Chapter 1

Simple Stresses and Strains

CHAPTER HIGHLIGHTS

- *Simple Stresses and Strain*
- *Stress–Strain Relationship*
- *Hooke's Law and Modulus of Elasticity*
- *Bars with a Continuously Varying Cross Section (width vary from b1 to b2)*
- *Volumetric Strain of Rectangular Bar with Triaxial Loading*
- *Relationship between Modulus of Elasticity and Modulus of Rigidity*
- *Thermal Stresses*
- *Compound Bars or Bars of Composite Section*
- *Bars with Varying Loads*
- *Thermal Stresses in Compound Bars*
- *Elongation due to Self-weight*
- *Strain Energy*

Stress

When a member is subjected to loads, resisting forces are developed.

Each member is in equilibrium under the action of the applied forces and the internal resisting forces. When a section of the member is considered, the intensity of the resisting force normal to the sectional plane is called intensity of normal stress or simply normal stress.

$$\text{Stress} = p = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A} = \frac{dR}{dA}$$

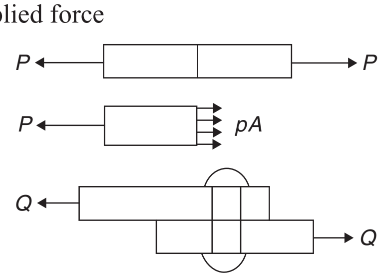
where R = resisting force
and A = cross-sectional area

$$\therefore \quad R = \int p\,dA \text{ and}$$

$$Q = \int q\,dA$$

where

Q = shear force
q = shear stress
P = applied force



Strain

Under the action of forces, members undergo a change in shape and size. This may be very minute or quite large depending upon the type of material. Under tensile forces, bars are elongated and under compressive forces, they are shortened.

The change in length per unit length is called strain.

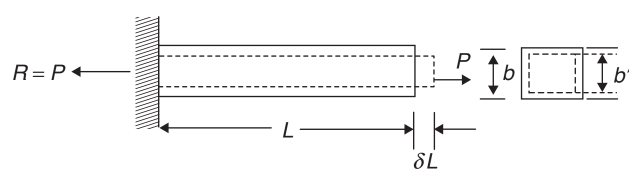
$$\text{Linear strain } e = \frac{\delta L}{L}$$

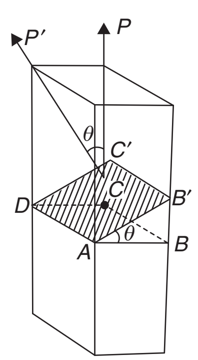
where L = original length
and δL = change in length

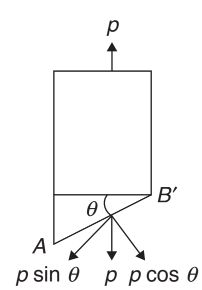
$$\text{Lateral strain} = \frac{\delta b}{b} = \frac{b - b'}{b} \text{ and } \frac{dt}{t} = \frac{t - t'}{t}$$

where, b is the breadth and t is the thickness.

Lateral dimension is the dimension perpendicular to the direction of load application.







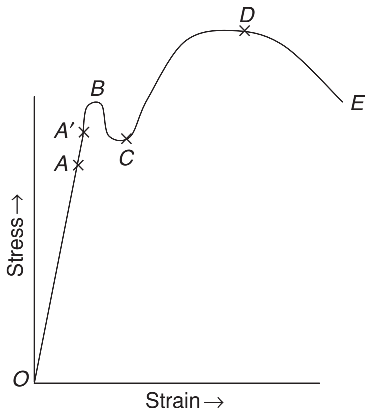
Area of $AB'C'D = \dfrac{ABCD}{\cos\theta}$

$$\sigma = \frac{p\cos^2\theta}{ABCD} = p\cos^2\theta$$

Net stress on the plane $AB'C'D = p\cos\theta$
where p is the stress normal to $ABCD$.

Stress–Strain Relationship

The stress–strain relationship can be plotted by conducting a test on a specimen using the universal testing machine (UTM). An extensometer can be used to measure the length variations.



In the initial portion OA, stress is directly proportional to strain. Point A is the limit of proportionality. Slightly beyond A, when the load is released, strain disappears completely and original length is regained. This point (A') is called as the elastic limit. Point B is the upper yield point and point C is the lower yield point. The horizontal portion, before point C is called as the yield plateau.

After the point C, strain hardening occurs. Point D represents the ultimate stress which is the maximum stress the material can resist. Here, the process of necking begins. Point E is the breaking point, the stress at which the specimen fails.

In some materials like aluminum and copper, there are no specific yield points.

In brittle materials, there are no yield points. For these materials, the ultimate point and the breaking point are same.

Factor of Safety

$$\text{Factor of safety} = FS = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

Factor of safety is used in a design process to avoid failures.

FS for steel = 1.85
FS for concrete = 3.00

Hooke's Law and Modulus of Elasticity

Hooke's law states that stress is proportional to strain up to the limit of proportionality in the elastic region.

i.e., $p \propto e$
$\therefore p = Ee$
where
E = constant of proportionality of the material
E is known as modulus of elasticity or Young's modulus.
From Hooke's law,

$$E = \frac{p}{e} = \frac{\frac{P}{A}}{\frac{\delta L}{L}} = \frac{PL}{A\delta L}$$

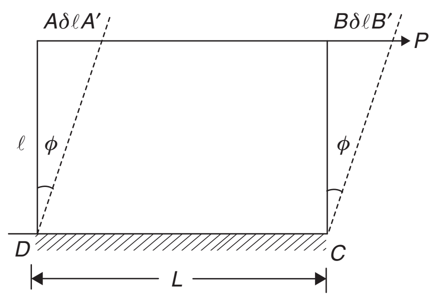
or
$$\delta L = \frac{PL}{AE}$$

Types of Strain

$$\text{Tensile strain } e_t = \frac{\text{increase in length}}{\text{original length}}$$

$$\text{Compressive strain } = e_c = \frac{\text{decrease in length}}{\text{original length}}$$

Shear strain is the angular deformation due to the shear forces.



Shear strain $\phi \simeq \tan\phi = \dfrac{\delta\ell}{\ell}$

Volumetric strain $= e_v = \dfrac{\delta v}{v} = e_x + e_y + e_z$

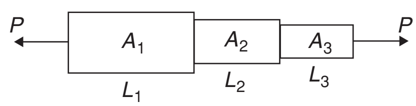
= sum of strains

in the x, y, and z directions of the body.

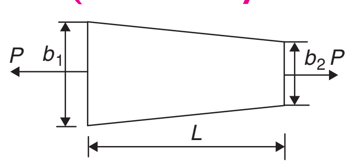
Bars of Varying Cross-sections

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3 + \cdots$$

$$= \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E} + \cdots$$



Bars with a Continuously Varying Cross-section (Width Vary from b_1 to b_2)



$$\delta L = \frac{PL}{tE(b_1 - b_2)} \log \frac{b_1}{b_2}$$

where t = thickness
b = breadth

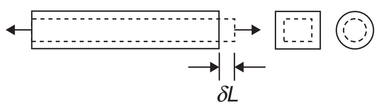
For a cylindrical rod when the diameter changes from d_1 to d_2,

$$\delta L = \frac{4PL}{\pi E d_1 d_2}$$

Poisson's Ratio

When a force is applied, there is a change in the dimension in the direction of application of the load. A change in dimension will occur in the lateral direction also.

If there is an expansion axially due to force acting in that direction, there is a contraction laterally and vice versa.



Within elastic limit, there is a constant ratio between lateral strain and longitudinal strain. This is called as Poisson's ratio.

That is, Poisson's ratio, $\frac{1}{m}$ or $\mu = \frac{\text{lateral strain}}{\text{linear strain}}$

Elastic Constants

The elastic constants are modulus of elasticity (or Young's modulus), modulus of rigidity and bulk modulus.

Modulus of elasticity is already explained along with Hook's law as the ratio of linear stress to linear strain with in elastic limit. It is denoted by letter E.

Modulus of rigidity is the ratio of shear stress to shear strain with in elastic limit. It is denoted by letter G.

Therefore,

$$G = \frac{q}{\phi}$$

where q = shear stress (sometimes denoted as τ)
and φ = shear strain

As already explained, shear strain is the angular deformation due to shear forces.

Bulk modulus is the ratio of identical stresses p acting in three mutually perpendicular directions on a body to the corresponding volumetric strain e_v. It is denoted by the letter K.

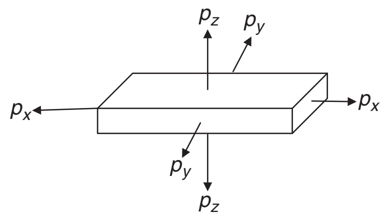
Therefore, $K = \frac{p}{e_v}$

where e_v = volumetric strain $= \frac{\Delta v}{V}$

$= \frac{\text{change in volume}}{\text{original volume}}$

$= e_x + e_y + e_z$

Volumetric Strain of Rectangular Bar with Triaxial Loading



Let stresses p_x, p_y and p_z act on 3 mutually perpendicular directions x, y, z as shown in the figure.

Change in length in the x direction is due to strain due to p_x and lateral strains due to p_y and p_z.

$$\frac{-\text{lateral strain}}{\text{linear strain}} = \mu$$

(– sign due to reduction in length)

Change in length due to p_x

$$\delta \ell_1 = \frac{p_x}{E} \times \ell$$

Change in length due to p_y

$$\delta \ell_2 = -\mu \frac{p}{E} \ell$$

Similarly, change in length due to p_z

$$\delta \ell_3 = -\mu \frac{p_z}{E} \ell$$

Net change in length

$$\delta \ell = \delta \ell_1 + \delta \ell_2 + \delta \ell_3$$

$$= \frac{\ell}{E}(p_x - \mu p_y - \mu p_z)$$

$$e_x = \frac{\delta \ell}{\ell} = \frac{1}{E}(p_x - \mu p_y - \mu p_z)$$

Similarly, $e_y = \frac{1}{E}[-\mu p_x + p_y - \mu p_z]$

and $e_z = \frac{1}{E}[-\mu p_x - \mu p_y + p_z]$

Volumetric strain

$$\frac{\delta V}{V} = e_v = e_x + e_y + e_z$$

$$= \frac{p_x}{E}(1 - 2\mu) + \frac{p_y}{E}(1 - 2\mu) + \frac{p_z}{E}(1 - 2\mu)$$

$$= (1 - 2\mu)\left(\frac{p_x + p_y + p_z}{E}\right)$$

In the case of uni axial loading,

$$p_y = p_z = 0$$

$$e_v = \frac{p_x}{E}(1-2\mu)$$

when $p_x = p_y = p_z = p$,

$$e_v = \frac{3p}{E}(1-2\mu)$$

From the definition of bulk modulus,

$$K = \frac{p}{e_v}$$

$$= \frac{p}{\frac{3p}{E}(1-2\mu)}$$

$$= \frac{E}{3(1-2\mu)}$$

or $E = 3K\,(1 - 2\mu)$

The above equation is the relationship between modulus of elasticity and bulk modulus.

Another conclusion is that if $(p_x + p_y + p_z) = 0$, $\frac{dv}{v} = 0$

∴ There is no change in volume.

Relationship between Modulus of Elasticity and Modulus of Rigidity

E = modulus of elasticity
G = modulus of rigidity
μ = Poisson's ratio
$E = 2G\,(1 + \mu)$

Relationship among Bulk Modulus, Modulus of Elasticity and Modulus of Rigidity

K = bulk modulus

$$E = 3\,K(1-2\,\mu) = \frac{9\,KG}{G+3\,K}$$

$$\frac{9}{E} = \frac{3}{G} + \frac{1}{K}$$

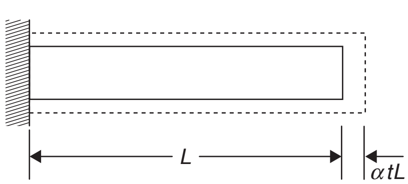
This is the relation connecting E, K and G.

Thermal Stresses

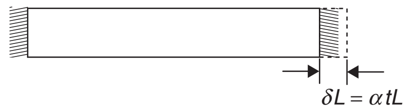
Materials expand on heating and contract on cooling. The change in dimension is found to be proportional to the length of the member and also to the change in temperature, i.e., $\delta L = \alpha\, tL$

where, t = change in temperature
L = length
α = constant of proportionality

α is called as the coefficient of linear thermal expansion.



No stresses will be developed if the bar is free to expand. But if the free expansion is prevented, then thermal stresses will be developed.



The stress developed is compressive.

As $$\delta L = \frac{PL}{AE}$$

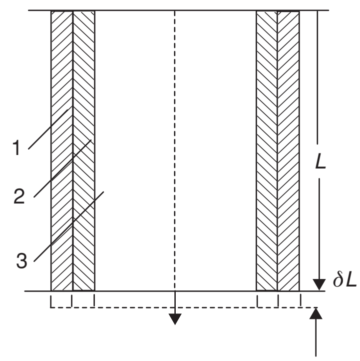
$$\frac{PL}{AE} = \alpha t L, \text{ i.e., } \frac{p\ell}{E} = \ell\alpha t$$

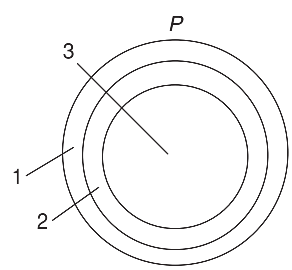
$\therefore p = E\alpha\, t$

where $p = \frac{P}{A}$, the thermal stress.

Compound Bars or Bars of Composite Section

These consist of parts of different materials joined together and loaded commonly. Therefore the elongation is same in all the materials.





$$\sigma_1 = \frac{P_1}{A_1},\ \sigma_2 = \frac{P_2}{A_2},\ \sigma_3 = \frac{P_3}{A_3}a$$

$$P = P_1 + P_2 + P_3 = \sigma_1 A_1 + \sigma_2 A_2 + \sigma_3 A_3$$

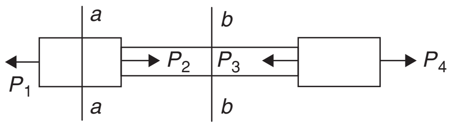
$$\text{Strain } = e = \frac{\delta L}{L} = \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} = \frac{\sigma_3}{E_3}$$

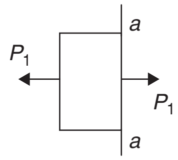
$$\therefore\ \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2},\ \frac{\sigma_1}{\sigma_3} = \frac{E_1}{E_3},\ \frac{\sigma_2}{\sigma_3} = \frac{E_2}{E_3}$$

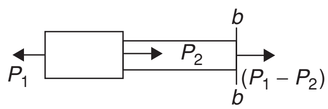
σ = stress

Bars with Varying Loads

In this case, loads may vary from portion to portion. Loads acting on each portion are found out. By finding the elongation of each portion, the total elongation is found out.

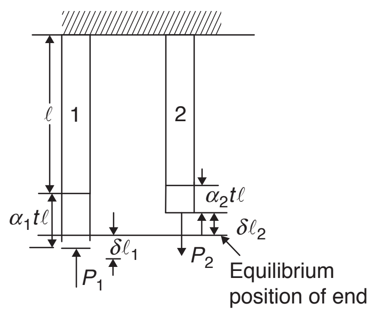






For equilibrium, $-P_1 + P_2 - P_3 + P_4 = 0$

Thermal Stresses in Compound Bars



Free expansion of bar 1 = $\alpha_1 tL$
Free expansion of bar 2 = $\alpha_2 tL$

As free expansion is prevented due to the compounding of the bars, the end of bars will have an equilibrium position as shown in the above figure and stresses will be developed in the bars.

At equilibrium condition, bar 1 shortens by $\delta\ell_1$ and bar 2 elongates by $\delta\ell_2$.

But $P_1 = P_2 = P$

$$\alpha_2 tL + \delta\ell_2 = \alpha_1 tL - \delta\ell_1$$

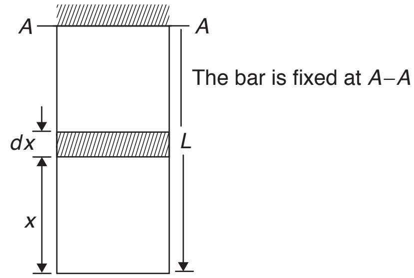
$$\delta\ell_1 = \frac{PL}{A_1 E_1}$$

$$\delta\ell_2 = \frac{PL}{A_2 E_2}$$

Solving for P, the stresses in the bars can be found out.

Elongation due to Self Weight

Bar of uniform cross-section



Weight below the elemental length = $wx \cdot A$

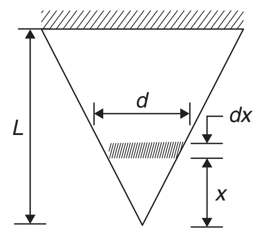
where w = specific weight

A = area of cross-section

$$\text{Elongation of elemental length} = \frac{Pdx}{AE} = \frac{wxA \cdot dx}{AE} = \frac{wx}{E}dx$$

$$\text{Total elongation due to self weight} = \int_o^L \frac{w}{E} xdx = \frac{wL^2}{2E}$$

Solid conical bar



Consider an element of length dx and diameter d at a distance x from the free end.

$$\text{Extension of } dx = \frac{Pdx}{AE} = \frac{1}{3}\frac{w}{E}xdx$$

$$\left(as\ P = \frac{1}{3}\frac{\pi d^2 wx}{4} \text{ and } A = \frac{\pi d^2}{4} \right)$$

$$\text{Total extension} = \int_o^L \frac{1}{3}\frac{w}{E}xdx = \frac{w}{3E}\left[\frac{x^2}{2}\right]_o^L = \frac{wL^2}{6E}.$$

Solved Examples

Example 1: With a steel tape of 30 m long and 15 mm × 0.8 mm cross-section a length was measured. The measured length was 120 m. During measurement a force of 100 N more than the normal was applied. What is the actual length of the line? Modulus of elasticity $= 2 \times 10^5$ N/mm^2

Solution: Elongation of 30 m tape during measurement was $\delta L = \dfrac{PL}{AE}$

$$= \frac{100 \times (30 \times 1000)}{(15 \times 0.8) \times (2 \times 10^5)} = 1.25 \text{ mm}$$

∴ If measured length is 30 m, the actual length is $30 + \dfrac{1.25}{1000} = 30.00125$ m.

If measured length is 120 m, actual length is $120 \times \dfrac{30.00125}{30} = 120.005$ m.

Example 2: A steel pipe is to be used to support a load of 150 kN. Pipes having outside diameter of 101.6 mm are available in different thicknesses of 3 mm, 3.5 mm, 3.65 mm, and 3.85 mm. Assuming a factor of safety of 1.8, choose the most economical thickness. (yield stress = 250 N/mm^2)

Solution: Permissible stress p

$$= \frac{250}{1.8} = 138.9 \text{ N/mm}^2$$

$$p = \frac{P}{A}$$

$$\therefore \quad A = \frac{P}{p} = \frac{150}{138.9} \times 10^3 = 1080 \text{ mm}^2$$

$$A = \frac{\pi}{4}(D^2 - d^2) = 1080$$

$$D^2 - d^2 = 1080 \times \frac{4}{\pi}$$

$$101.6^2 - d^2 = 1375.8$$

$$d^2 = 10{,}322.56 - 1{,}375.8 = 8{,}946.76$$

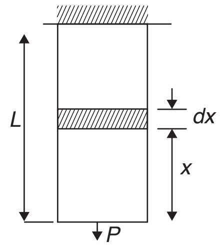
$$d = 94.59 \text{ mm}$$

$$\therefore \quad t = \frac{D-d}{2} = 3.505$$

∴ 3.65 mm thick pipe is sufficient.

Example 3: Find the extension of a bar of length L and weight w/unit length having uniform cross-section area, A suspended from top, due to its self weight and a load P applied at bottom. What is the extension if P = weight of the bar?

Solution:



Weight of the bar $= wAL$

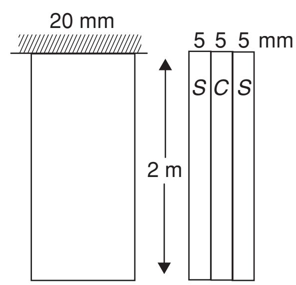
Extension due to $P = \dfrac{PL}{AE}$

Extension of the bar due to self weight $= \dfrac{wL^2}{2E}$

Total extension $= \dfrac{PL}{AE} + \dfrac{wL^2}{2E}$

when $P = wAL$, total extension $= \dfrac{3}{2}\dfrac{wL^2}{E}$

Direction for questions 4 to 6: A composite bar is made from one copper strip of 5 mm thickness in between 2 steel strips of 5 mm thickness each. The length of the bar is 2 m. Width of the bar is 20 mm. The composite bar hung vertical, is subjected to an axial load of $60\ kN \cdot E_s = 2 \times 10^5$ N/mm$^2 \cdot E_c = 1 \times 10^5$ N/mm^2



Example 4: Find the relation between stresses.

Solution: $\dfrac{\sigma}{E} = e$ (same for copper and steel)

That is, $\dfrac{\sigma_c}{E_c} = \dfrac{\sigma_s}{E_s}$

$$\frac{\sigma_c}{\sigma_s} = \frac{E_c}{E_s} = \frac{1}{2}$$

$$\sigma_s = 2\sigma_c.$$

Example 5: Find the value of stress in copper.

Solution: $P_C + P_s = 60{,}000$ N (1)

$$\delta L = \frac{P_c L}{A_c E_c} = \frac{P_s L}{A_s E_s}$$

$$\frac{P_c}{A_c E_c} = \frac{P_s}{A_s E_s}$$

$$\frac{P_c}{P_s} = \frac{A_c E_c}{A_s E_s}$$

$$= \frac{5 \times 20 \times 1 \times 10^5}{10 \times 20 \times 2 \times 10^5} = \frac{1}{4}$$

$$P_s = 4P_c \qquad (2)$$

Solving (1) and (2)

$$P_c = 12{,}000,\ P_s = 48{,}000$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{12000}{20 \times 5} = 120\ \text{N/mm}^2$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{48000}{20 \times 10} = 240\ \text{N/mm}^2.$$

Example 6: Find the elongation of the bar.

Solution:

$$\delta L = \frac{P_c L}{A_c E_c} \text{ or } = \frac{P_s L}{A_s E_s}$$

$$= \frac{12000 \times 2000}{20 \times 5 \times 1 \times 10^5} = 2.4\ \text{mm}$$

Strain Energy

When a body is subjected to a load within the elastic limit, it undergoes deformation. But its original shape is regained as soon as the load is released. At the loaded condition, it has got stored energy which is called elastic strain energy or resilience.

When an external force is applied, a resisting force starts developing gradually.

Work done by resisting force

= Average resistance × Δ

$$= \frac{1}{2} PeL$$

as $\Delta = eL$

$$= \frac{1}{2} pAeL$$

$$= \frac{1}{2} peV$$

$$= \frac{1}{2} \text{stress} \ \ \text{strain} \ \ \text{volume}$$

$$= \frac{p^2}{2E} V \left(\text{as } e = \frac{p}{E} \right)$$

= The strain energy

Maximum amount of strain energy that can be stored within elastic limit is called *proof resilience*.

$$\therefore \qquad \frac{\text{proof resilience}}{\text{unit volume}} = \frac{p_y^2}{2E}$$

where p_y = stress at elastic limit.
Stress analysis for various types of loads.

Gradually Applied Load

$$\text{Work done by load } = \frac{P}{2} \times eL$$

$$= \text{strain energy}$$

$$\text{That is, } \frac{1}{2} peV = \frac{P}{2} eL$$

$$\frac{1}{2} pe\ AL = \frac{P}{2} eL \text{ or } p = \frac{P}{A}$$

Suddenly Applied Load

Work done by load = $P \times eL$

$$\therefore \quad p = 2\frac{P}{A}$$

Impact Loading

Weight W falling from height h

Work done by load = $W(h + eL)$

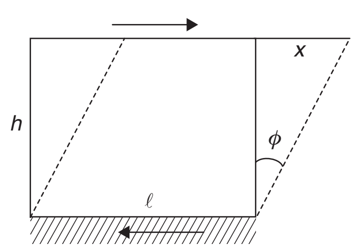
$$= W\left(h + \frac{p}{E} L \right)$$

If $\Delta = EL$ being very small is neglected,

$$\frac{p^2}{2E} AL = Wh$$

$$p = \sqrt{\frac{2EhW}{AL}}$$

Strain energy due to shear stress



$$\text{shear strain } = \frac{x}{h}$$

b = breadth

Shear strain energy

= Average shear resistance × x

$$= \frac{1}{2} q \times bL \times x$$

$$= \frac{1}{2} q\phi bLh \ (\text{as } x = h\phi)$$

$$= \frac{1}{2} q \frac{q}{G} blh \left(as\ \phi = \frac{q}{G} \right)$$

$$= \frac{1}{2} \frac{q^2}{G} V$$

It can be seen that strain energy

$$= \frac{1}{2} \text{ shear stress} \times \text{shear strain} \times \text{volume}$$

Example 7: What is the strain energy stored in steel specimen of 1.6 cm^2 cross-section and gauge length 6 cm if it stretches 0.005 cm under a load of 30,000 N?

Solution: Strain energy $= \frac{1}{2} p \times \Delta$

$= 75$ Ncm

$$\frac{1}{2} \times 30000 \times 0.005 = 75 \text{ Ncm.}$$

Example 8: In the previous example if the load at elastic limit is 48,000 N, find the proof resilience.

Solution: Elongation at elastic limit

$$= 0.005 \times \frac{48,000}{30,000} = 0.008 \text{ cm}$$

Proof resilience $= \frac{1}{2} \times 48,000 \times 0.008$

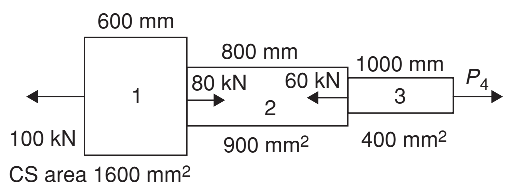
$= 192$ Ncm.

Exercises

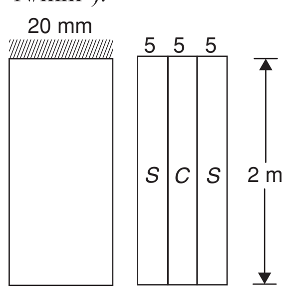
Practice Problems I

Direction for questions 1 to 15: Select the correct alternative from the given choices.

Direction for questions 1 and 2: A steel bar with different diameters at portions 1, 2 and 3 is loaded as shown in the sketch. Taking $E = 2 \times 10^5$ N/mm^2



1. Determine load P_4
   (A) 60 kN (B) 80 kN
   (C) 90 kN (D) 70 kN
2. Determine total elongation of the bar
   (A) 1.276 mm (B) 1.572 mm
   (C) 1.015 mm (D) 1.763 mm
3. A compound bar made of two steel strips (s) and one copper strip (c), as shown in figure is subjected to a temperature rise of 50°C. The load carried by copper strip due to the temperature rise is
   (Take $\alpha_s = 1.2 \times 10 - 5 / °C$,
   $E_s = 2 \times 10^5$ N/mm^2
   $\alpha_c = 1.8 \times 10 - 5 / °C$ and
   $E_c = 1 \times 10^5$ N/mm^2).



   (A) 1600 N (B) 1200 N
   (C) 2200 N (D) 2400 N

Direction for questions 4 and 5: A Metal block of 300 mm × 200 mm × 100 mm is subjected to loads of 10 kN, 20 kN and 15 kN respectively in the direction of length, breadth and thickness.

Taking $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio 0.25,

4. Stresses in the direction of length, breadth and thickness in N/mm^2 are
   (A) 0.7, 0.92, 0.3
   (B) 0.5, 0.67, 0.25
   (C) 0.8, 0.95, 0.4
   (D) 0.5, 0.82, 0.35
5. Change in the volume of the metal block in mm^3 is
   (A) 21.24 (B) 27.78
   (C) 24.63 (D) 20.97

Direction for questions 6 and 7: A metallic rod of diameter 1.25 cm stretches 0.3 cm when a steady load of 7500 N is applied. Taking $E = 2 \times 10^4$ kN/cm^2

6. The length of the rod is
   (A) 98.2 cm (B) 106.6 cm
   (C) 89.6 cm (D) 982 cm
7. The stress produced in a rod by a weight of 800 N when it falls through 8 cm and hits at bottom collar before the stretching is
   (A) 16,780 N/cm^2 (B) 15, 240 N/cm^2
   (C) 15970 N/cm^2 (D) 14, 360 N/cm^2
8. Modulus of rigidity of a material is 0.5×10^5 N/mm^2 and its bulk modulus is 0.8×10^5 N/mm^2. Determine its Poisson's ratio.
   (A) 0.36
   (B) 0.18
   (C) 0.24
   (D) 0.16

9. A steel specimen of 1.5 cm^2 cross-section stretches 0.005 cm over a gauge length of 5 cm under an axial load of 40,000 N. If load at the elastic limit for the specimen is 50,000 N, the proof resilience is

(A) 146 Ncm (B) 198 Ncm
(C) 184 Ncm (D) 156 Ncm

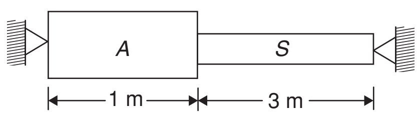
10. A composite bar is rigidly fitted between two supports A and B as shown in figure. The support reactions when the temperature rises 30°C is

(Take $E_a = 70$ GN/m^2

$E_s = 200$ GN/m^2,

$\alpha_a = 11 \times \frac{10^{-6}}{^\circ C}$

$\alpha_a = 12 \times \frac{10^{-6}}{^\circ C}$)



Cross-sectional area of $A = 600$ mm^2, $S = 300$ mm^2

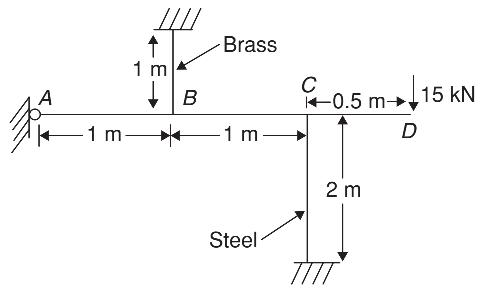
(A) 19103 N
(B) 18204 N
(C) 20205 N
(D) 17935 N

11. A RCC column of cross-section 450 mm × 450 mm is reinforced with 8 nos. of 30 mm diameter steel bars. The column is subjected to a vertical compressive load of 900 kN. If modulus of elasticity of steel and concrete are 2.1×10^5 N/mm^2 and 1.4×10^4 N/mm^2, respectively, stress in the steel is

(A) 56.26 N/mm^2
(B) 47.93 N/mm^2
(C) 57.56 N/mm^2
(D) 48.72 N/mm^2

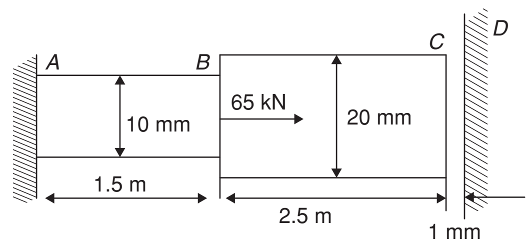
12. A rigid bar $ABCD$ is hinged at A and supported by a brass rod and a steel rod at B and C as shown in figure. When a load of 15 kN is applied at D, stress in brass rod is

[Given:
Area of cross-section of steel rod = 600 mm^2
Area of cross-section of brass rod = 1000 mm^2
E steel = 2×10^5 N/mm^2
E brass = 1×10^5 N/mm^2]



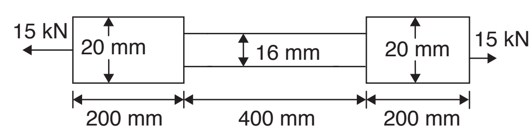
(A) 10.06 N/mm^2 (B) 9.85 N/mm^2
(C) 11.03 N/mm^2 (D) 12.92/mm^2

13. A steel bar ABC is placed on a smooth horizontal table is fixed at its left end A as shown in figure. Its right end C is 1 mm away from another support D. A load of 65 kN is applied axially at the cross-section B and acts from left to right. AB is 1 cm in diameter and 1.5 m long and BC is 2 cm in diameter and 2.5 m long. Young's modulus $E = 20 \times 10^6$ N/cm^2. The stress in portion BC will be



(A) 123 N/mm^2 (B) 228 N/mm^2
(C) 308 N/mm^2 (D) 417 N/mm^2

14. A bar with circular cross-section as shown in figure is subjected to a load of 15 kN. Modulus of elasticity E is 2×10^5 N/mm^2



The strain energy stored in the bar is

(A) 1968 N/mm (B) 2011 N/mm
(C) 1743 N/mm (D) 1835 N/mm

15. A steel rod of diameter 20 mm and length 400 mm has a collar at lower end and is fixed at top. A weight of 50 N falls freely along the rod and strikes the collar. If the instantaneous stress is not to exceed 250 N/mm^2 and Young's modulus is 2×10^5 N/mm^2 the maximum height from which the weight can be allowed to fall is

(A) 392.2 mm (B) 385.6 mm
(C) 405.5 mm (D) 410.3 mm

Practice Problems 2

Direction for questions 1 to 10: Select the correct alternative from the given choices.

1. Two steel rails of 12 m lengths were laid with a gap of 1.5 mm at ends at a temperature of 24°C. The thermal stress produced at a temperature of 40°C is
(Assume $E = 2\times10^5$ N/mm^2, coefficient of thermal expansion $= 12\times\dfrac{10^{-6}}{°C}$
(A) 10.5 N/mm^2 (B) 12.2 N/mm^2
(C) 13.4 N/mm^2 (D) 15.5 N/mm^2

2. Determine the extension of a rectangular bar 1.5 m long 8 mm thick and varying in width from 100 mm at top to 50 mm at bottom under an axial load of 800 kN
(A) 1.04 mm (B) 12.36 mm
(C) 10.40 mm (D) 9.48 mm

Direction for questions 3 and 4: A steel rod of diameter 60 mm and length 5 m is subjected to a tensile load of 120 kN. Taking Poisson's ratio as 0.25 and
$E = 2\times10^5$ N/mm^2

3. Determine the longitudinal strain produced in the rod.
(A) 1.92×10^{-3} (B) 2.79×10^{-3}
(C) 3.82×10^{-4} (D) 2.12×10^{-4}

4. Determine the change in diameter of the rod in mm
(A) 3.18×10^{-3} (B) 1.83×10^{-4}
(C) 2.57×10^{-3} (D) 2.12×10^{-4}

Direction for questions 5 to 7: A bar of diameter 30 mm and length 350 mm elongates by 0.3 mm under an axial load of 120 kN. The change in diameter is 0.0045 mm.

5. Determine Poisson's ratio
(A) 0.225 (B) 0.175
(C) 0.302 (D) 0.193

6. Determine Young's modulus in N/mm^2
(A) 1.98×10^5 (B) 2.02×10^5
(C) 2.51×10^5 (D) 1.73×10^5

7. Determine the bulk modulus in N/mm^2
(A) 1.56×10^5 (B) 2.16×10^5
(C) 1.02×10^5 (D) 2.02×10^5

8. A metallic bar of 2 m length is heated from 30°C to 50°C. Coefficient of linear expansion is 12×10^{-6} / °C and
$E = 2\times10^5$ MN/m^2. The stress in the bar is
(A) 48 N/mm^2 (B) 4.8 N/mm^2
(C) Zero (D) 3.6 N/mm^2

Direction for questions 9 and 10: A 50 N weight falls from a height of 50 mm on a collar attached to bar of 20 mm diameter and 300 mm long. Taking
$E = 2\times10^5$ N/mm^2,

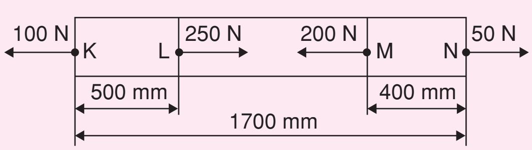
9. Determine instantaneous stress produced.
(A) 133.7 N/mm^2
(B) 103.2 N/mm^2
(C) 117.5 N/mm^2
(D) 98.3 N/mm^2

10. Determine the instantaneous extension produced.
(A) 0.155 mm (B) 1.55 mm
(C) 0.215 mm (D) 1.02 mm

Previous Years' Questions

1. In terms of Poission's ratio (μ) the ratio of Young's modulus (E) to shear modulus (G) of elastic materials is **[2004]**
(A) $2(1+\mu)$ (B) $2(1-\mu)$
(C) $\frac{1}{2}(1+\mu)$ (D) $\frac{1}{2}(1-\mu)$

2. The figure below shows a steel rod of 25 mm^2 cross-sectional area. It is loaded at four points, K, L, M and N. Assume E_{steel} = 200 GPa. The total change in length of the rod due to loading is **[2004]**



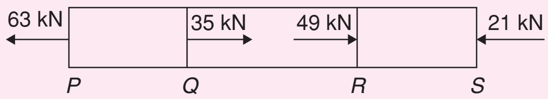
(A) 1 μm
(B) −10 μm
(C) 16 μm
(D) −20 μm

3. A uniform, slender cylindrical rod is made of a homogeneous and isotropic material. The rod rests on a frictionless surface. The rod is heated uniformly. If the radial and longitudinal thermal stresses are represented by σ_r and σ_z, respectively, then **[2005]**
(A) $\sigma_r = 0, \sigma_z = 0$ (B) $\sigma_r \neq 0, \sigma_z = 0$
(C) $\sigma_r = 0, \sigma_z \neq 0$ (D) $\sigma_r \neq 0, \sigma_z \neq 0$

4. A steel bar of 40 mm × 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and E = 200 GPa, the elongation of the bar will be **[2006]**

(A) 1.25 mm (B) 2.70 mm
(C) 4.05 mm (D) 5.40 mm

5. A bar having a cross-sectional area of 700 mm^2 is subjected to axial loads at the positions indicated. The value of stress in the segment QR is **[2006]**



(A) 40 MPa (B) 50 MPa
(C) 70 MPa (D) 120 MPa

6. A steel rod of length L and diameter D, fixed at both ends is uniformly heated to a temperature rise of ΔT. The Young's modulus is E and the coefficient of linear expansion is α. The thermal stress in the rod is **[2007]**

(A) 0 (B) $\alpha\Delta T$
(C) $E\alpha\Delta T$ (D) $E\alpha\Delta TL$

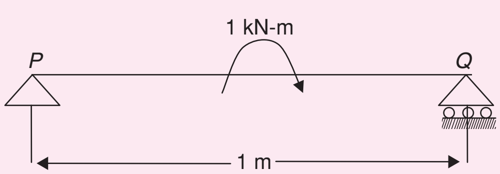
7. A 200 × 100 × 50 mm steel block is subjected to a hydrostatic pressure of 15 MPa. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3, respectively. The change in the volume of the block in mm^3 is **[2007]**

(A) 85 (B) 90
(C) 100 (D) 110

8. A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter? **[2008]**

(A) Young's modulus
(B) Shear modulus
(C) Poisson's ratio
(D) Both Young's modulus and shear modulus

9. A simply supported beam PQ is loaded by a moment of 1 kN-m at the mid-span of the beam as shown in the figure. The reaction forces R_P and R_Q at supports P and Q respectively are **[2011]**



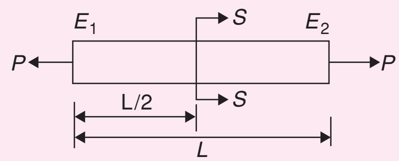
(A) 1 kN downward, 1 kN upward
(B) 0.5 kN upward, 0.5 kN downward
(C) 0.5 kN downward, 0.5 kN upward
(D) 1 kN upward, 1 kN upward

10. A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is **[2012]**

(A) $-\dfrac{\alpha(\Delta T)E}{(1-2v)}$ (B) $-\dfrac{2\alpha(\Delta T)E}{(1-2v)}$

(C) $-\dfrac{3\alpha(\Delta T)E}{(1-2v)}$ (D) $-\dfrac{\alpha(\Delta T)E}{3(1-2v)}$

11. A rod of length L having uniform cross-sectional area A is subjected to a tensile force P as shown in the figure below. If the Young's modulus of the material varies linearly from E_1 to E_2 along the length of the rod, the normal stress developed at the section-SS is **[2013]**



(A) $\dfrac{P}{A}$ (B) $\dfrac{P(E_1-E_2)}{A(E_1+E_2)}$

(C) $\dfrac{PE_2}{AE_1}$ (D) $\dfrac{PE_1}{AE_2}$

12. A circular rod of length L and area of cross-section A has a modulus of elasticity E and coefficient of thermal expansion 'α'. One end of the rod is fixed and other end is free. If the temperature of the rod is increased by ΔT, then **[2014]**

(A) stress developed in the rod is $E\alpha\Delta T$ and strain developed in the rod is $\alpha\,\Delta T$.
(B) both stress and strain developed in the rod are zero.
(C) stress developed in the rod is zero and strain developed in the rod is $\alpha\,\Delta T$.
(D) stress developed in the rod is $E\alpha\Delta T$ and strain developed in the rod is zero.

13. A metallic rod of 500 mm length and 50 mm diameter, when subjected to a tensile force of 100 kN at the ends, experiences an increase in its length by 0.5 mm and a reduction in its diameter by 0.015 mm. The Poisson's ratio of the rod material is ______. **[2014]**

14. A 200 mm long, stress free rod at room temperature is held between two immovable rigid walls. The temperature of the rod is uniformly raised by 250°C. If the Young's modulus and coefficient of thermal expansion are 200 GPa and 1 × 10^{-5} /°C, respectively, the magnitude of the longitudinal stress (in MPa) developed in the rod is ______. **[2014]**

15. A steel cube, with all faces free to deform, has Young's modulus, E, Poisson's ratio, θ, and coefficient of thermal expansion, α. The pressure (hydrostatic stress) developed within the cube, when it is subjected to a uniform increase in temperature, ΔT, is given by ______ **[2014]**

(A) 0 (B) $\dfrac{\alpha(\Delta T)E}{1-2\vartheta}$

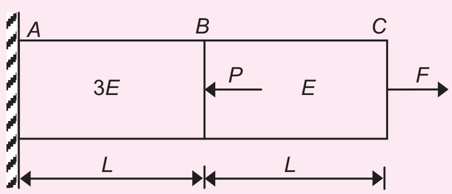
(C) $-\dfrac{\alpha(\Delta T)E}{1-2\vartheta}$ (D) $\dfrac{\alpha(\Delta T)E}{3(1-2\vartheta)}$

16. If the Poisson's ratio of an elastic material is 0.4, the ratio of modulus of rigidity to Young's modulus is **[2014]**

17. The number of independent elastic constants required to define the stress–strain relationship for an isotropic elastic solid is _____. **[2014]**

18. A rod is subjected to a uni-axial load within linear elastic limit. When the change in the stress is 200 MPa, the change in the strain is 0.001. If the Poisson's ratio of the rod is 0.3, the modulus of rigidity (in GPa) is _______. **[2015]**

19. A horizontal bar with a constant cross-section is subjected to loading as shown in the figure. The Young's moduli for the sections AB and BC are $3E$ and E, respectively.



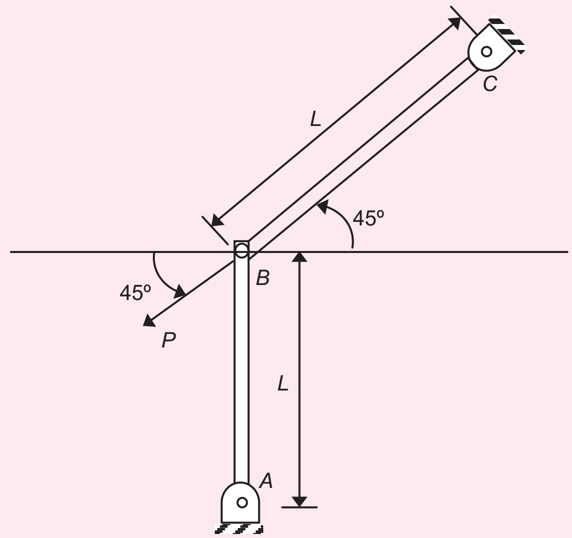
For the deflection at C to be zero, the ratio P/F is _______

[2016]

20. In the figure, the load $P = 1$ N, length $L = 1$ m, Young's modulus $E = 70$ GPa, and the cross-section of the links is a square with dimension 10 mm × 10 mm. All joints are pin joints.

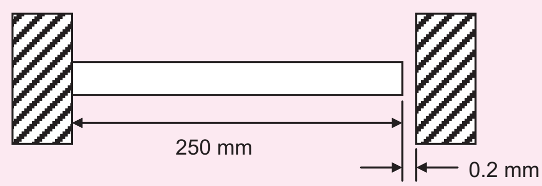
The stress (in Pa) in the link AB is _____. **[2016]**

(Indicate compressive stress by a negative sign and tensile stress by a positive sign.)



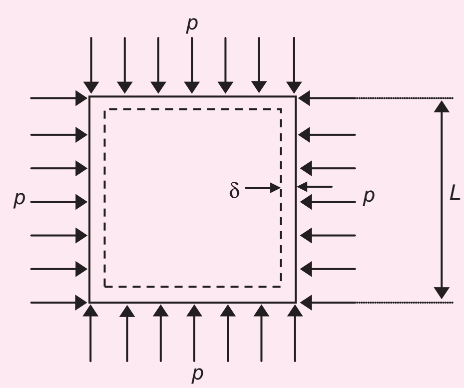
21. A circular metallic rod of length 250 mm is placed between two rigid immovable walls as shown in the figure. The rod is in perfect contact with the wall on the left side and there is a gap of 0.2 mm between the rod and the wall on the right side. If the temperature of the rod is increased by 200°C, the axial stress developed in the rod is ______ MPa. **[2016]**

Young's modulus of the material of the rod is 200 GPa and the coefficient of thermal expansion is 10^{-5} per °C.



22. A square plate of dimension $L \times L$ is subjected to a uniform pressure load $p = 250$ MPa on its edges as shown in the figure given on next page. Assume plane stress conditions. The Young's modulus $E = 200$ GPa (see figure).

The deformed shape is a square of dimension $L - 2\delta$. If $L = 2$ m and $\delta = 0.001$ m, the Poisson's ratio of the plate material is ______. **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** D	**4.** B	**5.** A	**6.** D	**7.** B	**8.** C	**9.** D	**10.** A
11. B	**12.** C	**13.** A	**14.** D	**15.** A					

Practice Problems 2

1. C	**2.** C	**3.** D	**4.** A	**5.** B	**6.** A	**7.** C	**8.** C	**9.** B	**10.** A

Previous Years' Questions

1. A **2.** B **3.** A **4.** A **5.** A **6.** C **7.** B **8.** D **9.** A **10.** A
11. A **12.** C **13.** 0.29 to 0.31 **14.** 499 to 501 **15.** A **16.** 0.35 to 0.36
17. 1.29 to 2.1 **18.** 76 to 78 **19.** 4 **20.** 0 **21.** 240 **22.** 0.2