10. Real Numbers

Questions Pg-181

1. Question

Using the common form of rational numbers, prove that the sum, difference, product and quotient of any two rational numbers is again a rational number.

Answer

Sum of two rational numbers is a rational number:

As we know that, any rational number exists in the form of $\frac{p}{q}$ where p is the numerator and q is the denominator (q≠0), p and q are both integers.

Let us take two rational numbers as ‘a/b’ and ‘c/d’ where (b,d ≠0).

‘a’ and ‘c’ are the numerators while ‘b’ and ‘d’ are the

denominators. a,b,c and d are integers.

Sum of the above rational numbers = $\frac{a}{b} + \frac{c}{d}$

= $\frac{ad + cb}{bd}$ ....eq(1)

As we know that sum, product and division of two integers are

always integers.

So, (ad), (bc),(bd) and (ad + bc) are integer values.

Therefore, $\frac{ad + cb}{bd}$ is fraction with integers in the numerator

and denominator.

As we know that, by definition, a rational number can be expressed as a fraction with integer values in the numerator and denominator (denominator not zero).

So, $\frac{ad + cb}{bd}$ is a rational number (bd ≠0).

Therefore, Sum of two rational numbers is a rational number.

Difference of two rational numbers is a rational number.

Let us take two rational numbers as ‘$\frac{a}{b}$’ and ‘$\frac{c}{d}$’ where (b, d ≠0).

‘a’ and ‘c’ are the numerators while ‘b’ and ‘d’ are the

denominators. a,b,c and d are integers

Difference of the above rational numbers = $\frac{a}{b} - \frac{c}{d}$

= $\frac{ad-cb}{bd}$...........eq(1)

As we know that sum, product and division of two integers are

always integers.

So, (ad), (bc), (bd) and (ad-bc) are integer values.

Therefore, $\frac{ad-cb}{bd}$ is fraction with integer values in the numerator

and denominator.

By definition, a rational number can be expressed as a fraction with integer values in the numerator and denominator (denominator not zero).

So, $\frac{ad-cb}{bd}$ is a rational number (bd ≠0).

Therefore, difference of two rational numbers is a rational number.

Product of two rational numbers is a rational number.

Let us take two rational numbers as ‘a/b’ and ‘c/d’ where(b,d≠0).

‘a’ and ‘c’ are the numerators while ‘b’ and ‘d’ are the

denominators. a,b,c and d are integers.

Product of the numbers = $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right)$

$= \frac{ac}{bd}$

From the above statements, we can say that (ac) and (bd) are also integers with (bd ≠0).

So, ac/bd is a fraction with integer values in the numerator and denominator (denominator not zero) making it a rational number.

Quotient of any two rational numbers is again a rational number:

Let us take two rational numbers as ‘$\frac{a}{b}$’ and ‘$\frac{c}{d}$’ where(b, d≠0).

‘a’ and ‘c’ are the numerators while ‘b’ and ‘d’ are the

denominators. a,b,c and d are integers.

By, dividing the rational numbers we have = $\frac{\left(\frac{a}{b}\right)}{\frac{c}{d}}$

$= \left(\frac{a}{b}\right) \times \left(\frac{d}{c}\right)$

$= \frac{ad}{bc}$

From the above statements, we can say that (ad) and (bc) are also integers with (b,c ≠0).

So, ad/bc is a fraction with integer values.

Let $\frac{X}{Y} = \frac{ad}{bc}$.

Thus, X /Y can be expressed as a quotient of two integers and by definition, a rational number .

2. Question

Prove that the product of any irrational number and non-zero rational number is an irrational number.

Answer

Let us take an irrational number ‘k’ and a non-zero rational number $\frac{a}{b}$’ where (a,b≠ 0).

Assume that the product of an irrational number and non-zero rational number is an rational number.

Therefore, let $k \times \frac{a}{b} = \frac{c}{d}$

where c/d is another rational number.

$k \times \frac{a}{b} = \frac{c}{d}$

$\Rightarrow k = \frac{\left(\frac{c}{d}\right)}{\left(\frac{a}{b}\right)}$

$\Rightarrow k = \left(\frac{c}{d}\right) \times \left(\frac{b}{a}\right)$

$\Rightarrow k = \frac{cb}{ad} \Rightarrow$

We can say that (bc) and (ad) are also integers with (ad $\neq$0).

So, bc/ad is a fraction with integer values in the numerator and denominator (denominator not zero) making it a rational number.

This is a contradiction to the fact that ‘k’ as an irrational number.

So, the assumption is wrong.

Therefore, the product of any irrational number and non-zero rational number is an irrational number.

3. Question

Give an example of two different irrational numbers whose product is a rational number.

Answer

$8\sqrt{2}$ and $\sqrt{2}$

are two different irrational numbers.

Product of the two = $8\sqrt{2} \times \sqrt{2}$

= $8 \times (\sqrt{2} \times \sqrt{2})$

= $8 \times 2 = 16$

16 is a rational number.

Questions Pg-187

1. Question

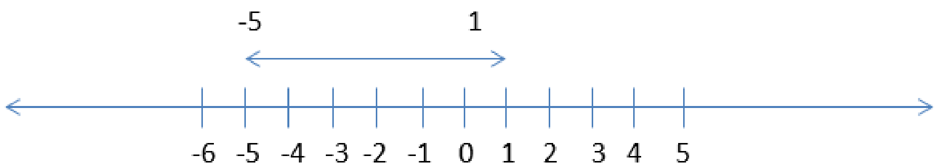
Find the distance between the two points on the number line, denoted by each pair of numbers given below:

i) 1, -5 ii) $\frac{1}{2}, \frac{2}{3}$ iii) $-\frac{1}{2}, -\frac{1}{3}$

iv) $-\frac{1}{2}, \frac{3}{4}$ v) $-\sqrt{2}, -\sqrt{3}$

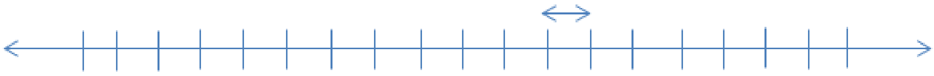
Answer

i) 1, -5 -5 1



As we know that the distance between the two points on the number line is the difference between the larger and the smaller number. Here, the larger no. = 1 the smaller no. = -5 Therefore, the distance between the points (1,-5) = {1-(-5)} = 1 + 5 = 6

ii) $\frac{1}{2}, \frac{2}{3}$

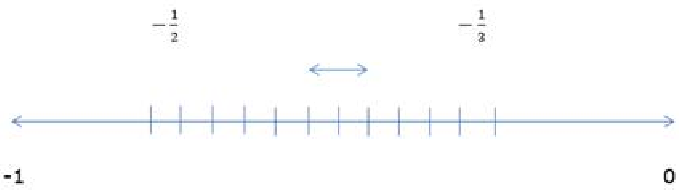


$-1\ 0\ \frac{1}{2}\ \frac{2}{3}\ 1\ 2$

As we know that the distance between the two points on the number line is the difference between the larger and the smaller number. Here, the larger no. = $\frac{2}{3}$ the smaller no. = $\frac{1}{2}$ Therefore, the distance between the points $(\frac{1}{2}, \frac{2}{3}) = (\frac{2}{3} - \frac{1}{2})$

$= \frac{4-3}{6} = \frac{1}{6}$

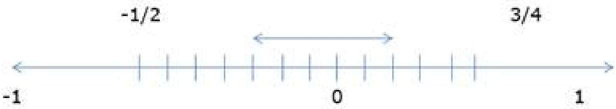
iii) $-\frac{1}{2}, -\frac{1}{3}$



As we know that the distance between the two points on the number line is the difference between the larger and the smaller number. Here, the larger no. = $-\frac{1}{3}$ the smaller no. = $-\frac{1}{2}$ Therefore, the distance between the points $(-\frac{1}{2}, -\frac{1}{3}) = \{-\frac{1}{3} - (-\frac{1}{2})\}$

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

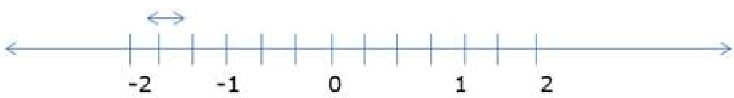
iv) $-\frac{1}{2}, \frac{3}{4}$



As we know that the distance between the two points on the number line is the difference between the larger and the smaller number. Here, the larger no. = $\frac{3}{4}$ the smaller no. = $-\frac{1}{2}$ Therefore, the distance between the points $(-\frac{1}{2}, \frac{3}{4}) = \{\frac{3}{4} - (-\frac{1}{2})\}$

$= \left(\frac{3}{4} + \frac{1}{2}\right) = \frac{5}{4}$

v) $-\sqrt{2}$, $-\sqrt{3}$



As we know that the distance between the two points on the number line is the difference between the larger and the smaller number. Here, the larger no. = $-\sqrt{2}$ = -1.414 (upto 3 decimal places) the smaller no. = $-\sqrt{3}$ = -1.732 (upto 3 decimal places) Therefore, the distance between the points ($-\sqrt{2}$, $-\sqrt{3}$) = $\{-\sqrt{2} - (-\sqrt{3})\}$

$= (-\sqrt{2} + \sqrt{3}) = -1.414 + 1.732$

= 0.318

2. Question

Find the midpoint of each pair of points in the first problem.

Answer

i) 1, -5

As we know that the midpoint of two points on the number line is half the sum of the numbers denoting these points.

Therefore midpoint of (1,-5) = $\frac{1}{2}$\{1 + (-5)\}

$= \frac{1}{2}(1-5)$

$= \frac{1}{2} \times (-4) = -2$

ii) $\frac{1}{2}, \frac{2}{3}$

As we know that the midpoint of two points on the number line is half the sum of the numbers denoting these points.

Therefore, midpoint of $(\frac{1}{2}, \frac{2}{3}) = \frac{1}{2}\{\frac{1}{2} + \frac{2}{3}\}$

$= \frac{1}{2}\{\frac{3 + 4}{6}\}$

$= \frac{1}{2} \times (\frac{7}{6}) = \frac{7}{12}$

iii) $-\frac{1}{2}, -\frac{1}{3}$

As we know that the midpoint of two points on the number line is half the sum of the numbers denoting these points.

Therefore, midpoint of $(-\frac{1}{2}, -\frac{1}{3}) = \frac{1}{2}\{-\frac{1}{2} - \frac{1}{3}\}$

$= \frac{1}{2}\{\frac{-3-2}{6}\}$

$= \frac{1}{2} \times (-\frac{5}{6}) = -\frac{5}{12}$

iv) $-\frac{1}{2}, \frac{3}{4}$

As we know that the midpoint of two points on the number line is half the sum of the numbers denoting these points.

Therefore, midpoint of $\left(-\frac{1}{2}, \frac{3}{4}\right) = \frac{1}{2}\{-\frac{1}{2} + \frac{3}{4}\}$

$= \frac{1}{2}\{\frac{-2 + 3}{4}\}$

$= \frac{1}{2} \times (\frac{1}{4}) = \frac{1}{8}$

v) $-\sqrt{2}, -\sqrt{3}$

As we know that the midpoint of two points on the number line is half the sum of the numbers denoting these points.

Therefore, midpoint of $(-\sqrt{2}, -\sqrt{3}) = \frac{1}{2}\{(-\sqrt{2} - \sqrt{3}\}$

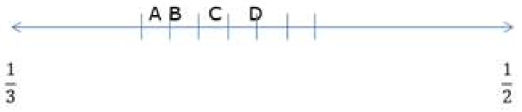
$= \frac{1}{2}(-1.414 - 1.732)$

$= \frac{1}{2} \times (-3.146) \ = \ 1.573$

3. Question

The part of the number line between the points denoted by the numbers $\frac{1}{3}$ & $\frac{1}{2}$ is divided into 4 parts. Find the numbers denoting the ends of such parts.

Answer



As we know that the distance between the two points on the number line is the difference between the larger and the smaller number.

So, distance between points $\left(\frac{1}{2}, \frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3}$

$= \frac{3-2}{6}$

$= \frac{1}{6}$

As per the question, this distance is to be divided into 4 parts.

So, each part would be of length $= \frac{\frac{1}{6}}{4}$

$= \frac{1}{24}$

1^{st} part $(\frac{1}{3}, A)$, where A $= \frac{1}{3} + \frac{1}{24}$

$= \frac{9}{24} \ (\frac{1}{2}, \frac{9}{24})$

2^{nd} part(A,B), where B $= \frac{9}{24} + \frac{1}{24}$

$= \frac{10}{24} \ (\frac{9}{24}, \frac{10}{24})$

3^{rd} part (B,C), where C $= \frac{10}{24} + \frac{1}{25}$

$= \frac{11}{24} \ (\frac{10}{24}, \frac{11}{24})$

4^{th} part (C,D), where D $= \frac{11}{24} + \frac{1}{24}$

$= \frac{12}{24}$

$= \frac{1}{2} \ (\frac{11}{24}, \frac{1}{2})$

Questions Pg-191

1 A. Question

Find those x satisfying each of the equations below:

|x – 1| = |x – 3|

Answer

.|x – 1| = |x – 3|

This can be solved in the following cases:

Case1 : when x>1, |x -1| = x -1 and x>3, |x-3| = x-3

x-1 = x-3 ⇒ no solution as x gets cancelled out on both sides……….eq(1)

Case2 : when x>1, |x-1| = x -1 and x<3, |x-3| = -(x-3)

x-1 = -(x-3)

⇒ x-1 = 3-x

⇒ 2x = 3 + 1

⇒ 2x = 4

⇒ x = 2………………..eq(2)

Case3 : when x<1, |x-1| = -(x-1) and x>3, |x-3| = x-3

-(x-1) = (x-3)

⇒ -x + 1 = x-3

⇒ -2x = -3-1 = -4

⇒ 2x = 4

⇒ x = 2………………..eq(3)

Case4 : when x<1, |x-1| = -(x-1) and x<3, |x-3| = -(x-3)

-(x-1) = -(x-3)

⇒ -x + 1 = -x + 3 ⇒ no solution as x gets cancelled out on both sides…………………..eq(4)

Now from eq(2) ans eq(3), we have x = 2 as the solution of the equation.

1 B. Question

Find those x satisfying each of the equations below:

|x – 3| = |x – 4|

Answer

This can be solved in the following cases:

Case1 : when x>3, |x-3| = x-3 and x>4, |x-4| = x-4

x-3 = x-4 ⇒ no solution as x gets cancelled out on both sides……….eq(1)

Case2 : when x>3, |x-3| = x-3 and x<4, |x-4| = -(x-4)

x-3 = -(x-4)

⇒ x-3 = 4-x

⇒ 2x = 4 + 3

⇒ 2x = 7

⇒ x = $\frac{7}{2}$………………..eq(2)

Case3 : when x<3, |x-3| = -(x-3) and x>4, |x- 4| = x-4

-(x-3) = (x-4)

⇒ -x + 3 = x-4

⇒ -2x = -4-3 = -7

⇒ 2x = 7

⇒ x = $\frac{7}{2}$...................eq(3)

Case4 : when x<3, |x-3| = -(x-3) and x<4, |x-4| = -(x-4)

-(x- 3) = -(x-4)

⇒ -x + 3 = -x + 4 ⇒ no solution as x gets cancelled out on both sides......................eq(4)

Now from eq(2) ans eq(3), we have x = $\frac{7}{2}$ as the solution of the equation.

1 C. Question

Find those x satisfying each of the equations below:

|x + 1| = |x – 5|s

Answer

This can be solved in the following cases:

Case1 : when x>-1, |x + 1| = x + 1 and x>5, |x-5| = x-5

x + 1 = x-5 ⇒ no solution as x gets cancelled out on both sides.........eq(1)

Case2 : when x>-1, |x + 1| = x + 1 and x<5, |x-5| = -(x-5)

x + 1 = -(x-5)

⇒ x + 1 = 5-x

⇒ 2x = 5-1

⇒ 2x = 4

⇒ x = 2...................eq(2)

Case3 : when x<-1, |x + 1| = -(x + 1) and x>5, |x-5| = x-5

-(x + 1) = (x-5)

⇒ -x-1 = x-5

⇒ -2x = -5-1 = -6

⇒ 2x = 6

⇒ x = 3...................eq(3)

Case4 : when x<-1, |x + 1| = -(x + 1) and x<5, |x-5| = -(x-5)

-(x + 1) = -(x-5)

⇒ -x-1 = -x + 5 ⇒ no solution as x gets cancelled out on both sides......................eq(4)

Now from eq(2) ans eq(3), we have x = 2 and x = 3 as the solution of the equation.

1 D. Question

Find those x satisfying each of the equations below:

|x| = |x + 1|

Answer

This can be solved in the following cases:

Case1 : when x>-1, |x + 1| = x + 1 and x>0, |x| = x

x = x + 1 ⇒ no solution as x gets cancelled out on both sides.........eq(1)

Case2 : when x>-1, |x + 1| = x + 1 and x<0, |x| = -x

x + 1 = -x

⇒ 2x = -1

⇒ x = -1/2..................eq(2)

Case3 : when x<-1, |x + 1| = -(x + 1) and x>0, |x| = x

-(x + 1) = x

⇒ -x-1 = x

⇒ -2x = 1

⇒ x = $-\frac{1}{2}$..................eq(3)

Case4 : when x<-1, |x + 1| = -(x + 1) and x<0, |x| = -x

-(x + 1) = -x

⇒ -x-1 = -x ⇒ no solution as x gets cancelled out on both sides.....................eq(4)

Now from eq(2) ans eq(3), we have x = $-\frac{1}{2}$ as the solution of the equation.

2. Question

Prove that if 1 < x < 4 and 1 < y < 4, then |x – y| < 3

Answer

Given that 1 < x < 4 and 1 < y < 4

1<x<4.........eq(1)

1<y<4

Multiplying by (-) sign to the above inequality

As we know that the inequality sign changes when multiplied by (-) sign.

Therefore, -1>-y>-4.........eq(2)

We can write eq(2) as -4<-y<-1

Now, adding eq(2) and eq(1)

We have, 1-4<x-y<4-1

Therefore, -3<x-y<3

So, by taking mod value of x-y we can write |x-y|<3.

3. Question

Prove that if x < 3 and y > 7, then |x – y| > 4

Answer

Given that x < 3 and y > 7

We have, y>7

Multiplying by (-) sign to the above inequality

As we know that the inequality sign changes when multiplied by (-) sign.

Therefore, -y<-7........eq(1)

Also, $x<3$........eq(2)

Now, adding eq(1) and eq(2).

We have, $x-y< -4$..........eq(3)

Again multiplying by (-) sign to the above inequality

We have, $-(x-y) > 4$.........eq(4)

By taking mod of x-y, we can say that $|x - y| > 4$.

4. Question

Find two numbers x, y such that $|x + y| = |x| + |y|$

Answer

Given, $|x + y| = |x| + |y|$

The above equation is valid for all $x,y \geq 0$.

Therefore any positive values of x and y will satisfy the above equation .

Let us take x = 2 and y = 3

LHS = $|x + y| = |2 + 3| = |5| = 5$

RHS = $|x| + |y| = |2| + |3| = 2 + 3 = 5$

Therefore, x = 2 and y = 3 are two values.

5. Question

Are there numbers x, y such that $|x + y| < |x| + |y|$?

Answer

To prove : $|x + y| < |x| + |y|$

We know that, $|x| \geq x$ and $|y| \geq y$

Therefore, $2|x||y| \geq 2xy$

Adding $x^2 + y^2$ to both sides,

We have, $x^2 + y^2 + 2|x||y| \geq x^2 + y^2 + 2xy$

$\Rightarrow |x|^2 + |y|^2 + 2|x||y| \geq x^2 + y^2 + 2xy$

$\Rightarrow (|x| + |y|)^2 \geq (x + y)^2$

$\Rightarrow (|x| + |y|)^2 \geq (|x + y|)^2$

$\Rightarrow |x| + |y| \geq |x + y|$

We can also say that $|x| + |y| > |x + y|$

Therefore, this inequality holds true for all x and y.

6. Question

Are there numbers x, y such that $|x + y| > |x| + |y|$?

Answer

There are no any numbers such that $|x + y| >|x| + |y|$

As the correct inequality is $|x| + |y| \geq |x + y|$ which is true for all x and y values.

7. Question

What are the numbers x, for which |x – 2| + |x – 8 | = 6?

Answer

To find the value of x we will take different cases.

Case1 : when x>2, |x -2| = x-2 and x>8, |x – 8| = x-8

x-2 + x-8 = 6

⇒ 2x-10 = 6

⇒ 2x = 6 + 10

⇒ 2x = 16

⇒ x = 8 ……….eq(1)

Case2 : when x>2, |x -2| = x-2 and x<8,|x – 8| = -(x-8)

(x-2) + (-x + 8) = 6

No solution as x gets cancelled out………..eq(2)

Case3 : when x<2, |x -2| = -(x-2) and x>8,|x – 8| = x-8

⇒ -x + 2 + x-8 = 6

No solution as x gets cancelled out ………………..eq(3)

Case4 : when x<2, |x -2| = -(x-2) and x<8,|x – 8| = -(x-8)

(-x + 2) + (-x + 8) = 6

⇒ -2x + 10 = 6

⇒ -2x = 6-10

⇒ -2x = -4 ⇒ x = 2 …………………..eq(4)

Now from eq(1) ans eq(4), we have x = 8 and x = 2 as the solution of the equation.

8. Question

What are the numbers x, for which |x – 2| + |x – 8| = 10?

Answer

To find the value of x we will take different cases.

Case1 : when x>2, |x -2| = x-2 and x>8, |x – 8| = x-8

x-2 + x-8 = 10

⇒ 2x-10 = 10

⇒ 2x = 10 + 10

⇒ 2x = 20

⇒ x = 10 ……….eq(1)

Case2 : when x>2, |x -2| = x-2 and x<8,|x – 8| = -(x-8)

(x-2) + (-x + 8) = 10

No solution as x gets cancelled out………..eq(2)

Case3 : when x<2, |x -2| = -(x-2) and x>8,|x – 8| = x-8

⇒ -x + 2 + x-8 = 10

No solution as x gets cancelled out ………………..eq(3)

Case4 : when $x<2$, $|x -2| = -(x-2)$ and $x<8$, $|x - 8| = -(x-8)$

$(-x + 2) + (-x + 8) = 10$

$\Rightarrow -2x + 10 = 10$

$\Rightarrow -2x = 10-10$

$\Rightarrow -2x = 0 \Rightarrow x = 0$ ……………………eq(4)

Now from eq(1) ans eq(4), we have $x = 10$ and $x = 0$ as the solution of the equation.