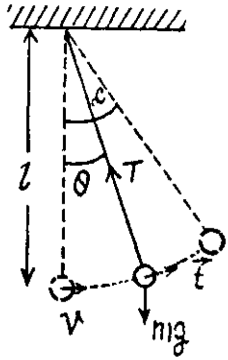
So, $$w_{t(y)} + w_{n(y)} = 0$$

or, $$g\sin\theta\sin\theta + 2g\cos^2\theta\,(-\cos\theta) = 0$$

or, $$\cos\theta = \frac{1}{\sqrt{3}} \text{ or, } \theta = 54{\cdot}7°$$

1.86 The ball has only normal acceleration at the lowest position and only tangential acceleration at any of the extreme position. Let v be the speed of the ball at its lowest position and l be the length of the thread, then according to the problem

$$\frac{v^2}{l} = g \sin \alpha \qquad (1)$$



where α is the maximum deflection angle

From Newton's law in projection form : $F_t = m w_t$

$$-mg \sin \theta = mv \frac{dv}{l\, d\theta}$$

or, $\quad -g\, l \sin \theta\, d\theta = v\, dv$

On integrating both the sides within their limits.

$$-gl \int_0^{\alpha} \sin \theta\, d\theta = \int_v^0 v\, dv$$

or, $\quad v^2 = 2gl\, (1 - \cos \alpha) \qquad (2)$

Note : Eq. (2) can easily be obtained by the conservation of mechanical energy of the ball in the uniform field of gravity.

From Eqs. (1) and (2) with $\theta = \alpha$

$$2gl\,(1 - \cos \alpha) = lg \cos \alpha$$

or, $\quad \cos \alpha = \frac{2}{3}$ so, $\alpha = 53°$

1.87 Let us depict the forces acting on the body A (which are the force of gravity $m\vec{g}$ and the normal reaction $\vec{N}$) and write equation $\vec{F} = m\vec{w}$ via projection on the unit vectors $\hat{u}_t$ and $\hat{u}_n$ (Fig.)

From $F_t = m w_t$

$$mg \sin \theta = m \frac{dv}{dt}$$

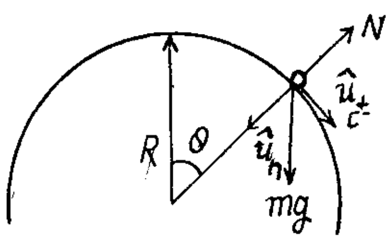
$$= m \frac{v\, dv}{ds} = m \frac{v\, dv}{R\, d\theta}$$

or, $\quad g R \sin \theta\, d\theta = v\, dv$

Integrating both side for obtaining $v(\theta)$

$$\int_0^{\theta} g R \sin \theta\, d\theta = \int_0^{v} v\, dv$$

or, $\quad v^2 = 2\, gR\, (1 - \cos \theta) \qquad (1)$



From $F_n = m w_n$

$$mg \cos \theta - N = m \frac{v^2}{R} \qquad (2)$$

At the moment the body loses contact with the surface, $N = 0$ and therefore the Eq. (2) becomes

$$v^2 = gR \cos \theta \qquad (3)$$

where v and θ correspond to the moment when the body loses contact with the surface. Solving Eqs. (1) and (3) we obtain $\cos\theta = \frac{2}{3}$ or, $\theta = \cos^{-1}(2/3)$ and $v = \sqrt{2gR/3}$.

1.88 At first draw the free body diagram of the device as, shown. The forces, acting on the sleeve are it's weight, acting vertically downward, spring force, along the length of the spring and normal reaction by the rod, perpendicular to its length.

Let F be the spring force, and Δl be the elongation.

From, $F_n = mw_n$:

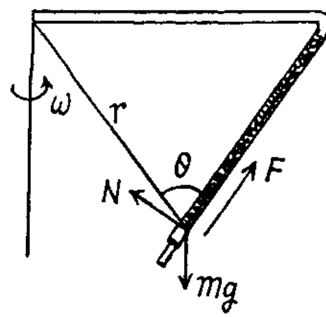
$$N\sin\theta + F\cos\theta = m\,\omega^2 r \tag{1}$$

where $r\cos\theta = (l_0 + \Delta l)$.

Similarly from $F_t = mw_t$

$$N\cos\theta - F\sin\theta = 0 \quad \text{or,} \quad N = F\sin\theta/\cos\theta \tag{2}$$

From (1) and (2)



$$F(\sin\theta/\cos\theta)\cdot\sin\theta + F\cos\theta = m\,\omega^2 r$$

$$= m\,\omega^2(l_0 + \Delta l)/\cos\theta$$

On putting $F = \kappa\,\Delta\,l$,

$$\kappa\,\Delta\,l\sin^2\theta + \kappa\,\Delta\,l\cos^2\theta = m\,\omega^2(l_0 + \Delta\,l)$$

on solving, we get,

$$\Delta l = m\,\omega^2\frac{l_0}{\kappa - m\,\omega^2} = \frac{l_0}{(\kappa/m\,\omega^2 - 1)}$$

and it is independent of the direction of rotation.

1.89 According to the question, the cyclist moves along the circular path and the centripetal force is provided by the frictional force. Thus from the equation $F_n = m\,w_n$

$$fr = \frac{m\,v^2}{r} \quad \text{or} \quad kmg = \frac{m\,v^2}{r}$$

or

$$k_0\left(1 - \frac{r}{R}\right)g = \frac{v^2}{r} \quad \text{or} \quad v^2 = k_0\,(r - r^2/R)\,g \tag{1}$$

For v_{max}, we should have $\dfrac{d\left(r - \dfrac{r^2}{R}\right)}{dr} = 0$

or,

$$1 - \frac{2r}{R} = 0, \quad \text{so } r = R/2$$

Hence $v_{max} = \frac{1}{2}\sqrt{k_0\,gR}$

1.90 As initial velocity is zero thus

$$v^2 = 2\,w_t\,s \tag{1}$$

As $w_t > 0$ the speed of the car increases with time or distance. Till the moment, sliding starts, the static friction provides the required centripetal acceleration to the car.

Thus

$$fr = mw, \quad \text{but} \quad fr \le kmg$$

So, $$w^2 \le k^2 g^2 \text{ or, } w_t^2 + \frac{v^2}{R} \le k^2 g^2$$

or, $$v^2 \le (k^2 g^2 - w_t^2) R$$

Hence $$v_{max} = \sqrt{(k^2 g^2 - w_t^2) R}$$

so, from Eqn. (1), the sought distance $s = \dfrac{v_{max}^2}{2 w_t} = \dfrac{1}{2}\sqrt{\left(\dfrac{kg}{w_t}\right)^2 - 1} = 60$ m.

1.91 Since the car follows a curve, so the maximum velocity at which it can ride without sliding at the point of minimum radius of curvature is the sought velocity and obviously in this case the static friction between the car and the road is limiting.

Hence from the equation $F_n = mw$

$$kmg \ge \frac{m v^2}{R} \quad \text{or} \quad v \le \sqrt{kRg}$$

so $$v_{max} = \sqrt{k R_{min} g}\,. \tag{1}$$

We know that, radius of curvature for a curve at any point (x, y) is given as,

$$R = \left| \frac{[1 + (dy/dx)^2]^{3/2}}{(d^2 y)/dx^2} \right| \tag{2}$$

For the given curve,

$$\frac{dy}{dx} = \frac{a}{\alpha}\cos\left(\frac{x}{\alpha}\right) \text{ and } \frac{d^2 y}{dx^2} = \frac{-a}{\alpha^2}\sin\frac{x}{\alpha}$$

Substituting this value in (2) we get,

$$R = \frac{[1 + (a^2/\alpha^2)\cos^2(x/\alpha)]^{3/2}}{(a/\alpha^2)\sin(x/\alpha)}$$

For the minimum R, $\dfrac{x}{\alpha} = \dfrac{\pi}{2}$

and therefore, corresponding radius of curvature

$$R_{min} = \frac{\alpha^2}{a} \tag{3}$$

Hence from (1) and (2)

$$v_{max} = \alpha\sqrt{kg/a}$$

1.92 The sought tensile stress acts on each element of the chain. Hence divide the chain into small, similar elements so that each element may be assumed as a particle. We consider one such element of mass dm, which subtends angle $d\alpha$ at the centre. The chain moves along a circle of known radius R with a known angular speed ω and certain forces act on it. We have to find one of these forces.

From Newton's second law in projection form, $F_x = mw_x$ we get

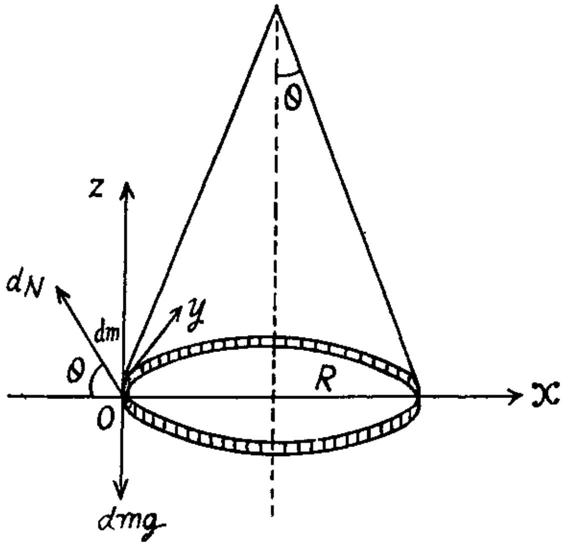
$$2T\sin(d\alpha/2) - dN\cos\theta = dm\,\omega^2 R$$

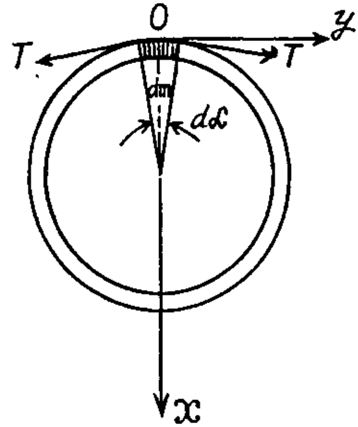
and from $F_Z = mw_Z$ we get

$$dN\sin\theta = g\,dm$$

Then putting $dm = m\,d\alpha/2\pi$ and $\sin(d\alpha/2) = d\alpha/2$ and solving, we get,

$$T = \frac{m(\omega^2 R + g\cot\theta)}{2\pi}$$





1.93 Let, us consider a small element of the thread and draw free body diagram for this element.

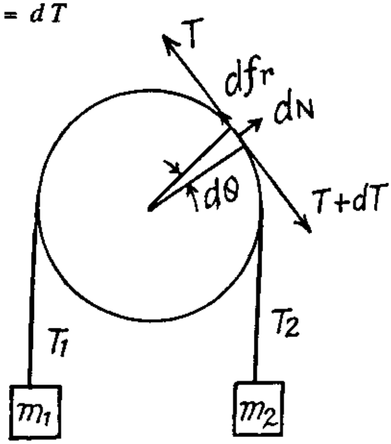
(a) Applying Newton's second law of motion in projection form, $F_n = mw_n$ for this element,

$(T + dT)\sin(d\theta/2) + T\sin(d\theta/2) - dN = dm\,\omega^2 R = 0$

or, $\quad 2T\sin(d\theta/2) = dN$, [negelecting the term$(dT\sin d\theta/2)$]

or, $$T\,d\theta = dN, \text{ as } \sin\frac{d\theta}{2} = \frac{d\theta}{2} \qquad (1)$$

Also, $$d\,fr = k\,dN = (T + dT) - T = dT \qquad (2)$$



From Eqs. (1) and (2),

$k\,T\,d\theta = d\,T$ or $\frac{dT}{T} = k\,d\theta$

In this case $Q = \pi$ so,

or, $\quad$ or, $\ln\frac{T_2}{T_1} = k\pi \qquad (3)$

So, $\quad k = \frac{1}{\pi}\ln\frac{T_2}{T_1} = \frac{1}{\pi}\ln\eta_0$

as $\quad \frac{T_2}{T_1} = \frac{m_2 g}{m_1 g} = \frac{m_2}{m_1} = \eta_0$

(b) When $\frac{m_2}{m_1} = \eta$, which is greater than η_0, the blocks will move with same value of acceleration. (say w) and clearly m_2 moves downward. From Newton's second law in projection form (downward for m_2 and upward for m_1) we get :

$$m_2 g - T_2 = m_2 w \qquad (4)$$

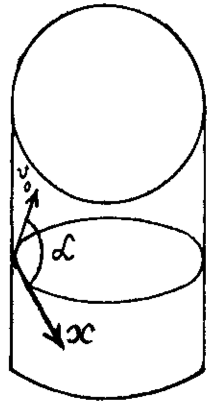
and $$T_1 - m_1 g = m_1 w \qquad (5)$$

Also $$\frac{T_2}{T_1} = \eta_0 \tag{6}$$

Simultaneous solution of Eqs. (4), (5) and (6) yields :

$$w = \frac{(m_2 - \eta_0 m_1)\,g}{(m_2 + \eta_0 m_1)} = \frac{(\eta - \eta_0)}{(\eta + \eta_0)}\,g \left(\text{as } \frac{m_2}{m_1} = \eta\right)$$

1.94 The force with which the cylinder wall acts on the particle will provide centripetal force necessary for the motion of the particle, and since there is no acceleration acting in the horizontal direction, horizontal component of the velocity will remain constant througout the motion.



So $$v_x = v_0 \cos\alpha$$

Using, $F_n = m\,w_n$, for the particle of mass m,

$$N = \frac{m\,v_x^2}{R} = \frac{m\,v_0^2 \cos^2\alpha}{R},$$

which is the required normal force.

1.95 Obviously the radius vector describing the position of the particle relative to the origin of coordinate is

$$\vec{r} = x\vec{i} + y\vec{j} = a\sin\omega t\,\vec{i} + b\cos\omega t\,\vec{j}$$

Differentiating twice with respect the time :

$$\vec{w} = \frac{d^2\vec{r}}{dt^2} = -\omega^2 (a\sin\omega t\,\vec{i} + b\cos\omega t\,\vec{j}) = -\omega^2\vec{r} \tag{1}$$

Thus $$\vec{F} = m\vec{w} = -m\omega^2\vec{r}$$

1.96 (a) We have $\Delta\vec{p} = \int \vec{F}\,dt$

$$= \int_0^t m\vec{g}\,dt = m\vec{g}\,t \tag{1}$$

(b) Using the solution of problem 1.28 (b), the total time of motion, $\tau = -\frac{2(\vec{v}_0\cdot\vec{g})}{g^2}$

Hence using $t = \tau$ in (1)

$$|\Delta\vec{p}| = mg\tau$$

$$= -2m(\vec{v}_0\cdot\vec{g})/g \quad (\vec{v}_0\cdot\vec{g} \text{ is -ve})$$

1.97 From the equation of the given time dependence force $\vec{F} = \vec{a}\,t(\tau - t)$ at $t = \tau$, the force vanishes,

(a) Thus $$\Delta\vec{p} = \vec{p} = \int_0^\tau \vec{F}\,dt$$

or, $$\vec{p} = \int_0^{\tau} \vec{a}\, t\,(\tau - t)\, dt \;\; \frac{\vec{a}\,\tau^3}{6}$$

but $$\vec{p} = m\,\vec{v} \text{ so } \vec{v} = \frac{\vec{a}\,\tau^3}{6m}$$

(b) Again from the equation $\vec{F} = m\,\vec{w}$

$$\vec{a}t\,(\tau - t) = m\,\frac{d\vec{v}}{dt}$$

or, $$\vec{a}(t\tau - t^2)\,dt = m\,d\vec{v}$$

Integrating within the limits for $\vec{v}(t)$,

$$\int_0^{t} \vec{a}(t\tau - t^2)\,dt = m\int_0^{\vec{v}} d\vec{v}$$

or, $$\vec{v} = \frac{\vec{a}}{m}\left(\frac{\tau\,t^2}{2} - \frac{t^3}{3}\right) = \frac{\vec{a}\,t^2}{m}\left(\frac{\tau}{2} - \frac{t}{3}\right)$$

Thus $$v = \frac{a\,t^2}{m}\left(\frac{\tau}{2} - \frac{t}{3}\right) \text{ for } t \le \tau$$

Hence distance covered during the time interval $t = \tau$,

$$s = \int_0^{\tau} v\,dt$$

$$= \int_0^{\tau} \frac{a\,t^2}{m}\left(\frac{\tau}{2} - \frac{t}{3}\right) dt = \frac{a}{m}\,\frac{\tau^4}{12}$$

1.98 We have $F = F_0 \sin \omega t$

or $$m\,\frac{d\vec{v}}{dt} = \vec{F}_0 \sin \omega t \text{ or } m\,d\vec{v} = \vec{F}_0 \sin \omega t\, dt$$

On integrating,

$$m\vec{v} = \frac{-\vec{F}_0}{\omega}\cos \omega t + C, \text{ (where } C \text{ is integration constant)}$$

When $$t = 0,\ v = 0, \text{ so } C = \frac{\vec{F}_0}{m\omega}$$

Hence, $\vec{v} = \dfrac{-\vec{F}_0}{m\,\omega}\cos \omega t + \dfrac{\vec{F}_0}{m\,\omega}$

As $|\cos \omega t \le 1$ so, $v = \dfrac{F_0}{m\,\omega}(1 - \cos \omega t)$

Thus $$s = \int_0^t v\,dt$$

$$= \frac{F_0 t}{m\omega} - \frac{F_0 \sin \omega t}{m\omega^2} = \frac{F_0}{m\omega^2}(\omega t - \sin \omega t)$$

(Figure in the answer sheet).

1.99 According to the problem, the force acting on the particle of mass m is, $\vec{F} = \vec{F}_0 \cos \omega t$

So, $$m\frac{d\vec{v}}{dt} = \vec{F}_0 \cos \omega t \quad \text{or} \quad d\vec{v} = \frac{\vec{F}_0}{m}\cos \omega t\, dt$$

Integrating, within the limits.

$$\int_0^{\vec{v}} d\vec{v} = \frac{\vec{F}_0}{m}\int_0^t \cos \omega t\, dt \quad \text{or} \quad \vec{v} = \frac{\vec{F}_0}{m\omega}\sin \omega t$$

It is clear from equation (1), that after starting at $t = 0$, the particle comes to rest fro the first time at $t = \frac{\pi}{\omega}$.

From Eq. (1), $v = |\vec{v}| = \frac{F_0}{m\omega}\sin \omega t$ for $t \le \frac{\pi}{\omega}$ (2)

Thus during the time interval $t = \pi/\omega$, the sought distance

$$s = \frac{F_0}{mw}\int_0^{\pi/\omega} \sin \omega t\, dt = \frac{2F}{m\omega^2}$$

From Eq. (1)

$$v_{max} = \frac{F_0}{m\omega} \text{ as } |\sin \omega t| \le 1$$

1.100 (a) From the problem $\vec{F} = -r\vec{v}$ so $m\frac{d\vec{v}}{dt} = -r\vec{v}$

Thus $$m\frac{dv}{dt} = -rv \; [\text{as } d\vec{v} \uparrow\downarrow \vec{v}]$$

or, $$\frac{dv}{v} = -\frac{r}{m}dt$$

On integrating $$\ln v = -\frac{r}{m}t + C$$

But at $t = 0$, $v = v_0$, so, $C = \ln v_0$

or, $$\ln\frac{v}{v_0} = -\frac{r}{m}t \quad \text{or,} \quad v = v_0 e^{-\frac{r}{m}t}$$

Thus for $t \to \infty$, $v = 0$

(b) We have $m\frac{dv}{dt} = -rv$ so $dv = \frac{-r}{m}ds$

Integrating within the given limits to obtain $v(s)$:

or, $$\int_{v_0}^{v} dv = -\frac{r}{m}\int_0^s ds \quad \text{or} \quad v = v_0 - \frac{rs}{m} \tag{1}$$

Thus for $$v = 0,\ s = s_{total} = \frac{m v_0}{r}$$

(c) Let we have $$\frac{m\,dv}{v} = -r\,v \quad \text{or} \quad \frac{dv}{v} = \frac{-r}{m}dt$$

or, $$\int_0^{v_0/\eta} \frac{dv}{v} = \frac{-r}{m}\int_0^t dt, \quad \text{or, } \ln\frac{v_0}{\eta v_0} = -\frac{r}{m}t$$

So $$t = \frac{-m\ln(1/\eta)}{r} = \frac{m\ln\eta}{r}$$

Now, average velocity over this time interval,

$$<v> = \frac{\int v\,dt}{\int dt} = \frac{\int_0^{\frac{m}{r}\ln\eta} v_0 e^{-\frac{rt}{m}}\,dt}{\frac{m}{r}\ln\eta} = \frac{v_0(\eta-1)}{\eta\ln\eta}$$

1.101 According to the problem

$$m\frac{dv}{dt} = -k\,v^2 \quad \text{or, } m\frac{dv}{v^2} = -k\,dt$$

Integrating, withing the limits,

$$\int_{v_0}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt \quad \text{or, } t = \frac{m}{k}\frac{(v_0 - v)}{v_0 v} \tag{1}$$

To fin. the value of k, rewrite

$$mv\frac{dv}{ds} = -k\,v^2 \quad \text{or, } \frac{dv}{v} = -\frac{k}{m}ds$$

On integrating

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{k}{m}\int_0^h ds$$

So, $$k = \frac{m}{h}\ln\frac{v_0}{v} \tag{2}$$

Putting the value of k from (2) in (1), we get

$$t = \frac{h(v_0 - v)}{v_0 v\ln\frac{v_0}{v}}$$

1.102 From Newton's second law for the bar in projection from, $F_x = m\,w_x$ along x direction we get

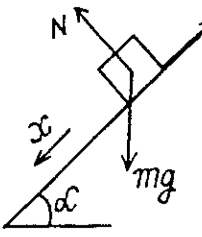
$$mg\sin\alpha - kmg\cos\alpha = mw$$

or, $$v\frac{dv}{dx} = g\sin\alpha - ax\,g\cos\alpha, \text{ (as } k = ax\text{)},$$

or, $$v\,dv = (g\sin\alpha - ax\,g\cos\alpha)\,dx$$

or, $$\int_0^v v\,dv = g\int_0^x (\sin\alpha - x\cos\alpha)\,dx$$

So, $$\frac{v^2}{2} = g\left(\sin\alpha\,x - \frac{x^2}{2}a\cos\alpha\right) \qquad (1)$$



From (1) $v = 0$ at either

$$x = 0, \text{ or } x = \frac{2}{a}\tan\alpha$$

As the motion of the bar is unidirectional it stops after going through a distance of $\frac{2}{a}\tan\alpha$.

From (1), for v_{max},

$$\frac{d}{dx}\left(\sin\alpha x - \frac{x^2}{2}a\cos\alpha\right) = 0, \text{ which yields } x = \frac{1}{a}\tan\alpha$$

Hence, the maximum velocity will be at the distance, $x = \tan\alpha/a$

Putting this value of x in (1) the maximum velocity,

$$v_{max} = \sqrt{\frac{g\sin\alpha\tan\alpha}{a}}$$

1.103 Since, the applied force is proportional to the time and the frictional force also exists, the motion does not start just after applying the force. The body starts its motion when F equals the limiting friction.

Let the motion start after time t_0, then

$$F = at_0 = kmg \text{ or, } t_0 = \frac{kmg}{a}$$

So, for $t = \le t_0$, the body remains at rest and for $t > t_0$ obviously

$$\frac{mdv}{dt} = a(t - t_0) \quad \text{or,} \quad m\,dv = a(t - t_0)\,dt$$

Integrating, and noting $v = 0$ at $t = t_0$, we have for $t > t_0$

$$\int_0^v m\,dv = a\int_{t_0}^t (t - t_0)\,dt \quad \text{or} \quad v = \frac{a}{2m}(t - t_0)^2$$

Thus $$s = \int v\,dt = \frac{a}{2m}\int_{t_0}^t (t - t_0)^2\,dt = \frac{a}{6m}(t - t_0)^3$$

1.104 While going upward, from Newton's second law in vertical direction :

$$m\,\frac{v\,dv}{ds} = -(mg + kv^2) \quad \text{or} \quad \frac{v\,dv}{\left(g + \frac{kv^2}{m}\right)} = -ds$$

At the maximum height h, the speed $v = 0$, so

$$\int_{v_0}^{0} \frac{v\,dv}{g + (kv^2/m)} = -\int_0^h ds$$

Integrating and solving, we get,

$$h = \frac{m}{2k}\ln\left(1 + \frac{kv_o^2}{mg}\right) \qquad (1)$$

When the body falls downward, the net force acting on the body in downward direction equals $(mg - kv^2)$,

Hence net acceleration, in downward direction, according to second law of motion

$$\frac{v\,dv}{ds} = g - \frac{kv^2}{m} \quad \text{or,} \quad \frac{v\,dv}{g - \frac{kv^2}{m}} = ds$$

Thus
$$\int_0^{v'} \frac{v\,dv}{g - kv^2/m} = \int_0^h ds$$

Integrating and putting the value of h from (1), we get,

$$v' = v_0 \Big/ \sqrt{1 + k\,v_0^2/mg}.$$

1.105 Let us fix $x - y$ co-ordinate system to the given plane, taking x-axis in the direction along which the force vector was oriented at the moment $t = 0$, then the fundamental equation of dynamics expressed via the projection on x and y-axes gives,

$$F\cos\omega t = m\,\frac{d\,v_x}{dt} \qquad (1)$$

and
$$F\sin\omega t = m\,\frac{d\,v_y}{dt} \qquad (2)$$

(a) Using the condition $v(0) = 0$, we obtain $v_x = \frac{F}{m\,\omega}\sin\omega t$ (3)

and

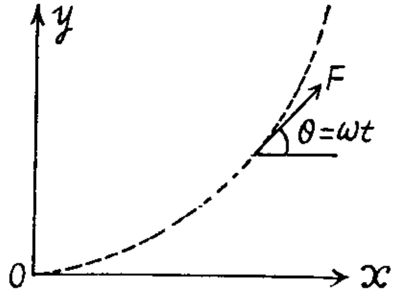
$$v_y = \frac{F}{m\,\omega}(1 - \cos\omega t) \qquad (4)$$

Hence,
$$v = \sqrt{v_x^2 + v_y^2} = \left(\frac{2F}{m\,\omega}\right)\left|\sin\left(\frac{\omega t}{2}\right)\right|$$

(b) It is seen from this that the velocity v turns into zero after the time interval Δt, which can be found from the relation, $\omega \frac{\Delta t}{2} = \pi$. Consequentely,

the sought distance, is

$$s = \int_0^{\Delta t} v\, dt = \frac{8F}{m\,\omega^2}$$



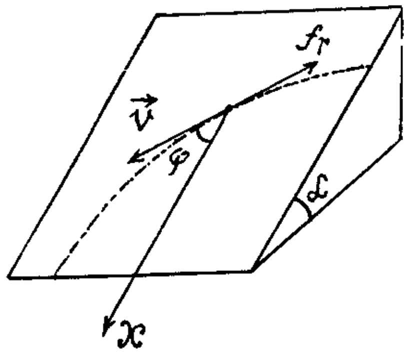
Average velocity, $< v > = \frac{\int v\, dt}{\int dt}$

So, $<v> = \int_0^{2\pi/\omega} \frac{2F}{m\,\omega} \sin\left(\frac{\omega\, t}{2}\right) dt \,/\, (2\,\pi\,\omega) = \frac{4F}{\pi\, m\,\omega}$

1.106 The acceleration of the disc along the plane is determined by the projection of the force of gravity on this plane $F_x = mg \sin\alpha$ and the friction force $fr = kmg \cos\alpha$. In our case $k = \tan\alpha$ and therefore

$$fr = F_x = mg \sin\alpha$$

Let us find the projection of the acceleration on the derection of the tangent to the trajectory and on the *x-axis* :



$m\, w_t = F_x \cos\varphi - fr = mg \sin\alpha\, (\cos\varphi - 1)$

$m\, w_x = F_x - fr \cos\varphi = mg \sin\alpha\, (1 - \cos\varphi)$

It is seen fromthis that $w_t = -w_x$, which means that the velocity v and its projection v_x differ only by a constant value C which does not change with time, i.e.

$$v = v_x + C,$$

where $v_x = v\cos\varphi$. The constant C is found from the initial condition $v = v_0$, whence $C = v_0$ since $\varphi = \frac{\pi}{2}$ initially. Finally we obtain

$$v = v_0 / (1 + \cos\varphi).$$

In the cource of time $\varphi \to 0$ and $v \to v_0/2$. (Motion then is unaccelerated.)

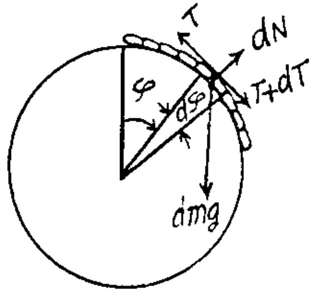
1.107 Let us consider an element of length ds at an angle φ from the vertical diameter. As the speed of this element is zero at initial instant of time, it's centripetal acceleration is zero, and hence, $dN - \lambda\, ds \cos\varphi = 0$, where λ is the linear mass density of the chain Let T and $T + dT$ be the tension at the upper and the lower ends of ds. we have from, $F_t = m\, w_t$

$$(T + dT) + \lambda\, ds\, g \sin\varphi - T = \lambda ds\, w_t$$

or, $$dT + \lambda\, Rd\varphi\, g \sin\varphi = \lambda ds\, w_t$$

If we sum the above equation for all elements, the term $\int dT = 0$ because there is no tension at the free ends, so

$$\lambda gR \int_0^{l/R} \sin\varphi \, d\varphi = \lambda w_t \int ds = \lambda l w_t$$



Hence $\quad w_t = \frac{gR}{l}\left(1 - \cos\frac{l}{R}\right)$

As $w_n = a$ at initial moment

So, $w = |w_t| = \frac{gR}{l}\left(1 - \cos\frac{l}{R}\right)$

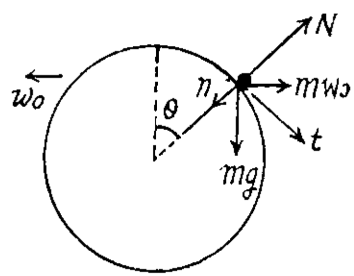
1.108 In the problem, we require the velocity of the body, realtive to the sphere, which itself moves with an acceleration w_0 in horizontal direction (say towards left). Hence it is advisible to solve the problem in the frame of sphere (non-inertial frame).

At an arbitary moment, when the body is at an angle θ with the vertical, we sketch the force diagram for the body and write the second law of motion in projection form $F_n = mw_n$

or, $mg\cos\theta - N - mw_0\sin\theta = \frac{mv^2}{R}$ (1)

At the break off point, $N = 0$, $\theta = \theta_0$ and let $v = v_0$, so the Eq. (1) becomes,

$$\frac{v_0^2}{R} = g\cos\theta_0 - w_0\sin\theta_0 \qquad (2)$$



From, $F_t = mw_t$

$$mg\sin\theta - mw_0\cos\theta = m\frac{v\,dv}{ds} = m\frac{v\,dv}{R\,d\theta}$$

or, $v\,dv = R\,(g\sin\theta + w_0\cos\theta)\,d\theta$

Integrating, $\int_0^{v_0} v\,dv = \int_0^{\theta_0} R\,(g\sin\theta + w_0\cos\theta)\,d\theta$

$$\frac{v_0^2}{2R} = g(1 - \cos\theta_0) + w_0\sin\theta_0 \qquad (3)$$

Note that the Eq. (3) can also be obtained by the work-energy theorem $A = \Delta T$ (in the frame of sphere)

therefore, $\quad mgR\,(1 - \cos\theta_0) + mw_0 R\sin\theta_0 = \frac{1}{2}mv_0^2$

[here $mw_0 R\sin\theta_0$ is the work done by the pseudoforce $(-m\vec{w}_o)$]

or, $$\frac{v_0^2}{2R} = g\,(1 - \cos\theta_0) + w_0\sin\theta_0$$

Solving Eqs. (2) and (3) we get,

$$v_0 = \sqrt{2gR/3} \text{ and } \theta_0 = \cos^{-1}\left[\frac{2+k\sqrt{5+9k^2}}{3(1+k^2)}\right], \text{ where } k = \frac{w_0}{g}$$

Hence $$\theta_0 \Big|_{w_0=g} = 17°$$

1.109 This is not central force problem unless the path is a circle about the said point. Rather here F_t (tangential force) vanishes. Thus equation of motion becomes,

$$v_t = v_0 = \text{constant}$$

and, $$\frac{mv_0^2}{r} = \frac{A}{r^2} \text{ for } r = r_0$$

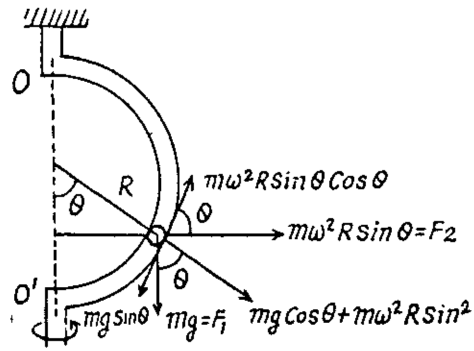
We can consider the latter equation as the equilibrium under two forces. When the motion is perturbed, we write $r = r_0 + x$ and the net force acting on the particle is,

$$-\frac{A}{(r_0+x)^n} + \frac{mv_0^2}{r_0+x} = \frac{-A}{r_0^n}\left(1-\frac{nx}{r_0}\right) + \frac{m\,v_0^2}{r_0}\left(1-\frac{x}{r_0}\right) = -\frac{m\,v_0^2}{r_0^2}(1-n)x$$

This is opposite to the displacement x, if $n < 1\cdot$ $\left(\frac{mv_0^2}{r}\right.$ is an outward directed centrifugul force while $\frac{-A}{r^n}$ is the inward directed external force).

1.110 There are two forces on the sleeve, the weight F_1 and the centrifugal force F_2. We resolve both forces into tangential and normal component then the net downward tangential force on the sleeve is,

$$mg\sin\theta\left(1-\frac{\omega^2 R}{g}\cos\theta\right)$$



This vanishes for $\theta = 0$ and for $\theta = \theta_0 = \cos^{-1}\left(\frac{g}{\omega^2 R}\right)$, which is real if $\omega^2 R > g$. If $\omega^2 R < g$, then $1 - \frac{\omega^2 R}{g}\cos\theta$ is always positive for small values of θ and hence the net tangential force near $\theta = 0$ opposes any displacement away from it. $\theta = 0$ is then stable.

If $\omega^2 R > g$, $1-\frac{\omega^2 R}{g}\cos\theta$ is negative for small θ near $\theta = 0$ and $\theta = 0$ is then unstable.

However $\theta = \theta_0$ is stable because the force tends to bring the sleeve near the equilibrium position $\theta = \theta_0$.

If $\omega^2 R = g$, the two positions coincide and becomes a stable equilibrium point.

1.111 Define the axes as shown with z along the local vertical, x due east and y due north. (We assume we are in the northern hemisphere). Then the Coriolis force has the components.

$$\vec{F}_{cor} = -2m(\vec{\omega} \times \vec{v})$$

$= 2m\omega\left[v_y\cos\theta - v_z\sin\theta)\vec{i} - v_x\cos\theta\,\vec{j} + v_x\cos\theta\,\vec{k}\right] = 2m\omega\,(v_y\cos\theta - v_z\sin\theta)\,\vec{i}$

since v_x is small when the direction in which the gun is fired is due north. Thus the equation of motion (neglecting centrifugal forces) are

$\ddot{x} = 2m\omega\,(v_y\sin\varphi - v_z\cos\varphi), \ddot{y} = 0$ and $\ddot{z} = -g$

Integrating we get $\dot{y} = v$ (constant), $\dot{z} = -gt$

and $\dot{x} = 2\omega v\sin\varphi\, t + \omega g\, t^2\cos\varphi$

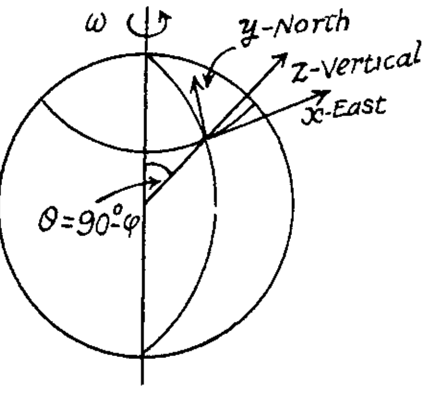
Finally,

$x = \omega v t^2\sin\varphi + \frac{1}{3}\omega g t^3\cos\varphi$

Now $v >> gt$ in the present case. so,

$$x = \omega v\sin\varphi\left(\frac{s}{v}\right)^2 = \omega\sin\varphi\frac{s^2}{v}$$

$$= 7 \text{ cm (to the east).}$$



1.112 The disc exerts three forces which are mutually perpendicular. They are the reaction of the weight, mg, vertically upward, the Coriolis force $2mv'\omega$ perpendicular to the plane of the vertical and along the diameter, and $m\omega^2 r$ outward along the diameter. The resultant force is,

$$F = m\sqrt{g^2 + \omega^4 r^2 + (2v'\omega)^2}$$

1.113 The sleeve is free to slide along the rod AB. Thus only the centrifugal force acts on it. The equation is,

$$m\dot{v} = m\omega^2 r \quad \text{where } v = \frac{dr}{dt}.$$

$$\text{But } \dot{v} = v\frac{dv}{dr} = \frac{d}{dr}\left(\frac{1}{2}v^2\right)$$

so, $$\frac{1}{2}v^2 = \frac{1}{2}\omega^2 r^2 + \text{constant}$$

or, $$v^2 = v_0^2 + \omega^2 r^2$$

v_0 being the initial velocity when $r = 0$. The Coriolis force is then,

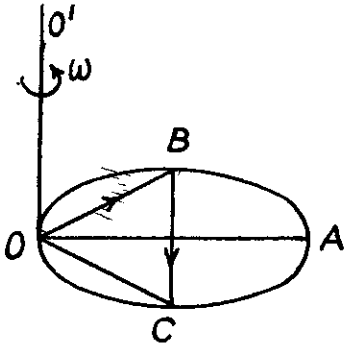
$$2m\omega\sqrt{v_0^2 + \omega^2 r^2} = 2m\omega^2 r\sqrt{1 + v_0^2/\omega^2 r^2}$$

$$= 2{\cdot}83 \text{ N on putting the values.}$$

1.114 The disc $OBAC$ is rotating with angular velocity ω about the axis OO' passing through the edge point O. The equation of motion in rotating frame is,

$$m\vec{w}' = \vec{F} + m\omega^2 \vec{R} + 2m\vec{v}' \times \vec{\omega} = \vec{F} + \vec{F}_{in}$$

where $\vec{F}_{in}$ is the resultant inertial forc. (pseudo force) which is the vector sum of centrifugal and Coriolis forces.



(a) At A, F_{in} vanishes. Thus $0 = -2m\omega^2 R\,\hat{n} + 2mv'\,\omega\,\hat{n}$

where $\hat{n}$ is the inward drawn unit vector to the centre from the point in question, here A.

Thus, $$v' = \omega R$$

so, $$w = \frac{v'^2}{\rho} = \frac{v'^2}{R} = \omega^2 R.$$

(b) At B $$\vec{F}_{in} = m\omega^2 \vec{OC} + m\omega^2 \vec{BC}$$

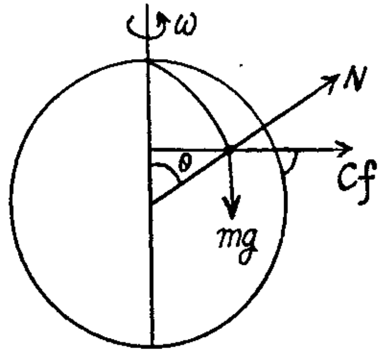
its magnitude is $m\omega^2\sqrt{4R^2 - r^2}$, where $r = OB$.

1.115 The equation of motion in the rotating coordinate system is,

$$m\,\vec{w}' = \vec{F} + m\omega^2 \vec{R} + 2m\,(\vec{v}' \times \vec{\omega})$$

Now, $$\vec{v}' = R\,\dot{\theta}\,\vec{e}_\theta + R\sin\theta\,\dot{\varphi}\,\vec{e}_\varphi$$

and $$\vec{w}' = w'\cos\theta\,\vec{e}_r - w'\sin\theta\,\vec{e}_\theta$$



$$\frac{1}{2m}\vec{F}_{cor} = \begin{vmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_\varphi \\ 0 & R\,\dot{\theta} & R\sin\theta\,\dot{\varphi} \\ \omega\cos\theta & -\omega\sin\theta & 0 \end{vmatrix}$$

$$= \vec{e}_r(\omega R\sin^2\theta\,\dot{\varphi}) + \omega R\sin\theta\cos\theta\,\dot{\varphi}\,\vec{e}_\theta - \omega R\,\dot{\theta}\cos\theta\,\vec{e}_\theta$$

Now on the sphere,

$$\vec{v}' = (-R\,\dot{\theta}^2 - R\sin^2\theta\,\varphi^2)\,\vec{e}_r$$
$$+ (R\,\ddot{\theta} - R\sin\theta\cos\theta\,\dot{\varphi}^2)\,\vec{e}_\theta$$
$$+ (R\sin\theta\ddot{\varphi} + 2R\cos\theta\,\dot{\theta}\,\dot{\varphi})\,\vec{e}_\varphi$$

Thus the equation of motion are,

$$m(-R\,\dot{\theta}^2 - R\sin^2\theta\,\dot{\varphi}^2) = N - mg\cos\theta + m\omega^2 R\sin^2\theta + 2m\omega R\sin^2\theta\,\dot{\varphi}$$

$$m(R\,\ddot{\theta} - R\sin\theta\cos\theta\,\dot{\varphi}^2) = mg\sin\theta + m\omega^2 R\sin\theta\cos\theta + 2m\omega R\sin\theta\cos\theta\,\dot{\varphi}$$

$$m(R\sin\theta\,\ddot{\varphi} + 2R\cos\theta\,\dot{\theta}\,\dot{\varphi}) = -2\,m\omega R\,\dot{\theta}\cos\theta$$

From the third equation, we get, $\dot{\varphi} = -\omega$

A result that is easy to understant by considering the motion in non-rotating frame. The eliminating $\dot{\varphi}$ we get,

$$mR\,\dot{\theta}^2 = mg\cos\theta - N$$
$$mR\,\ddot{\theta} = mg\sin\theta$$

Integrating the last equation,

$$\frac{1}{2}mR\,\dot{\theta}^2 = mg(1 - \cos\theta)$$

Hence $$N = (3 - 2\cos\theta)\, mg$$

So the body must fly off for $\theta = \theta_0 = \cos^{-1}\frac{2}{3}$, exactly as if the sphere were nonrotating.

Now, at this point F_{cf} = centrifugal force = $m\omega^2 R \sin\theta_0 = \sqrt{\frac{5}{9}}\, m\omega^2 R$

$$F_{cor} = \sqrt{\omega^2 R^2 \dot\theta^2 \cos^2\theta + (\omega^2 R^2)^2 \sin^2\theta} \times 2m$$

$$= \sqrt{\frac{5}{9}(\omega^2 R)^2 + \omega^2 R^2 \times \frac{4}{9} \times \frac{2g}{3R}} \times 2m = \frac{2}{3} m\omega^2 R \sqrt{5 + \frac{8g}{3\,\omega^2 R}}$$

1.116 (a) When the train is moving along a meridian only the Coriolis force has a lateral component and its magnitude (see the previous problem) is,

$$2m\,\omega\, v\cos\theta = 2m\,\omega\sin\lambda$$

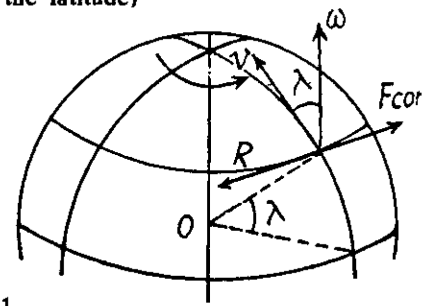
(Here we have put $R\,\dot\theta \to v$)

So, $$F_{lateral} = 2 \times 2000 \times 10^3 \times \frac{2\pi}{86400} \times \frac{54000}{3600} \times \frac{\sqrt{3}}{2}$$

$= 3{\cdot}77$ kN, (we write λ for the latitude)

(b) The resultant of the inertial forces acting on the train is,

$$\vec{F}_{in} = -2m\omega R\,\dot\theta\cos\theta\,\vec{e}_\varphi$$
$$+ (m\omega^2 R\sin\theta\cos\theta + 2m\,\omega R\sin\theta\cos\theta\,\dot\varphi)\,\vec{e}_\theta$$
$$+ (m\omega^2 R\sin^2\theta + 2m\,\omega R\sin^2\theta\,\dot\varphi)\,\vec{e}_r$$



This vanishes if $\dot\theta = 0,\ \dot\varphi = -\frac{1}{2}\omega$

Thus $$\vec{v} = v_\varphi\,\vec{e}_\varphi,\ v_\varphi = -\frac{1}{2}\omega R\sin\theta = -\frac{1}{2}\omega R\cos\lambda$$

(We write λ for the latitude here)

Thus the train must move from the east to west along the 60th parallel with a speed,

$$\frac{1}{2}\omega R\cos\lambda = \frac{1}{4} \times \frac{2\pi}{8{\cdot}64} \times 10^{-4} \times 6{\cdot}37 \times 10^6 = 115{\cdot}8 \text{ m/s} \approx 417 \text{ km/hr}$$

1.117 We go to the equation given in 1.111. Here $v_y = 0$ so we can take $y = 0$, thus we get for the motion in the $x\,z$ plane.

$$\ddot{x} = -2\omega\, v_z\cos\theta$$

and $$\ddot{z} = -g$$

Integrating, $$z = -\frac{1}{2}gt^2$$

$$\dot{x} = \omega\, g\cos\varphi t^2$$

So $$x = \frac{1}{3}\omega\, g\cos\varphi\, t^3 = \frac{1}{3}\omega\, g\cos\varphi\left(\frac{2h}{g}\right)^{3/2}$$

$$= \frac{2\,\omega h}{3}\cos\varphi\sqrt{\frac{2h}{g}}$$

There is thus a displacement to the east of

$$\frac{2}{3} \times \frac{2\pi}{8} 64 \times 500 \times 1 \times \sqrt{\frac{400}{9{\cdot}8}} \approx 26 \text{ cm.}$$