HEAT & THERMODYNAMICS

Total translational K.E. of gas = $\frac{1}{2} M < V^2 > = \frac{3}{2} PV = \frac{3}{2} nRT$

$$< V^2 > = \frac{3P}{\rho} \qquad V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mol}}} = \sqrt{\frac{3KT}{m}}$$

Important Points :

– $V_{rms} \propto \sqrt{T}$ $\qquad \overline{V} = \sqrt{\frac{8KT}{\pi m}} = 1.59\sqrt{\frac{KT}{m}}$ $\qquad V_{rms} = 1.73\sqrt{\frac{KT}{m}}$

Most probable speed $V_p = \sqrt{\frac{2KT}{m}} = 1.41\sqrt{\frac{KT}{m}}$ $\quad \therefore V_{rms} > \overline{V} > V_{mp}$

Degree of freedom :

Mono atomic f = 3

Diatomic f = 5

polyatomic f = 6

Maxwell's law of equipartition of energy :
Total K.E. of the molecule = 1/2 f KT
For an ideal gas :

Internal energy $U = \frac{f}{2}\, nRT$

Workdone in isothermal process : $W = [2.303\, nRT \log_{10} \frac{V_f}{V_i}]$

Internal energy in isothermal process : $\Delta U = 0$

Work done in isochoric process : $dW = 0$

Change in int. energy in isochoric process :

$$\Delta U = n\, \frac{f}{2} R \Delta T \quad = \text{heat given}$$

Isobaric process :

Work done $\Delta W = nR(T_f - T_i)$
change in int. energy $\Delta U = nC_V\, \Delta T$
heat given $\Delta Q = \Delta U + \Delta W$

Specific heat : $C_V = \frac{f}{2} R$ $\quad Cp = \left(\frac{f}{2} + 1\right) R$

Molar heat capacity of ideal gas in terms of R :

(i) for monoatomic gas : $\frac{C_p}{C_v} = 1.67$

(ii) for diatomic gas : $\frac{C_p}{C_v} = 1.4$

(iii) for triatomic gas : $\frac{C_p}{C_v} = 1.33$

In general : $\gamma = \frac{C_p}{C_v} = \left[1 + \frac{2}{f}\right]$

Mayer's eq. $\Rightarrow C_p - C_v = R$ for ideal gas only

Adiabatic process :

Work done $\Delta W = \frac{nR(T_i - T_f)}{\gamma - 1}$

In cyclic process :

$\Delta Q = \Delta W$

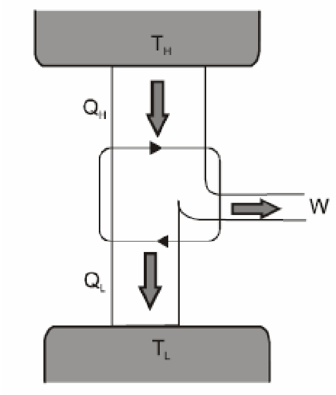
In a mixture of non-reacting gases :

$$\text{Mol. wt.} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2}$$

$$\gamma = \frac{C_{p(mix)}}{C_{v(mix)}} = \frac{n_1C_{p_1} + n_2C_{p_2} + ...}{n_1C_{v_1} + n_2C_{v_2} + ...}$$

Heat Engines



$$\text{Efficiency}, \ \eta = \frac{\text{work done by the engine}}{\text{heat supplied to it}}$$

$$= \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Second law of Thermodynamics

- **Kelvin- Planck Statement**

It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

• **Rudlope Classius Statement**

It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance

Entropy

• change in entropy of the system is $\Delta S = \frac{\Delta Q}{T} \Rightarrow S_f - S_i = \int_i^f \frac{\Delta Q}{T}$

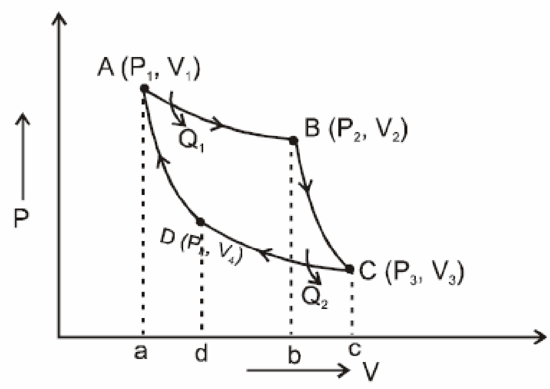
• In an adiabatic reversible process, entropy of the system remains constant.

Efficiency of Carnot Engine

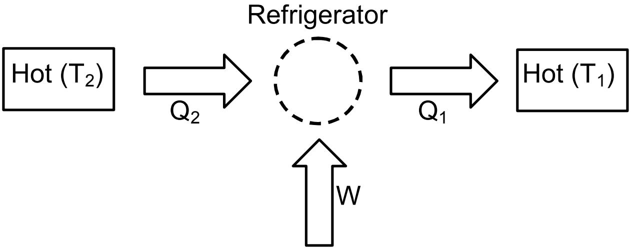
(1) Operation I (Isothermal Expansion)
(2) Operation II (Adiabatic Expansion)
(3) Operation III (Isothermal Compression)
(4) Operation IV (Adiabatic Compression)

Thermal Efficiency of a Carnot engine

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \eta = 1 - \frac{T_2}{T_1}$$



Refrigerator (Heat Pump)



- Coefficient of performance, $\beta = \frac{Q_2}{W} = \; = \frac{1}{\frac{T_1}{T_2} - 1} = \; = \frac{1}{\frac{T_1}{T_2} - 1}$

Calorimetry and thermal expansion

Types of thermometers :

(a) Liquid Thermometer : $T = \left[\frac{\ell - \ell_0}{\ell_{100} - \ell_0}\right] \times 100$

(b) Gas Thermometer :

Constant volume : $T = \left[\frac{P - P_0}{P_{100} - P_0}\right] \times 100$; $P = P_0 + \rho g h$

Constant Pressure : $T = \left[\frac{V}{V - V'}\right] T_0$

(c) Electrical Resistance Thermometer :

$$T = \left[\frac{R_t - R_0}{R_{100} - R_0}\right] \times 100$$

Thermal Expansion :

(a) Linear :

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \quad \text{or} \quad L = L_0(1 + \alpha \Delta T)$$

(b) Area/superficial :

$$\beta = \frac{\Delta A}{A_0 \Delta T} \qquad \text{or} \qquad A = A_0(1 + \beta\,\Delta T)$$

(c) volume/ cubical :

$$r = \frac{\Delta V}{V_0 \Delta T} \qquad \text{or} \qquad V = V_0(1 + \gamma\,\Delta T)$$

$$\boxed{\alpha = \frac{\beta}{2} = \frac{\gamma}{3}}$$

Thermal stress of a material :

$$\frac{F}{A} = Y\frac{\Delta \ell}{\ell}$$

Energy stored per unit volume :

$$E = \frac{1}{2}K(\Delta L)^2 \qquad \text{or} \qquad E = \frac{1}{2}\frac{AY}{L}(\Delta L)^2$$

Variation of time period of pendulum clocks :

$$\Delta T = \frac{1}{2}\,\alpha\,\Delta\theta T$$

$T' < T$ - clock-fast : time-gain

$T' > T$ - clock slow : time-loss

CALORIMETRY :

Specific heat $S = \dfrac{Q}{m.\Delta T}$

Molar specific heat $C = \dfrac{\Delta Q}{n.\Delta T}$

Water equivalent $= m_W S_W$

HEAT TRANSFER

Thermal Conduction : $\dfrac{dQ}{dt} = -KA\dfrac{dT}{dx}$

Thermal Resistance : $R = \dfrac{\ell}{KA}$

Series and parallel combination of rod :

(i) **Series :** $\dfrac{\ell_{eq}}{K_{eq}} = \dfrac{\ell_1}{K_1} + \dfrac{\ell_2}{K_2} + \cdots\cdots$ (when $A_1 = A_2 = A_3 = \ldots\ldots\ldots$)

(ii) **Parallel :** $K_{eq} A_{eq} = K_1 A_1 + K_2 A_2 + \ldots\ldots$ (when $\ell_1 = \ell_2 = \ell_3 = \ldots\ldots\ldots$)

for absorption, reflection and transmission

$$r + t + a = 1$$

Emissive power : $E = \dfrac{\Delta U}{\Delta A\,\Delta t}$

Spectral emissive power : $E_\lambda = \dfrac{dE}{d\lambda}$

Emissivity : $e = \dfrac{\text{E of a body at T temp.}}{\text{E of a black body at T temp.}}$

Kirchoff's law : $\dfrac{E(\text{body})}{a(\text{body})} = E$ (black body)

Wein's Displacement law : $\lambda_m \,.\, T = b.$

$b = 0.282$ cm-k

Stefan Boltzmann law :

$u = \sigma T^4$ $\quad s = 5.67 \times 10^{-8}$ W/m^2 k^4

$\Delta u = u - u_0 = e\,\sigma A\,(T^4 - T_0^4)$

Newton's law of cooling : $\dfrac{d\theta}{dt} = k\,(\theta - \theta_0)\,; \quad \theta = \theta_0 + (\theta_i - \theta_0)\,e^{-k\,t}$