11.9 Differentiation and Integration of Power Series

Continuous function: $f(x)$

Power series: $\sum_{n=0}^{\infty} a_n x^n$

Whole number: n

Radius of Convergence: R

1223. Differentiation of Power Series

Let $f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$ for $|x| < R$.

Then, for $|x| < R$, $f(x)$ is continuous, the derivative $f'(x)$ exists and

$$f'(x) = \frac{d}{dx} a_0 + \frac{d}{dx} a_1 x + \frac{d}{dx} a_2 x^2 + \ldots$$

$$= a_1 + 2a_2 x + 3a_3 x^2 + \ldots = \sum_{n=1}^{\infty} n a_n x^{n-1}.$$

1224. Integration of Power Series

Let $f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$ for $|x| < R$.

Then, for $|x| < R$, the indefinite integral $\int f(x)dx$ exists and

$$\int f(x)dx = \int a_0 dx + \int a_1 x dx + \int a_2 x^2 dx + \ldots$$

$$= a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + \ldots = \sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1} + C.$$