Chapter 8
Gravitation

Introduction

Newton at the age of twenty-three is said to have seen an apple falling down from tree in his orchid. This was the year 1665. He started thinking about the role of earth's attraction in the motion of moon and other heavenly bodies.

Fig. 8.1

By comparing the acceleration due to gravity due to earth with the acceleration required to keep the moon in its orbit around the earth, he was able to arrive the Basic Law of Gravitation.

Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 are separated by a distance r exert on each other is given by $F \propto \frac{m_1 m_2}{r^2}$

or $F = G\frac{m_1 m_2}{r^2}$

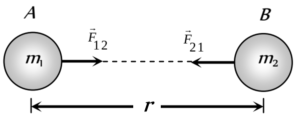


Fig. 8.2

Vector form : According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$

$$= \frac{Gm_1m_2}{r^2}\hat{r}_{21} \quad [\because \hat{r}_{12} = -\hat{r}_{21}]$$

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

If $m_1 = m_2$ and $r = 1$ then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart.

(i) The value of G in the laboratory was first determined by Cavendish using the torsional balance.

(ii) The value of G is $6.67\times10^{-}$ *N–m kg* in S.I. and $6.67\times10^{-}$ *dyne-cm-g*2 – t––– in C.G.S. system.

(iii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iv) The value of G does not depend upon the nature and size of the bodies.

(v) It also does not depend upon the nature of the medium between the two bodies.

(vi) As G is very small, hence gravitational forces are very small, unless one (or both) of the mass is huge.

Properties of Gravitational Force

(1) It is always attractive in nature while electric and magnetic force can be attractive or repulsive.

(2) It is independent of the medium between the particles while electric and magnetic force depend on the nature of the medium between the particles.

(3) It holds good over a wide range of distances. It is found true for interplanetary to inter atomic distances.

(4) It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.

(5) It is a two-body interaction *i.e.* gravitational force between two particles is independent of the presence or absence of other particles; so the principle of superposition is valid *i.e.* force on a particle due to number of particles is the resultant of forces due to individual particles *i.e.* $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

While nuclear force is many body interaction

(6) It is the weakest force in nature : As $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$.

(7) The ratio of gravitational force to electrostatic force between two electrons is of the order of 10^{-43} .

(8) It is a conservative force *i.e.* work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.

(9) It is an action reaction pair *i.e.* the force with which one body (say earth) attracts the second body (say moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

Note : ❑ The law of gravitation is stated for two point masses, therefore for any two arbitrary finite size bodies, as shown in the figure, It can not be applied as there is not unique value for the separation.

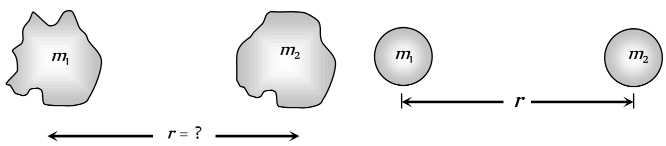


Fig. 8.3

But if the two bodies are uniform spheres then the separation *r* may be taken as the distance between their centres because a sphere of uniform mass behave as a point mass for any point lying outside it.

Acceleration Due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by *g*.

Consider a body of mass *m* is lying on the surface of earth then gravitational force on the body is given by

$$F = \frac{GMm}{R^2} \quad ...(i)$$

Where *M* = mass of the earth and *R* = radius of the earth.

If *g* is the acceleration due to gravity, then the force on the body due to earth is given by

Force = mass × acceleration

or $F = mg$ ...(ii)

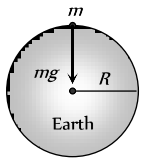


Fig. 8.4

From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

$$\therefore \; g = \frac{GM}{R^2} \quad ...(iii)$$

$$\Rightarrow g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

[As mass (M) = volume ($\frac{4}{3}\pi R^3$) × density (ρ)]

$$\therefore \; g = \frac{4}{3}\pi \rho G R \quad ...(iv)$$

(i) From the expression $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho G R$ it is clear that its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. *i.e.* a given planet (reference body) produces same acceleration in a light as well as heavy body.

(ii) The greater the value of (M/R^2) or ρR, greater will be value of *g* for that planet.

(iii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

(iv) Dimension $[g] = [LT^{-2}]$

(v) it's average value is taken to be 9.8 *m/s* or 981 *cm/sec* or 32 *feet/sec*, on the surface of the earth at mean sea level.

(vi) The value of acceleration due to gravity vary due to the following factors : (a) Shape of the earth, (b) Height above the earth surface, (c) Depth below the earth surface and (d) Axial rotation of the earth.

Variation in g Due to Shape of Earth

Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 *km* longer than polar radius, from $g = \frac{GM}{R^2}$

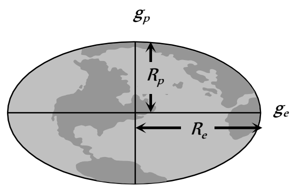


Fig. 8.5

At equator $g_e = \frac{GM}{R_e^2}$ ...(i)

At poles $g_p = \frac{GM}{R_p^2}$ ...(ii)

From (i) and (ii) $\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$

Since $R_{equator} > R_{pole}$

$\therefore \; g_{pole} > g_{equator}$ and $g_p = g_e + 0.018 \; ms^{-2}$

Therefore the weight of body increases as it is taken from equator to the pole.

Variation in *g* With Height

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \quad ...(i)$$

Acceleration due to gravity at height *h* from the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \quad ...(ii)$$

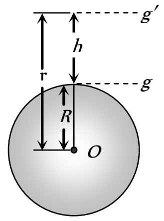


Fig. 8.6

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ ...(iii)

$= g\frac{R^2}{r^2}$ ...(iv)

[As $r = R + h$]

(i) As we go above the surface of the earth, the value of *g* decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of *g* becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right]$$

[As $h << R$]

(iv) If $h << R$ then decrease in the value of *g* with height :

Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

Percentage decrease $\frac{\Delta g}{g}\times 100\% = \frac{2h}{R}\times 100\%$

Variation in *g* With Depth

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho G R \quad ...(i)$$

Acceleration due to gravity at depth *d* from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d) \quad ...(ii)$$

From (i) and (ii) $g' = g\left[1-\frac{d}{R}\right]$

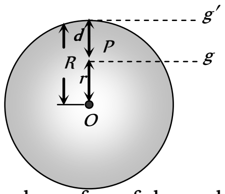


Fig. 8.7

(i) The value of *g* decreases on going below the surface of the earth. From equation (ii) we get $g' \propto (R-d)$.

So it is clear that if *d* increase, the value of *g* decreases.

(ii) At the centre of earth $d = R$ $\therefore$ $g' = 0$, *i.e.,* the acceleration due to gravity at the centre of earth becomes zero.

(iii) Decrease in the value of *g* with depth

Absolute decrease $\Delta g = g - g' = \frac{dg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$

Percentage decrease $\frac{\Delta g}{g}\times 100\% = \frac{d}{R}\times 100\%$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

Variation in *g* Due to Rotation of Earth

As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.

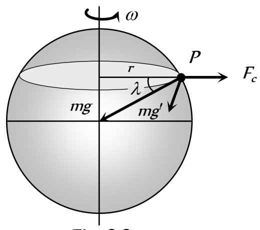


Fig. 8.8

Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass *m* lying at point *P*, whose latitude is λ, then due to rotation of earth its apparent weight can be given by $\overrightarrow{mg'} = \overrightarrow{mg} + \overrightarrow{F_c}$

or $mg' = \sqrt{(mg)^2 + (F_c)^2 + 2mg\, F_c \cos(180-\lambda)}$

$\Rightarrow mg' = \sqrt{(mg)^2 + (m\omega^2 R\cos\lambda)^2 + 2mg\, m\omega^2 R\cos\lambda\,(-\cos\lambda)}$

[As $F_c = m\omega^2 r = m\omega^2 R\cos\lambda$]

By solving we get $g' = g - \omega^2 R\cos^2\lambda$

Note : ❑ The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.

❑ For the poles $\lambda = 90^o$ and for equator $\lambda = 0^o$

(i) Substituting $\lambda = 90^o$ in the above expression we get $g_{pole} = g - \omega^2 R\cos^2 90^o$

$\therefore$ $g_{pole} = g$ ...(i)

i.e., there is no effect of rotational motion of the earth on the value of g at the poles.

(ii) Substituting $\lambda = 0^o$ in the above expression we get $g_{eqator} = g - \omega^2 R\cos^2 0^o$

$\therefore$ $g_{equator} = g - \omega^2 R$ ...(ii)

i.e., the effect of rotation of earth on the value of g at the equator is maximum.

From equation (i) and (ii)

$$g_{pole} - g_{equator} = R\omega^2 = 0.034 m/s^2$$

(iii) When a body of mass m is moved from the equator to the poles, its weight increases by an amount

$m(g_p - g_e) = m\omega^2 R$

(iv) Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$$g' = g - \omega^2 R\cos^2 \lambda$$

$$\Rightarrow 0 = g - \omega^2 R\cos^2 0^o \quad \text{[As } \lambda = 0^o \text{ for equator]}$$

$$\Rightarrow g - \omega^2 R = 0$$

$$\therefore \omega = \sqrt{\frac{g}{R}}$$

or time period of rotation of earth $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

Substituting the value of $R = 6400 \times 10^3 m$ and $g = 10 m/s^2$ we get

$$\omega = \frac{1}{800} = 1.25 \times 10^{-3} \frac{rad}{sec} \text{ and } T = 5026.5\ sec = 1.40\ hr.$$

Note : ❑ This time is about $\frac{1}{17}$ times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.

❑ If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.

❑ After considering the effect of rotation and elliptical shape of the earth, acceleration due to gravity at the poles and equator are related as

$$g_p = g_e + 0.034 + 0.018 m/s^2 \qquad \therefore g_p = g_e + 0.052 m/s^2$$

Mass and Density of Earth

Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \frac{GM}{R^2}$, so we have $M = \frac{gR^2}{G}$

$$\therefore M = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.98 \times 10^{24} kg \approx 10^{25} kg$$

and as we know $g = \frac{4}{3}\pi\rho GR$, so we have $\rho = \frac{3g}{4\pi GR}$

$$\therefore \rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^6} = 5478.4\ kg/m^3$$

Inertial and Gravitational Masses

(1) **Inertial mass** : It is the mass of the material of the body, which measures its inertia.

If an external force F acts on a body of mass m_i then according to Newton's second law of motion

$$F = m_i a \text{ or } m_i = \frac{F}{a}$$

Hence inertial mass of a body may be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.

(i) It is the measure of ability of the body to oppose the production of acceleration in its motion by an external force.

(ii) Gravity has no effect on inertial mass of the body.

(iii) It is proportional to the quantity of matter contained in the body.

(iv) It is independent of size, shape and state of body.

(v) It does not depend on the temperature of body.

(vi) It is conserved when two bodies combine physically or chemically.

(vii) When a body moves with velocity v, its inertial mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$, where m_0 = rest mass of body, c = velocity of light in vacuum,

(2) **Gravitational Mass :** It is the mass of the material of body, which determines the gravitational pull acting upon it.

If M is the mass of the earth and R is the radius, then gravitational pull on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \text{ or } m_g = \frac{F}{GM/R^2} = \frac{F}{I}$$

Here m_g is the gravitational mass of the body, if $I = 1$ then $m_g = F$

Thus the gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity,

(3) **Comparison between inertial and gravitational mass**

(i) Both are measured in the same units.

(ii) Both are scalar.

(iii) Both do not depend on the shape and state of the body

(iv) Inertial mass is measured by applying Newton's second law of motion where as gravitational mass is measured by applying Newton's law of gravitation.

(v) Spring balance measure gravitational mass and inertial balance measure inertial mass.

(4) **Comparison between mass and weight of the body**

Mass *(m)*	Weight *(W)*
It is a quantity of matter contained in a body.	It is the attractive force exerted by earth on any body.
Its value does not change with *g*	Its value changes with *g*.
Its value can never be zero for any material particle.	At infinity and at the centre of earth its value is zero.
Its unit is kilogram and its	Its unit is Newton or *kg-wt* and

dimension is $[M]$.	dimension are $[MLT^{-2}]$
It is determined by a physical balance.	It is determined by a spring balance.
It is a scalar quantity.	It is a vector quantity.

Gravitational Field

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body.

So if a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then

$$\vec{I} = \frac{\vec{F}}{m}$$

(i) It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field is considered.

(ii) Units : *Newton/kg* or *m/s*

(iii) Dimension : $[MLT^{-2}]$

(iv) If the field is produced by a point mass M and the test mass m is at a distance r from it then by Newton's law of gravitation $F = \frac{GMm}{r^2}$, then intensity of gravitational field

$$I = \frac{F}{m} = \frac{GMm/r^2}{m}$$

$$\therefore \quad I = \frac{GM}{r^2}$$

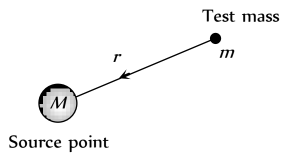


Fig. 8.9

(v) As the distance (r) of test mass from the point mass (M), increases, intensity of gravitational field decreases

$$I = \frac{GM}{r^2};$$

$$\therefore \quad I \propto \frac{1}{r^2}$$

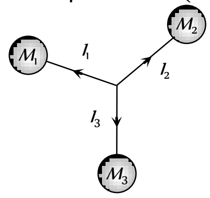


Fig. 8.10

(vi) Intensity of gravitational field $I = 0$, when $r = \infty$.

(vii) Intensity at a given point (P) due to the combined effect of different point masses can be calculated by vector sum of different intensities

$$\overrightarrow{I_{net}} = \vec{I_1} + \vec{I_2} + \vec{I_3} + ...$$

(viii) Point of zero intensity : If two bodies A and B of different masses m_1 and m_2 are d distance apart.

Let P be the point of zero intensity *i.e.*, the intensity at this point is equal and opposite due to two bodies A and B and if any test mass placed at this point it will not experience any force.

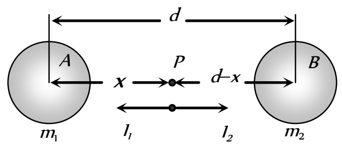


Fig. 8.11

For point P, $\vec{I_1} + \vec{I_2} = 0 \quad \Rightarrow \frac{-Gm_1}{x^2} + \frac{Gm_2}{(d-x)^2} = 0$

By solving $\quad x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$ and $(d - x) = \frac{\sqrt{m_2}\, d}{\sqrt{m_1} + \sqrt{m_2}}$

(ix) Gravitational field line is a line, straight or curved such that a unit mass placed in the field of another mass would always move along this line. Field lines for an isolated mass m are radially inwards.

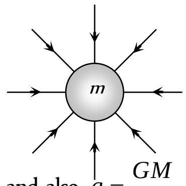


Fig. 8.12

(x) As $I = \frac{GM}{r^2}$ and also $g = \frac{GM}{R^2}$ $\therefore \quad I = g$

Thus the intensity of gravitational field at a point in the field is equal to acceleration of test mass placed at that point.

Gravitational Field Intensity for Different Bodies

(1) **Intensity due to uniform solid sphere**

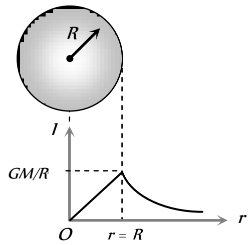


Fig. 8.13

Outside the surface **r > R**	On the surface **r = R**	Inside the surface **r < R**
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$

(2) **Intensity due to spherical shell**

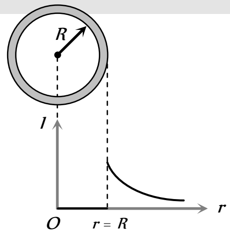


Fig. 8.14

Outside the surface **r > R**	On the surface **r = R**	Inside the surface **r < R**
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$

(3) **Intensity due to uniform circular ring**

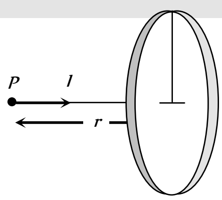


Fig. 8.15

At a point on its axis	At the centre of the ring
$I=\dfrac{GMr}{(a^2+r^2)^{3/2}}$	$I = 0$

(4) **Intensity due to uniform disc**

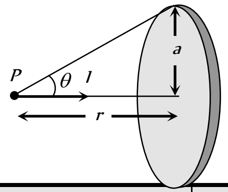


Fig. 8.16

At a point on its axis	At the centre of the disc
$I=\dfrac{2GMr}{a^2}\left[\dfrac{1}{r}-\dfrac{1}{\sqrt{r^2+a^2}}\right]$ or $I=\dfrac{2GM}{a^2}(1-\cos\theta)$	$I = 0$

Gravitational Potential

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.,*

$$V=-\frac{W}{m}=-\int\frac{\vec{F}.\vec{dr}}{m}=-\int\vec{I}.\vec{dr} \qquad \left[\text{As } \frac{F}{m}=I\right]$$

$$\therefore\ I=-\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit : *Joule/kg* or *m/sec*

(iii) Dimension : $[MLT]$

(iv) If the field is produced by a point mass then

$$V=-\int I\,dr=-\int\left(-\frac{GM}{r^2}\right)dr \qquad \left[\text{As } I=-\frac{GM}{r^2}\right]$$

$$\therefore\ V=-\frac{GM}{r}+c \qquad [\text{Here } c = \text{constant of integration}]$$

Assuming reference point at ∞ and potential to be zero there we get

$$0=-\frac{GM}{\infty}+c \Rightarrow c=0$$

$$\therefore\ \text{Gravitational potential } V=-\frac{GM}{r}$$

(v) Gravitational potential difference : It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is

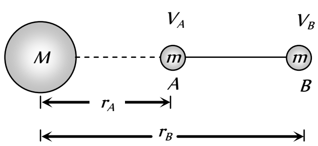


Fig. 8.17

$$\Delta V=V_B-V_A=\frac{W_{A\to B}}{m}=-GM\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$$

(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.

$$V=V_1+V_2+V_3+\ldots\ldots\ldots$$

$$=-\frac{GM}{r_1}-\frac{GM}{r_2}-\frac{GM}{r_3}\ldots\ldots\ldots$$

$$=-G\sum_{i=1}^{i=n}\frac{M_i}{r_i}$$

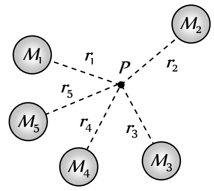


Fig. 8.18

Gravitational Potential for Different Bodies

(1) **Potential due to uniform ring**

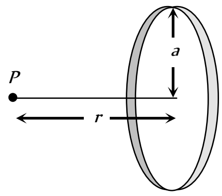


Fig. 8.19

At a point on its axis	At the centre
$V=-\dfrac{GM}{\sqrt{a^2+r^2}}$	$V=-\dfrac{GM}{a}$

(2) **Potential due to spherical shell**

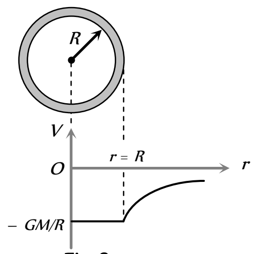


Fig. 8.20

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V=\dfrac{-GM}{r}$	$V=\dfrac{-GM}{R}$	$V=\dfrac{-GM}{R}$

(3) **Potential due to uniform solid sphere**

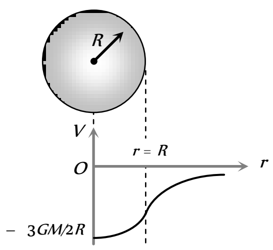


Fig. 8.21

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \frac{-GM}{r}$	$V_{surface} = \frac{-GM}{R}$	$V = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$ at the centre $(r = 0)$ $V_{centre} = \frac{-3}{2}\frac{GM}{R}$ (max.) $V_{-} = \frac{3}{2}V_{surface}$

Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r}\frac{GMm}{x^2}dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

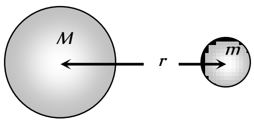


Fig. 8.22

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

(i) Potential energy is a scalar quantity.

(ii) *Unit* : Joule

(iii) *Dimension* : $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy

$$U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy

$$\Delta U = \int_{r_1}^{r_2}\frac{GMm}{x^2}dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

(ix) Relation between gravitational potential energy and potential

$$U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$$

$$\therefore U = mV$$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = m\,V_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

(xi) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$

Work Done Against Gravity

If the body of mass m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \quad [\text{As } r_1 = R \text{ and } r_2 = R+h]$$

$$\Rightarrow W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}} \quad \left[\text{As } \frac{GM}{R^2} = g\right]$$

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

(iii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

From $W = \frac{mgh}{1+h/R} = mgh \quad \left[\text{As } \frac{h}{R} \to 0\right]$

Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth $(r = R)$ to infinity $(r = \infty)$ is

$$W = \int_{R}^{\infty}\frac{GMm}{x^2}dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]\text{s}$$

$$\text{or } v_e = \sqrt{2\times\frac{4}{3}\pi\rho GR\times R} \Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho}$$

$$[\text{As } g = \frac{4}{3}\pi\rho GR]$$

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.

(iii) For the earth as $g = 9.8 m/s^2$ and $R = 6400 km$

$$\therefore v_e = \sqrt{2\times 9.8\times 6.4\times 10^6} = 11.2 km/\text{sec}$$

(iv) A planet will have atmosphere if the velocity of molecule in its atmosphere $\left[v_{rms} = \sqrt{\frac{3RT}{M}}\right]$ is lesser than escape velocity. This is why earth has atmosphere (as at earth $v_{rms} < v_e$) while moon has no atmosphere (as at moon $v_{rms} > v_e$)

(v) If a body projected with velocity lesser than escape velocity ($v < v_e$), it will reach a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.

(vi) Maximum height attained by body : Let a projection velocity of body (mass m) is v, so that it attains a maximum height h. At maximum height, the velocity of particle is zero, so kinetic energy is zero.

By the law of conservation of energy

Total energy at surface = Total energy at height h.

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$$\Rightarrow \frac{v^2}{2} = GM\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMh}{R(R+h)}$$

$$\Rightarrow \frac{2GM}{v^2R} = \frac{R+h}{h} = 1 + \frac{R}{h}$$

$$\Rightarrow h = \frac{R}{\left(\frac{2GM}{v^2R} - 1\right)} = \frac{R}{\frac{v_e^2}{v^2} - 1} = R\left[\frac{v^2}{v_e^2 - v^2}\right]$$

$$[\text{As } v_e = \sqrt{\frac{2GM}{R}} \quad \therefore \frac{2GM}{R} = v_e^2]$$

(vii) If a body is projected with velocity greater than escape velocity $(v > v_e)$ then by conservation of energy.

Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R} \Rightarrow v'^2 = v^2 - v_e^2 \qquad [\text{As } \frac{2GM}{R} = v_e^2]$

$$\therefore v' = \sqrt{v^2 - v_e^2}$$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

(viii) Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

Total energy at the surface of the earth $= KE + PE = 0 - \frac{GMm}{R}$

$$\therefore \text{Escape energy} = \frac{GMm}{R}$$

(ix) If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called black holes.

The radius of a black hole is given as

$$R = \frac{2GM}{C^2}$$

[As $C = \sqrt{\frac{2GM}{R}}$, where C is the velocity of light]

Kepler's Laws of Planetary Motion

Planets are large natural bodies rotating around a star in definite orbits. The planetary system of the star sun called solar system consists of nine planets, viz., Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Out of these planets Mercury is the smallest and closest to the sun and so hottest. Jupiter is largest and has maximum moons (12). Venus is closest to Earth and brightest. Kepler after a life time study, work out three empirical laws which govern the motion of these planets and are known as *Kepler's laws of planetary motion*. These are,

(1) **The law of Orbits** : Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

(2) **The law of Area** : The line joining the sun to the planet sweeps out equal areas in equal interval of time. *i.e.* areal velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

$$\text{Areal velocity} = \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$$

$$\therefore \frac{dA}{dt} = \frac{L}{2m}$$

$$[\text{As } L = mvr \; ; \; rv = \frac{L}{m}]$$

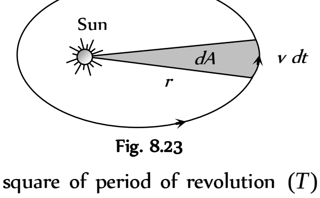


Fig. 8.23

(3) **The law of periods** : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

Proof : From the figure $AB = AF + FB$

$$2a = r_1 + r_2 \quad \therefore a = \frac{r_1 + r_2}{2}$$

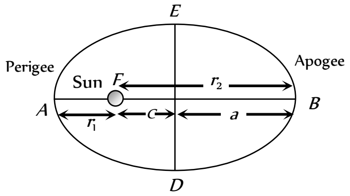


Fig. 8.24

where a = semi-major axis

r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

Important data

Planet	Semi-major axis a (10^{10} meter)	Period T(year)	T^2/a^3 (10^{-34} year/meter)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Note : ❑ Kepler's laws are valid for satellites also.

Solar System

Fig. 8.25

Velocity of a Planet in Terms of Eccentricity

Applying the law of conservation of angular momentum at perigee and apogee

$$mv_p r_p = mv_a r_a$$

$$\Rightarrow \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{a+c}{a-c} = \frac{1+e}{1-e}$$

[As $r_p = a - c$, $r_a = a + c$ and eccentricity $e = \frac{c}{a}$]

Applying the conservation of mechanical energy at perigee and apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

$$\Rightarrow v_p^2 - v_a^2 = 2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]$$

$$\Rightarrow v_a^2\left[\frac{r_a^2 - r_p^2}{r_p^2}\right] = 2GM\left[\frac{r_a - r_p}{r_a r_p}\right] \qquad \left[\text{As } v_p = \frac{v_a r_a}{r_p}\right]$$

$$\Rightarrow v_a^2 = \frac{2GM}{r_a + r_p}\left[\frac{r_p}{r_a}\right] \Rightarrow v_a^2 = \frac{2GM}{a}\left(\frac{a-c}{a+c}\right) = \frac{GM}{a}\left(\frac{1-e}{1+e}\right)$$

Thus the speeds of planet at apogee and perigee are

$$v_a = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)},$$

$$v_p = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

Note: ❑ The gravitational force is a central force so torque on planet relative to sun is always zero, hence angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

Som

			th	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mea			0	228	778	1430	2870	4500	5900
Period of revolution, *year*	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Orbital speed, *km/s*	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Equatiorial diameter, *km*	4880	12100	12800	6790	143000	120000	51800	49500	2300
Mass (Earth =1)	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Density (Water =1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Surface value of *g*, *m/s*	3.78	8.60	9.78	3.72	22.9	9.05	7.77	11.0	0.5
Escape velocity, *km/s*	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.1
Known satellites	0	0	1	2	16+ring	18+rings	17+rings	8+rings	1

Orbital Velocity of Satellite

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

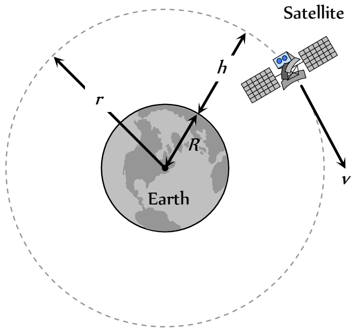


Fig. 8.26

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

[As $GM = gR^2$ and $r = R + h$]

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.,* satellites of diferent masses have same orbital velocity, if they are in the same orbit.

(ii) Orbital velocity depends on the mass of central body and radius of orbit.

(iii) For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v \propto 1/\sqrt{r}\right)$.

(iv) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \qquad \therefore\ v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

[As $h = 0$ and $GM = gR^2$]

For the earth $v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9\, km/s \approx 8\, km/\sec$

(v) Close to the surface of planet $v = \sqrt{\frac{GM}{R}}$

[As $v_e = \sqrt{\frac{2GM}{R}}$]

$$\therefore \quad v = \frac{v_e}{\sqrt{2}} \ \textit{i.e.,}\ v_{escape} = \sqrt{2}\ v_{orbital}$$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

(vi) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the orbital velocity varies as $v \propto \frac{1}{\sqrt{r^{n-1}}}$.

Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$$\therefore \quad T = \frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$$\Rightarrow T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} \qquad \left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

$$\Rightarrow T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}} \quad [\text{As } GM = gR^2]$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \quad [\text{As } r = R+h]$$

(i) From $T = 2\pi\sqrt{\frac{r^3}{GM}}$, it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

(ii) $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$\Rightarrow \quad T^2 = \frac{4\pi^2}{GM}r^3 \ \textit{i.e.,}\ T^2 \propto r^3$$

This is in accordance with Kepler's third law of planetary motion r becomes *a* (semi major axis) if the orbit is elliptic.

(iii) Time period of nearby satellite,

From $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$

[As $h = 0$ and $GM = gR^2$]

For earth $R = 6400 km$ and $g = 9.8 m/s^2$

$T = 84.6 \text{ minute} \approx 1.4\ hr$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi\left(R^3\right)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the time period varies as $T \propto r^{\frac{n+1}{2}}$

(vi) If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_S - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$ *i.e.* satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Height of Satellite

As we know, time period of satellite $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2} = (R+h)^3$

$$\Rightarrow h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period we can calculate the height of satellite from the surface of the earth.

Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity *w.r.t.* that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) Its sense of rotation should be same as that of earth about its own axis *i.e.,* in anti-clockwise direction (from west to east).

(iii) Its period of revolution around the earth should be same as that of earth about its own axis.

$\therefore$ $T = 24\ hr = 86400$ sec

(iv) Height of geostationary satellite

As $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 24hr$

Substituting the value of G and M we get $R + h = r = 42000\ km = 7R$

$\therefore$ height of geostationary satellite from the surface of earh $h = 6R = 36000\,km$

(v) Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\frac{GM}{r}}$

Substituting the value of G and M we get $v = 3.08\ km/\sec$

Angular Momentum of Satellite

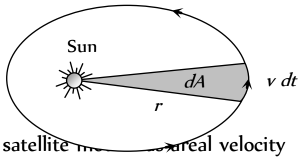
Angular momentum of satellite $L = mvr$

$$\Rightarrow L = m\sqrt{\frac{GM}{r}}\, r \quad \left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

$$\therefore \quad L = \sqrt{m^2 GMr}$$

i.e., Angular momentum of satellite depends on both the mass of orbiting and central body as well as the radius of orbit.

(i) In case of satellite motion, force is central so torque = 0 and hence angular momentum of satellite is conserved *i.e.,* $L =$ constant



(ii) In case of satellite motion, areal velocity

Fig. 8.27

$$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(vdt)}{dt} = \frac{1}{2}rv$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m} \qquad [\text{As } L = mvr\ ]$$

But as $L =$ constant, $\therefore$ areal velocity (dA/dt) = constant which is Kepler's II law

i.e., Kepler's II law or constancy of areal velocity is a consequence of conservation of angular momentum.

Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

(1) Potential energy : $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$

$$\left[\text{As } V = \frac{-GM}{r}, L^2 = m^2 GMr\right]$$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

$$\left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

(3) Total energy :

$$E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$$

(i) Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

(ii) From the above expressions we can say that

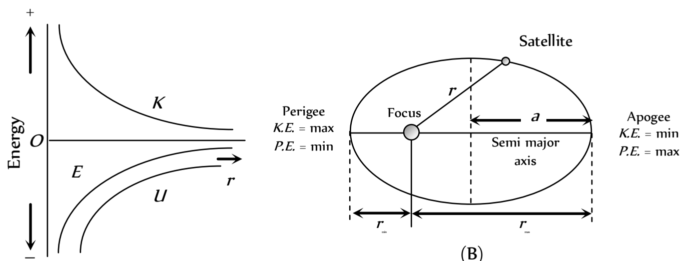
Kinetic energy (K) = – (Total energy)

Potential energy (U) = 2 (Total energy)

Potential energy (K) = – 2 (Kinetic energy)

(iii) Energy graph for a satellite

(iv) Energy distribution in elliptical orbit



(A) (B)

Fig. 8.28

(v) If the orbit of a satellite is elliptic then

(a) Total energy $(E) = \frac{-GMm}{2a}$ = constant ; where a is semi-major axis .

(b) Kinetic energy (K) will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee)

(c) Potential energy (U) will be minimum when kinetic energy = maximum i.e., the satellite is closest to the central body (at perigee) and maximum when kinetic energy = minimum i.e., the satellite is farthest from the central body (at apogee).

(vi) Binding Energy : Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*,

Binding Energy (B.E.) $= -E = \frac{GMm}{2r}$

Change in the Orbit of Satellite

When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different quantities can be shown by the following table

Quantities	Variation	Relation with r
Orbital velocity	Decreases	$v \propto \frac{1}{\sqrt{r}}$
Time period	Increases	$T \propto r^{3/2}$
Linear momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
Angular momentum	Increases	$L \propto \sqrt{r}$
Kinetic energy	Decreases	$K \propto \frac{1}{r}$
Potential energy	Increases	$U \propto -\frac{1}{r}$
Total energy	Increases	$E \propto -\frac{1}{r}$
Binding energy	Decreases	$BE \propto \frac{1}{r}$

Note:: ❑ Work done in changing the orbit

$W = E_2 - E_1$

$$W = \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$

$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

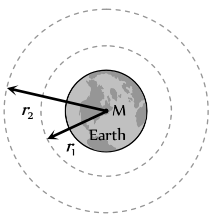


Fig. 8.29

Weightlessness

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

(1) **When objects fall freely under gravity** : For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.

(2) **When a satellite revolves in its orbit around the earth** : Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

(3) **When bodies are at null points in outer space** : On a body projected up, the pull of the earth goes on decreasing, but at the same time the gravitational pull of the moon on the body goes on increasing. At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body in question is said to appear weightless.

Weightlessness in a Satellite

A satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is $\frac{GM}{r^2}$ towards the centre of earth.

If a body of mass m placed on a surface inside a satellite moving around the earth. Then force on the body are

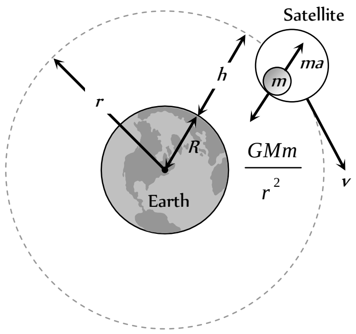


Fig. 8.30

(i) The gravitational pull of earth $= \frac{GMm}{r^2}$

(ii) The reaction by the surface $= R$

By Newton's law $\frac{GmM}{r^2} - R = m\,a$

$$\frac{GmM}{r^2} - R = m\left(\frac{GM}{r^2}\right)$$

$\therefore\ R = 0$

Thus the surface does not exert any force on the body and hence its apparent weight is zero.

A body needs no support to stay at rest in the satellite and hence all position are equally comfortable. Such a state is called weightlessness.

(i) One will find it difficult to control his movement, without weight he will tend to float freely. To get from one spot to the other he will have to push himself away from the walls or some other fixed objects.

(ii) As everything is in free fall, so objects are at rest relative to each other, *i.e.,* if a table is withdrawn from below an object, the object will remain where it was without any support.

(iii) If a glass of water is tilted and glass is pulled out, the liquid in the shape of container will float and will not flow because of surface tension.

(iv) If one tries to strike a match, the head will light but the stick will not burn. This is because in this situation convection currents will not be set up which supply oxygen for combustion

(v) If one tries to perform simple pendulum experiment, the pendulum will not oscillate. It is because there will not be any restoring torque and so $T = 2\pi\sqrt{(L/g')} = \infty$. [As $g' = 0$]

(vi) Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce its own gravity.

e.g. Moon is a satellite of earth but due to its own weight it applies gravitational force of attraction on the body placed on its surface and hence weight of the body will not be equal to zero at the surface of the moon.

Tips & Tricks

- ✍ The reference frame attached to the earth is non-inertial, because the earth revolves about its own axis as well as about the sun.
- ✍ Gravity holds the atmosphere around to the earth.
- ✍ If the earth were at one fourth the present distance from the sun, the duration of the year will be one eighth of the present year.
- ✍ If a packet is just released from an artificial satellite, it does not fall to the earth. On the other hand it will continue orbiting along with the satellite.
- ✍ Astronauts orbiting around the earth cannot use a pendulum clock. however, they can use spring clock
- ✍ To the astronauts in space, the sky appears black due to the absence of atmosphere above them.
- ✍ The gravitational force is much smaller than the electrical force because the value of *G* is very very small.
- ✍ The dimensional formula of gravitational field is same as that of acceleration due to gravity.
- ✍ A body in gravitational field has maximum binding energy when it is at rest.
- ✍ The moon is the natural satellite of the earth, but a man does not feel weightlessness on the surface of the moon. This is because, the mass of the moon is very large and it exerts a gravitational force on the man. On the other hand, the mass of the artificial satellite is very small and it exerts negligible or no gravitational force on the astronaut, so astronaut feels weightlessness in the artificial satellite but not on the moon.
- ✍ The planets are heavenly bodies revolving around the sun. The sun and the nine planets, revolving around it, constitute the solar system.
- ✍ All other planets except mercury and pluto revolve around the sun in almost circular orbits.
- ✍ If the radius of planet decreases by *x*% keeping the mass constant. The acceleration due to gravity on its surface increases by 2*x*%.
- ✍ If the mass of a planet increases by *x*% keeping radius constant, the acceleration due to gravity on its surface increases by *x*%.
- ✍ If the density of the planet decreases by *x*%, keeping the radius constant, the acceleration due to gravity decreases by *x*%.
- ✍ If the radius of the planet decreases by *x*%, keeping the density constant, the acceleration due to gravity decreases by *x*%.
- ✍ For the planets orbiting around the sun, angular speed, linear speed, kinetic energy etc. change with time but angular momentum remains constant.
- ✍ The ratio of inertial mass to gravitational mass is 1.
- ✍ Inertial mass *m* becomes infinite if the body moves with velocity of light.
- ✍ Intensity of gravitational field inside a shell is zero.
- ✍ If two spheres of same material, mass and radius are put in contact, the gravitational attraction between them is directly proportional to the fourth power of their radius.
- ✍ (a) There is no atmosphere on the moon because escape velocity on the moon is less than the *rms* velocity of the gas molecules.

  (b) Two satellites are orbiting in circular orbits of radii r_1 and r_2. Their orbital speeds are in the ratio : $v_1 / v_2 = (r_2 / r_1)^{1/2}$. It is independent to their masses
- ✍ Planets describe equal area around the sun in equal intervals of time.
- ✍ If the gravitational attraction of the sun on the planets varies as *n*th power of distance (of the planet from the sun), then year of the planet will be proportional to $R^{(n+1)/2}$.
- ✍ An object will experience weightlessness at equator, if the angular speed of the earth about its axis becomes more than (1/800) rad s^{-1}.
- ✍ Orbital velocity very near the surface of the earth is about 7.92 kms^{-1}.
- ✍ Greater the height of the satellite, smaller is the orbital velocity.
- ✍ Orbital velocity independent of the mass of the satellite.
- ✍ Orbital velocity is depends on the mass of the planet as well as radius of the orbit.
- ✍ If the altitude of the satellite is *n* times the radius of the earth, then the orbital velocity will be $(1/\sqrt{1+n})$ times the orbital velocity near the surface of the earth.
- ✍ If the radius of the orbit of a sattelite is *n* times the radius of the earth, then its orbital velocity will be $(1/\sqrt{n})$ the orbital velocity near the surface of the earth.
- ✍ The centripetal acceleration of the satellite is equal to the

acceleration due to gravity.

✍ The gravitational potential energy of a satellite of mass m is $U_p = -GMm/r$, where r is the radius of the orbit of satellite.

✍ Kinetic energy of the satellite

$$= \frac{GMm}{2r}$$

✍ Total energy of the satellite

$$E = U + K = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

✍ When velocity of the satellite increases, its kinetic energy increases and hence total energy becomes less negative. That is the satellite begins to revolve in orbit of greater radius.

✍ If the total energy of the satellite becomes $+ve$, the satellite escapes from the gravitational pull of the earth.

✍ When the satellite is taken to greater height the potential energy increases (becomes less negative) and kinetic energy decreases.

✍ For the orbiting satellite, the kinetic energy is less than potential energy. When $KE = PE$, the satellite escapes away from the gravitation pull of the earth.

✍ No energy is dissipated in keeping the satellite in orbit around a planet

✍ Time period of the satellite very near the surface of the earth is about 84.6 minutes or 1.4 *hr*.

✍ Geo-stationary satellite is a satellite which appear stationary to the observers on the earth. It is also called geosynchronous satellite.

✍ The orbit of a geostationery satellite is known as the parking orbit.

✍ To throw an ant or an elephant out of the gravitational field, the required velocity of projection is same !

✍ Escape velocity depends on the mass and size of the planet. That is why escape velocity on the planet Jupiter is more than on the earth and escape velocity on the Moon is less than that on the earth.

✍ If a body is orbiting around the earth, then it will escape away, when its velocity is increased by 41.8%.

✍ If the radius of earth is doubled keeping the density unchanged the escape velocity will be doubled.

✍ Escape velocity $= \sqrt{2} \times$ orbital velocity.

✍ If the body is at a height h above the surface of the earth, then escape velocity is given by

$$v_{es} = \sqrt{2g(R+h)}$$

✍ It is the least velocity required by a body to escape away from the gravitational pull of the earth.

✍ Escape velocity from the surface of the earth $= \sqrt{2gR} = 11.20 kms^{-1}$

✍ Body does not return to the earth when fired with escape velocity, irrespective of the angle of projection

✍ The escape velocity from the moon is 2.4 kms.

✍ When a projectile is fired with velocity less than the escape velocity, the sum of its gravitational potential and kinetic energy is negative.

✍ If ratio of the radii of two planets is r and the ratio of the acceleration due to gravity on the their surface is a, then ratio of escape velocities is $\sqrt{ar}$.

✍ If the radius of the earth is doubled keeping the mass unchanged, the escape velocity will becomes $(1/\sqrt{2})$ times the present value

✍ If a body falls freely from infinite height, then it reaches the surface of the earth with velocity 11.2 *km/s*

✍ When a body falls from a height h to the surface of the earth, its velocity on reaching the surface of the earth is given by

$$= \left[2gR\left(\frac{h}{R+h}\right)\right]^{1/2}$$

When $h << R$, we find : $v = \sqrt{2gh}$

✍ The tail of the comets points away from the sun due to the radiation pressure the sun.

Ordinary Thinking

Objective Questions

Newton's Law of Gravitation

1. The tidal waves in the sea are primarily due to
   (a) The gravitational effect of the moon on the earth
   (b) The gravitational effect of the sun on the earth
   (c) The gravitational effect of venus on the earth
   (d) The atmospheric effect of the earth itself

2. If there were a smaller gravitational effect, which of the following forces do you think would alter in some respect [NCERT 1978]
   (a) Viscous forces (b) Archimedes uplift
   (c) Electrostatic force (d) None of the above

3. A satellite of the earth is revolving in a circular orbit with a uniform speed *v*. If the gravitational force suddenly disappears, the satellite will [AIIMS 1982; AIEEE 2002]
   (a) Continue to move with velocity *v* along the original orbit
   (b) Move with a velocity *v*, tangentially to the original orbit
   (c) Fall down with increasing velocity
   (d) Ultimately come to rest somewhere on the original orbit

4. The atmosphere is held to the earth by [IIT 1986]
   (a) Winds (b) Gravity
   (c) Clouds (d) None of the above

5. The weight of a body at the centre of the earth is [AFMC 1988]
   (a) Zero
   (b) Infinite
   (c) Same as on the surface of earth
   (d) None of the above

6. If the distance between two masses is doubled, the gravitational attraction between them [CPMT 1973; AMU (Med.) 2000]
   (a) Is doubled (b) Becomes four times
   (c) Is reduced to half (d) Is reduced to a quarter

7. Which of the following is the evidence to show that there must be a force acting on earth and directed towards the sun [AIIMS 1980]
   (a) Deviation of the falling bodies towards east
   (b) Revolution of the earth round the sun
   (c) Phenomenon of day and night
   (d) Apparent motion of sun round the earth

8. The gravitational force between two stones of mass 1 *kg* each separated by a distance of 1 *metre* in vacuum is [DPMT 1984]
   (a) Zero (b) 6.675×10^{-5} *newton*
   (c) 6.675×10^{-11} *newton* (d) 6.675×10^{-8} *newton*

9. Two particles of equal mass go round a circle of radius *R* under the action of their mutual gravitational attraction. The speed of each particle is [CBSE PMT 1995; RPMT 2003]
   (a) $v=\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (b) $v=\sqrt{\frac{Gm}{2R}}$
   (c) $v=\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v=\sqrt{\frac{4Gm}{R}}$

10. The earth (mass $=6\times10^{24}\,kg$)) revolves round the sun with angular velocity $2\times10^{-7}\,rad/s$ in a circular orbit of radius $1.5\times10^{8}\,km$. The force exerted by the sun on the earth in *newtons*, is [CBSE PMT 1995; AFMC 1999; Pb. PMT 2003]
   (a) 18×10^{25} (b) Zero
   (c) 27×10^{39} (d) 36×10^{21}

11. Gravitational mass is proportional to gravitational [AIIMS 1998]
   (a) Field (b) Force
   (c) Intensity (d) All of these

12. The gravitational force between two point masses m_1 and m_2 at separation *r* is given by $F=k\frac{m_1m_2}{r^2}$
   The constant *k* [CPMT 1993]
   (a) Depends on system of units only
   (b) Depends on medium between masses only
   (c) Depends on both (a) and (b)
   (d) Is independent of both (a) and (b)

13. The distance of the centres of moon and earth is *D*. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero
   (a) $\frac{D}{2}$ (b) $\frac{2D}{3}$
   (c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

14. Who among the following gave first the experimental value of *G*
   (a) Cavendish (b) Copernicus
   (c) Brook Teylor (d) None of these

15. The mass of the moon is $7.34\times10^{22}\,kg$ and the radius is $1.74\times10^{6}\,m$. The value of gravitation force will be [AMU 1999]
   (a) 1.45 *N/kg* (b) 1.55 *N/kg*
   (c) 1.75 *N/kg* (d) 1.62 *N/kg*

16. The centripetal force acting on a satellite orbiting round the earth and the gravitational force of earth acting on the satellite both equal *F*. The net force on the satellite is [AMU 1999]
   (a) Zero (b) *F*
   (c) $F\sqrt{2}$ (d) 2 *F*

17. Reason of weightlessness in a satellite is [RPMT 2000]
   (a) Zero gravity
   (b) Centre of mass
   (c) Zero reaction force by satellite surface

(d) None

18. Mass M is divided into two parts xM and $(1-x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is **[EAMCET 2001]**

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$

(c) 1 (d) 2

19. The force of gravitation is **[AIIMS 2002]**

(a) Repulsive (b) Electrostatic

(c) Conservative (d) Non-conservative

20. The gravitational force F_g between two objects does not depend on **[RPET 2003]**

(a) Sum of the masses

(b) Product of the masses

(c) Gravitational constant

(d) Distance between the masses

21. Two sphere of mass m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be **[CBSE PMT 2003]**

(a) F (b) $\frac{F}{3}$

(c) $\frac{F}{9}$ (d) $3F$

22. Earth binds the atmosphere because of **[J&K CET 2005]**

(a) Gravity

(b) Oxygen between earth and atmosphere

(c) Both (a) and (b)

(d) None of these

23. Which of the following statements about the gravitational constant is true **[Kerala PET 2005]**

(a) It is a force

(b) It has no unit

(c) It has same value in all systems of units

(d) It depends on the value of the masses

(e) It does not depend on the nature of the medium in which the bodies are kept.

24. Two identical solid copper spheres of radius R placed in contact with each other. The gravitational attracton between them is proportional to **[Kerala PET 2005]**

(a) R^2 (b) R^{-2}

(c) R^4 (d) R^{-4}

Acceleration Due to Gravity

1. Weightlessness experienced while orbiting the earth in space-ship, is the result of **[NCERT 1978; DPMT 1982]**

(a) Inertia (b) Acceleration

(c) Zero gravity (d) Free fall towards earth

2. If the change in the value of 'g' at a height h above the surface of the earth is the same as at a depth x below it, then (both x and h being much smaller than the radius of the earth) **[NCERT 1983; BHU 2002]**

(a) $x = h$ (b) $x = 2h$

(c) $x = \frac{h}{2}$ (d) $x = h^2$

3. The time period of a simple pendulum on a freely moving artificial satellite is **[CPMT 1984; AFMC 2002]**

(a) Zero (b) 2 sec

(c) 3 sec (d) Infinite

4. Two planets have the same average density but their radii are R_1 and R_2. If acceleration due to gravity on these planets be g_1 and g_2 respectively, then **[AIIMS 1985]**

(a) $\frac{g_1}{g_2} = \frac{R_1}{R_2}$ (b) $\frac{g_1}{g_2} = \frac{R_2}{R_1}$

(c) $\frac{g_1}{g_2} = \frac{R_1^2}{R_2^2}$ (d) $\frac{g_1}{g_2} = \frac{R_1^3}{R_2^3}$

5. An iron ball and a wooden ball of the same radius are released from a height 'h' in vacuum. The time taken by both of them to reach the ground is **[NCERT 1975; AFMC 1998]**

(a) Unequal (b) Exactly equal

(c) Roughly equal (d) Zero

6. The correct answer to above question is based on **[NCERT 1975]**

(a) Acceleration due to gravity in vacuum is same irrespective of size and mass of the body

(b) Acceleration due to gravity in vacuum depends on the mass of the body

(c) There is no acceleration due to gravity in vacuum

(d) In vacuum there is resistance offered to the motion of the body and this resistance depends on the mass of the body

7. When a body is taken from the equator to the poles, its weight

(a) Remains constant

(b) Increases

(c) Decreases

(d) Increases at N-pole and decreases at S-pole

8. A body of mass m is taken to the bottom of a deep mine. Then

(a) Its mass increases (b) Its mass decreases

(c) Its weight increases (d) Its weight decreases

9. A body weighs 700 *gm wt* on the surface of the earth. How much will it weigh on the surface of a planet whose mass is $\frac{1}{7}$ and radius is half that of the earth **[CMC Vellore 1984; AFMC 2000]**

(a) 200 *gm wt* (b) 400 *gm wt*

(c) 50 *gm wt* (d) 300 *gm wt*

10. In order to find time, the astronaut orbiting in an earth satellite should use **[DPMT 1982]**

(a) A pendulum clock

(b) A watch having main spring to keep it going

(c) Either a pendulum clock or a watch

(d) Neither a pendulum clock nor a watch

11. A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to **[MP PMT 1987; DPMT 2002]**

(a) GM_0 / D_0^2 (b) $4mGM_0 / D_0^2$

(c) $4GM_0 / D_0^2$ (d) GmM_0 / D_0^2

12. If the earth stops rotating, the value of 'g' at the equator will [CPMT 1986]

(a) Increase (b) Remain same

(c) Decrease (d) None of the above

13. The mass and diameter of a planet have twice the value of the corresponding parameters of earth. Acceleration due to gravity on the surface of the planet is [NCERT 1971; Pb. PMT 2000]

(a) $9.8\, m/\sec^2$ (b) $4.9\, m/\sec^2$

(c) $980\, m/\sec^2$ (d) $19.6\, m/\sec^2$

14. As we go from the equator to the poles, the value of g [CPMT 1975; AFMC 1995; AFMC 2004]

(a) Remains the same

(b) Decreases

(c) Increases

(d) Decreases upto a latitude of 45°

15. Force of gravity is least at [CPMT 1992]

(a) The equator

(b) The poles

(c) A point in between equator and any pole

(d) None of these

16. The radius of the earth is 6400 *km* and $g = 10 m/\sec^2$. In order that a body of 5 *kg* weighs zero at the equator, the angular speed of the earth is [MP PMT 1985]

(a) 1/80 *radian/sec* (b) 1/400 *radian/sec*

(c) 1/800 *radian/sec* (d) 1/1600 *radian/sec*

17. The value of 'g' at a particular point is $9.8\, m/s^2$. Suppose the earth suddenly shrinks uniformly to half its present size without losing any mass. The value of 'g' at the same point (assuming that the distance of the point from the centre of earth does not shrink) will now be [NCERT 1984; DPMT 1999]

(a) $4.9\, m/\sec^2$ (b) $3.1\, m/\sec^2$

(c) $9.8\, m/\sec^2$ (d) $19.6\, m/\sec^2$

18. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is MH CET (Med.) 1999; CBSE PMT 1995]

(a) $4\pi G/3gR$ (b) $3\pi R/4gG$

(c) $3g/4\pi RG$ (d) $\pi RG/12G$

19. The weight of an object in the coal mine, sea level, at the top of the mountain are W_1, W_2 and W_3 respectively, then [EAMCET 1990]

(a) $W_1 < W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_1 > W_2 > W_3$

20. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is [MP PET 1994]

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$

(c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$ (d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

21. The mass of the earth is 81 times that of the moon and the radius of the earth is 3.5 times that of the moon. The ratio of the acceleration due to gravity at the surface of the moon to that at the surface of the earth is [MP PMT 1994]

(a) 0.15 (b) 0.04

(c) 1 (d) 6

22. Spot the *wrong* statement :

The acceleration due to gravity 'g' decreases if [MP PMT 1994]

(a) We go down from the surface of the earth towards its centre

(b) We go up from the surface of the earth

(c) We go from the equator towards the poles on the surface of the earth

(d) The rotational velocity of the earth is increased

23. Which of the following statements is true [Manipal MEE 1995]

(a) g is less at the earth's surface than at a height above it or a depth below it

(b) g is same at all places on the surface of the earth

(c) g has its maximum value at the equator

(d) g is greater at the poles than at the equator

24. A spring balance is graduated on sea level. If a body is weighed with this balance at consecutively increasing heights from earth's surface, the weight indicated by the balance

(a) Will go on increasing continuously

(b) Will go on decreasing continuously

(c) Will remain same

(d) Will first increase and then decrease

25. The value of g on the earth's surface is $980\, cm/\sec^2$. Its value at a height of 64 *km* from the earth's surface is [MP PMT 1995]

(a) $960.40\, cm/\sec^2$ (b) $984.90\, cm/\sec^2$

(c) $982.45\, cm/\sec^2$ (d) $977.55\, cm/\sec^2$

(Radius of the earth R = 6400 *kilometers*)

26. Choose the correct statement from the following :

[CPMT 1990; BHU 1998; Kerala PMT 2002;

Weightlessness of an astronaut moving in a satellite is a situation of

(a) Zero g (b) No gravity

(c) Zero mass (d) Free fall

27. If the earth rotates faster than its present speed, the weight of an object will [Haryana CEE 1996]

(a) Increase at the equator but remain unchanged at the poles

(b) Decrease at the equator but remain unchanged at the poles

(c) Remain unchanged at the equator but decrease at the poles

(d) Remain unchanged at the equator but increase at the poles

28. If the earth suddenly shrinks (without changing mass) to half of its present radius, the acceleration due to gravity will be [MNR 1998]

(a) $g/2$ (b) $4g$

(c) $g/4$ (d) $2g$

29. The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is

MP PET 2000, 01; RPET 2000; Pb. PET 2001]

(a) $g/4$ (b) $g/5$
(c) $g/6$ (d) $g/8$

30. R is the radius of the earth and ω is its angular velocity and g_p is the value of g at the poles. The effective value of g at the latitude $\lambda = 60°$ will be equal to [MP PMT 1999]

(a) $g_p - \frac{1}{4}R\omega^2$ (b) $g_p - \frac{3}{4}R\omega^2$
(c) $g_p - R\omega^2$ (d) $g_p + \frac{1}{4}R\omega^2$

31. The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth] [MP PMT 1999; Kerala PMT 2005]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$
(c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

32. At what height over the earth's pole, the free fall acceleration decreases by one percent (assume the radius of earth to be 6400 *km*) [KCET 1994]

(a) 32 *km* (b) 80 *km*
(c) 1.253 *km* (d) 64 *km*

33. The diameters of two planets are in the ratio 4 : 1 and their mean densities in the ratio 1 : 2. The acceleration due to gravity on the planets will be in ratio [ISM Dhanbad 1994]

(a) 1 : 2 (b) 2 : 3
(c) 2 : 1 (d) 4 : 1

34. At what altitude in metre will the acceleration due to gravity be 25% of that at the earth's surface (Radius of earth = *R metre*)

(a) $\frac{1}{4}R$ (b) R
(c) $\frac{3}{8}R$ (d) $\frac{R}{2}$

35. If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole [EAMCET (Med.) 1995]

(a) Doubles (b) Becomes half
(c) Remains same (d) Becomes zero

36. At the surface of a certain planet, acceleration due to gravity is one-quarter of that on earth. If a brass ball is transported to this planet, then which one of the following statements is not correct

(a) The mass of the brass ball on this planet is a quarter of its mass as measured on earth
(b) The weight of the brass ball on this planet is a quarter of the weight as measured on earth
(c) The brass ball has the same mass on the other planet as on earth
(d) The brass ball has the same volume on the other planet as on earth

37. Weight of 1 *kg* becomes 1/6 on moon. If radius of moon is $1.768 \times 10^6\ m$, then the mass of moon will be [RPET 1997]

(a) $1.99 \times 10^{30}\ kg$ (b) $7.56 \times 10^{22}\ kg$
(c) $5.98 \times 10^{24}\ kg$ (d) $7.65 \times 10^{22}\ kg$

38. Radius of earth is around 6000 *km*. The weight of body at height of 6000 *km* from earth surface becomes [RPMT 1997]

(a) Half (b) One-fourth
(c) One third (d) No change

39. Let g be the acceleration due to gravity at earth's surface and K be the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2% keeping all other quantities same, then

(a) g decreases by 2% and K decreases by 4%
(b) g decreases by 4% and K increases by 2%
(c) g increases by 4% and K increases by 4%
(d) g decreases by 4% and K increases by 4%

40. Where will it be profitable to purchase 1 kilogram sugar [RPET 1996]

(a) At poles (b) At equator
(c) At 45° latitude (d) At 40° latitude

41. If the radius of the earth shrinks by 1.5% (mass remaining same), then the value of acceleration due to gravity changes by

(a) 1% (b) 2%
(c) 3% (d) 4%

42. If radius of the earth contracts 2% and its mass remains the same, then weight of the body at the earth surface [CPMT 1997; KCET (Engg./Med.) 2001]

(a) Will decrease (b) Will increase
(c) Will remain the same (d) None of these

43. If mass of a body is M on the earth surface, then the mass of the same body on the moon surface is [AIIMS 1997; RPMT 1997; JIPMER 2000]

(a) $M/6$ (b) Zero
(c) M (d) None of these

44. Mass of moon is 7.34×10^{22} *kg*. If the acceleration due to gravity on the moon is $1.4\ m/s^2$ [ISM Dhanbad 1994], the radius of the moon is $(G = 6.667 \times 10^{-11}\ Nm^2/kg^2)$ [AFMC 1998]

(a) $0.56 \times 10^4\ m$ (b) $1.87 \times 10^6\ m$
(c) $1.92 \times 10^6\ m$ (d) $1.01 \times 10^8\ m$

45. What should be the velocity of earth due to rotation about its own axis so that the weight at equator become 3/5 of initial value. Radius of earth on equator is 6400 *km* [AMU 1999]

(a) $7.4 \times 10^{-4}\ rad/\sec$ (b) $6.7 \times 10^{-4}\ rad/\sec$
(c) $7.8 \times 10^{-4}\ rad/\sec$ (d) $8.7 \times 10^{-4}\ rad/\sec$

[SCRA 1994]

46. Acceleration due to gravity is 'g' on the surface of the earth. The value of acceleration due to gravity at a height of 32 *km* above earth's surface is (Radius of the earth = 6400 *km*) [KCET (Engg./Med.) 1999]

(a) 0.9 g (b) 0.99 g
(c) 0.8 g (d) 1.01 g

47. At what height from the ground will the value of 'g' be the same as that in 10 *km* deep mine below the surface of earth [RPET 1999]

(a) 20 *km* (b) 10 *km*
(c) 15 *km* (d) 5 *km*

48. If the Earth losses its gravity, then for a body [BHU 1999; MHCET 2003]

(a) Weight becomes zero, but not the mass

(b) Mass becomes zero, but not the weight

(c) Both mass and weight become zero

(d) Neither mass nor weight become zero

49. The height of the point vertically above the earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the earth = *R*) [EAMCET (Engg.) 2000]

(a) 8 *R* (b) 9 *R*

(c) 10 *R* (d) 20 *R*

50. An object weights 72 *N* on earth. Its weight at a height of *R*/2 from earth is [AIIMS 2000]

(a) 32 *N* (b) 56 *N*

(c) 72 *N* (d) Zero

51. The angular velocity of the earth with which it has to rotate so that acceleration due to gravity on 60° latitude becomes zero is (Radius of earth = 6400 *km*. At the poles $g = 10\,ms^{-2}$)

(a) $2.5\times10^{-3}\,rad/s$ (b) $5.0\times10^{-1}\,rad/s$

(c) $10\times10^{1}\,rad/s$ (d) $7.8\times10^{-2}\,rad/s$

52. Assuming earth to be a sphere of a uniform density, what is the value of gravitational acceleration in a mine 100 *km* below the earth's surface (Given *R* = 6400 *km*) [AFMC 2000; Pb. PMT 2000]

(a) $9.66\,m/s^2$ (b) $7.64\,m/s^2$

(c) $5.06\,m/s$ (d) $3.10\,m/s^2$

53. If radius of earth is *R* then the height '*h*' at which value of '*g*' becomes one-fourth is [BHU 2000]

(a) $\frac{R}{4}$ (b) $\frac{3R}{4}$

(c) R (d) $\frac{R}{8}$

54. *R* and *r* are the radii of the earth and moon respectively. ρ_e and ρ_m are the densities of earth and moon respectively. The ratio of the accelerations due to gravity on the surfaces of earth and moon is

(a) $\frac{R}{r}\frac{\rho_e}{\rho_m}$ (b) $\frac{r}{R}\frac{\rho_e}{\rho_m}$

(c) $\frac{r}{R}\frac{\rho_m}{\rho_e}$ (d) $\frac{R}{r}\frac{\rho_e}{\rho_m}$

55. If the mass of earth is 80 times of that of a planet and diameter is double that of planet and '*g*' on earth is $9.8\,m/s^2$, then the value of '*g*' on that planet is [Pb. PMT 1999; CPMT 2000]

(a) $4.9\,m/s^2$ (b) $0.98\,m/s^2$

(c) $0.49\,m/s^2$ (d) $49\,m/s^2$

56. Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface, what is the maximum range on the surface of the moon for the same velocity of projection [Kerala (Engg.) 2001]

(a) $0.2\,R_e$ (b) $2\,R_e$

(c) $0.5\,R_e$ (d) $5\,R_e$

57. The angular speed of earth, so that the object on equator may appear weightless, is ($g = 10\,m/s^2$, radius of earth 6400 *km*)

(a) $1.25\times10^{-3}\,rad/sec$ (b) $1.56\times10^{-3}\,rad/sec$

(c) $1.25\times10^{-1}\,rad/sec$ (d) 1.56 *rad/sec*

58. At what distance from the centre of the earth, the value of acceleration due to gravity *g* will be half that on the surface (*R* = radius of earth) [MP PMT 2001]

(a) 2 *R* (b) *R*

(c) 1.414 *R* (d) 0.414 *R*

59. If density of earth increased 4 times and its radius become half of what it is, our weight will [AMU (Engg.) 2001]

(a) Be four times its present value

(b) Be doubled

(c) Remain same

(d) Be halved [EAMCET 2000]

60. A man can jump to a height of 1.5 *m* on a planet *A*. What is the height he may be able to jump on another planet whose density and radius are, respectively, one-quarter and one-third that of planet *A*

(a) 1.5 *m* (b) 15 *m*

(c) 18 *m* (d) 28 *m*

61. Weight of a body is maximum at [AFMC 2001]

(a) Moon (b) Poles of earth

(c) Equator of earth (d) Centre of earth

62. What will be the acceleration due to gravity at height *h* if *h* >> *R*. Where *R* is radius of earth and *g* is acceleration due to gravity on the surface of earth [RPET 2001]

(a) $\frac{g}{\left(1+\frac{h}{R}\right)^2}$ (b) $g\left(1-\frac{2h}{R}\right)$

(c) $\frac{g}{\left(1-\frac{h}{R}\right)^2}$ (d) $g\left(1-\frac{h}{R}\right)$

63. The acceleration due to gravity near the surface of a planet of radius *R* and density *d* is proportional to [EAMCET 2000] [MP PET 2002; AIEEE 2004]

(a) $\frac{d}{R^2}$ (b) dR^2

(c) dR (d) $\frac{d}{R}$

64. The acceleration due to gravity is *g* at a point distant *r* from the centre of earth of radius *R*. If $r < R$, then [CPMT 2002]

(a) $g \propto r$ (b) $g \propto r^2$

(c) $g \propto r^{-1}$ (d) $g \propto r^{-2}$

65. A body weight *W* newton at the surface of the earth. Its weight at a height equal to half the radius of the earth will be [UPSEAT 2002]

(a) $\frac{W}{2}$ (b) $\frac{2W}{3}$

(c) $\frac{4W}{9}$ (d) $\frac{8W}{27}$

66. If the density of the earth is doubled keeping its radius constant then acceleration due to gravity will be $(g = 9.8\, m/s^2)$ [Pb. PMT 2002; Orissa 2002]

(a) $19.6\, m/s^2$ (b) $9.8\, m/s^2$

(c) $4.9\, m/s^2$ (d) $2.45\, m/s^2$

67. The acceleration due to gravity at pole and equator can be related as [DPMT 2002]

(a) $g_p < g_e$ (b) $g_p = g_e = g$

(c) $g_p = g_e < g$ (d) $g_p > g_e$

68. If the value of '*g*' acceleration due to gravity, at earth surface is $10\, m/s^2$, its value in m/s^2 at the centre of the earth, which is assumed to be a sphere of radius '*R*' *metre* and uniform mass density is [AIIMS 2002]

(a) 5 (b) 10/*R*

(c) 10/2*R* (d) Zero

69. A research satellite of mass 200 *kg* circles the earth in an orbit of average radius 3*R*/2 where *R* is the radius of the earth. Assuming the gravitational pull on a mass of 1 *kg* on the earth's surface to be 10 *N*, the pull on the satellite will be [Kerala (Engg.) 2002]

(a) 880 *N* (b) 889 *N*

(c) 890 *N* (d) 892 *N*

70. Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon R_m in terms of R_e will be [MP PMT 2003]

(a) $\frac{5}{18} R_e$ (b) $\frac{1}{6} R_e$

(c) $\frac{3}{18} R_e$ (d) $\frac{1}{2\sqrt{3}} R_e$

71. The acceleration of a body due to the attraction of the earth (radius *R*) at a distance 2 *R* from the surface of the earth is (*g* = acceleration due to gravity at the surface of the earth) [MP PET 2003]

(a) $\frac{g}{9}$ (b) $\frac{g}{3}$

(c) $\frac{g}{4}$ (d) g

72. The depth at which the effective value of acceleration due to gravity is $\frac{g}{4}$ is [MP PET 2003]

(a) R (b) $\frac{3R}{4}$

(c) $\frac{R}{2}$ (d) $\frac{R}{4}$

73. Weight of a body of mass *m* decreases by 1% when it is raised to height *h* above the earth's surface. If the body is taken to a depth *h* in a mine, change in its weight is [KCET 2003; MP PMT 2003]

(a) 2% decrease (b) 0.5% decrease

(c) 1% increase (d) 0.5% increase

74. If both the mass and the radius of the earth decrease by 1%, the value of the acceleration due to gravity will [MP PET 2004]

(a) Decrease by 1% (b) Increase by 1%

(c) Increase by 2% (d) Remain unchanged

75. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is *R*, the radius of the planet would be [CBSE PMT 2004]

(a) $2R$ (b) $4R$

(c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$

76. Two planets of radii in the ratio 2 : 3 are made from the material of density in the ratio 3 : 2. Then the ratio of acceleration due to gravity g_1 / g_2 at the surface of the two planets will be

(a) 1 (b) 2.25

(c) 4/9 (d) 0.12

77. A person will get more quantity of matter in *kg –wt.* at [J & K CET 2004]

(a) Poles (b) At latitude of 60°

(c) Equator (d) Satellite

78. At what depth below the surface of the earth, acceleration due to gravity *g* will be half its value 1600 *km* above the surface of the earth [Pb. PMT 2004]

(a) $4.2 \times 10^6\, m$ (b) $3.19 \times 10^6\, m$

(c) $1.59 \times 10^6\, m$ (d) None of these

79. What should be the angular speed of earth, so that body lying on equator may appear weightlessness

$(g = 10\, m/s^2,\ R = 6400\, km)$ [Pb. PET 2000]

(a) $\frac{1}{800} rad/s$ (b) $\frac{1}{400} rad/s$

(c) $\frac{1}{600} rad/s$ (d) $\frac{1}{100} rad/s$

80. A body weight 500 *N* on the surface of the earth. How much would it weigh half way below the surface of the earth [Pb. PET 2001; BHU 2004]

(a) 125 *N* (b) 250 *N*

(c) 500 *N* (d) 1000 *N*

81. If the density of a small planet is the same as that of earth, while the radius of the planet is 0.2 times that of the earth, the gravitational acceleration on the surface of that planet is [UPSEAT 2004; CBSE PMT 2005]

(a) 0.2 *g* (b) 0.4 *g*

(c) 2 *g* (d) 4 *g*

82. Acceleration due to gravity '*g*' for a body of mass '*m*' on earth's surface is proportional to (Radius of earth=*R*, mass of earth=*M*)

(a) GM/R^2 (b) m^0

(c) mM (d) $1/R^{3/2}$

83. A body has a weight 90 *kg* on the earth's surface, the mass of the moon is 1/9 that of the earth's mass and its radius is 1/2 that of the earth's radius. On the moon the weight of the body is

(a) 45 *kg* (b) 202.5 *kg*
(c) 90 *kg* (d) 40 *kg*

84. If it is assumed that the spinning motion of earth increases, then the weight of a body on equator [RPMT 2003]
(a) Decreases (b) Remains constant
(c) Increases (d) Becomes more at poles

85. The masses of two planets are in the ratio 1 : 2. Their radii are in the ratio 1 : 2. The acceleration due to gravity on the planets are in the ratio [MH CET 2004]
(a) 1 : 2 (b) 2 : 1
(c) 3 : 5 (d) 5 : 3

86. If earth is supposed to be a sphere of radius *R*, if g_{30} is value of acceleration due to gravity at latitude of 30° and *g* at the equator, the value of $g - g_{30^o}$ is [DCE 2005]
(a) $\frac{1}{4}\omega^2 R$ (b) $\frac{3}{4}\omega^2 R$
(c) $\omega^2 R$ (d) $\frac{1}{2}\omega^2 R$

87. If *M* the mass of the earth and *R* its radius, the ratio of the gravitational acceleration and the gravitational constant is [J&K CET 2005]
(a) $\frac{R^2}{M}$ (b) $\frac{M}{R^2}$
(c) MR^2 (d) $\frac{M}{R}$

Gravitation Potential, Energy and Escape Velocity

1. A body of mass *m* rises to height $h = R/5$ from the earth's surface, where *R* is earth's radius. If *g* is acceleration due to gravity at earth's surface, the increase in potential energy is [CPMT 1989; SCRA 1996; DPMT 2001]
(a) *mgh* (b) $\frac{4}{5}mgh$
(c) $\frac{5}{6}mgh$ (d) $\frac{6}{7}mgh$

2. In a gravitational field, at a point where the gravitational potential is zero [CPMT 1990]
(a) The gravitational field is necessarily zero
(b) The gravitational field is not necessarily zero
(c) Nothing can be said definitely about the gravitational field
(d) None of these

3. The gravitational field due to a mass distribution is $E = K/x^3$ in the *x*-direction. (*K* is a constant). Taking the gravitational potential to be zero at infinity, its value at a distance *x* is
(a) *K*/*x* (b) *K*/2x
(c) K/x^2 (d) $K/2x^2$

4. The mass of the earth is $6.00\times10^{24}\ kg$ and that of the moon is $7.40\times10^{22}\ kg$. The constant of gravitation $G = 6.67\times10^{-11}\ N-m^2/kg^2$. The potential energy of the system is $-7.79\times10^{28}\ joules$. The mean distance between the earth and moon is [MP PMT 1995]
(a) $3.80\times10^8\ metres$ (b) $3.37\times10^6\ metres$
(c) $7.60\times10^4\ metres$ (d) $1.90\times10^2\ metres$

5. The change in potential energy, when a body of mass *m* is raised to a height *nR* from the earth's surface is (*R* = Radius of earth)
(a) $mgR\frac{n}{n-1}$ (b) *nmgR*
(c) $mgR\frac{n^2}{n^2+1}$ (d) $mgR\frac{n}{n+1}$

6. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are distance *d* apart. The minimum velocity with which a particle of mass *m* should be projected from a point midway between their centres so that it escapes to infinity is [MP PET 1997]
(a) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (b) $2\sqrt{\frac{2G}{d}(M_1+M_2)}$
(c) $2\sqrt{\frac{Gm}{d}(M_1+M_2)}$ (d) $2\sqrt{\frac{Gm(M_1+M_2)}{d(R_1+R_2)}}$

7. If mass of earth is *M*, radius is *R* and gravitational constant is *G*, then work done to take 1 *kg* mass from earth surface to infinity will be [RPET 1997]
(a) $\sqrt{\frac{GM}{2R}}$ (b) $\frac{GM}{R}$
(c) $\sqrt{\frac{2GM}{R}}$ (d) $\frac{GM}{2R}$

8. A rocket is launched with velocity 10 *km*/*s*. If radius of earth is *R*, then maximum height attained by it will be [RPET 1997]
(a) 2*R* (b) 3*R*
(c) 4*R* (d) 5*R*

9. There are two bodies of masses 100 *kg* and 10000 *kg* separated by a distance 1 *m*. At what distance from the smaller body, the intensity of gravitational field will be zero [BHU 1997]
(a) $\frac{1}{9}m$ (b) $\frac{1}{10}m$
(c) $\frac{1}{11}m$ (d) $\frac{10}{11}m$

10. What is the intensity of gravitational field of the centre of a spherical shell [RPET 2000] [MP PET 1994]
(a) Gm/r^2 (b) *g*
(c) Zero (d) None of these

11. The gravitational potential energy of a body of mass '*m*' at the earth's surface $-mgR_e$. Its gravitational potential energy at a height R_e from the earth's surface will be (Here R_e is the radius of the earth) [AIIMS 2000; MP PET 2000; Pb. PMT 2004]
(a) $-2mgR_e$ (b) $2mgR_e$

(c) $\frac{1}{2}mgR_e$ (d) $-\frac{1}{2}mgR_e$

12. Escape velocity of a body of 1 kg mass on a planet is 100 *m/sec*. Gravitational Potential energy of the body at the Planet is

(a) – 5000 *J* (b) – 1000 *J*

(c) – 2400 *J* (d) 5000 *J*

13. A body of mass *m* is placed on the earth's surface. It is taken from the earth's surface to a height $h = 3R$. The change in gravitational potential energy of the body is [CBSE PMT 2002]

(a) $\frac{2}{3}mgR$ (b) $\frac{3}{4}mgR$

(c) $\frac{mgR}{2}$ (d) $\frac{mgR}{4}$

14. A body of mass *m kg.* starts falling from a point 2*R* above the Earth's surface. Its kinetic energy when it has fallen to a point '*R*' above the Earth's surface [*R*-Radius of Earth, *M*-Mass of Earth, *G*-Gravitational Constant] [MP PMT 2002]

(a) $\frac{1}{2}\frac{GMm}{R}$ (b) $\frac{1}{6}\frac{GMm}{R}$

(c) $\frac{2}{3}\frac{GMm}{R}$ (d) $\frac{1}{3}\frac{GMm}{R}$

15. A body is projected vertically upwards from the surface of a planet of radius *R* with a velocity equal to half the escape velocity for that planet. The maximum height attained by the body is [KCET (Engg./Med.) 2002]

(a) *R*/3 (b) *R*/2

(c) *R*/4 (d) *R*/5

16. Energy required to move a body of mass *m* from an orbit of radius 2*R* to 3*R* is [AIEEE 2002]

(a) $GMm/12R^2$ (b) $GMm/3R^2$

(c) $GMm/8R$ (d) $GMm/6R$

17. The kinetic energy needed to project a body of mass *m* from the earth surface (radius *R*) to infinity is [AIEEE 2002]

(a) *mgR*/2 (b) 2 *mgR*

(c) *mgR* (d) *mgR*/4

18. Radius of orbit of satellite of earth is *R*. Its kinetic energy is proportional to [BHU 2003; CPMT 2004]

(a) $\frac{1}{R}$ (b) $\frac{1}{\sqrt{R}}$

(c) R (d) $\frac{1}{R^{3/2}}$

19. In some region, the gravitational field is zero. The gravitational potential in this region [BVP 2003]

(a) Must be variable (b) Must be constant

(c) Cannot be zero (d) Must be zero

20. A particle falls towards earth from infinity. It's velocity on reaching the earth would be [Orissa JEE 2003]

(a) Infinity (b) $\sqrt{2gR}$

(c) $2\sqrt{gR}$ (d) Zero

21. Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity will depend on which of the following factors : [MP PMT 2002]

I. Mass of the planet

II. Mass of the particle escaping

III. Temperature of the planet

IV. Radius of the planet

Select the correct answer from the codes given below : [SCRA 1994]

(a) I and II (b) II and IV

(c) I and IV (d) I, III and IV

22. v_e and v_p denotes the escape velocity from the earth and another planet having twice the radius and the same mean density as the earth. Then [NCERT 1974; MP PMT 1994]

(a) $v_e = v_p$ (b) $v_e = v_p / 2$

(c) $v_e = 2v_p$ (d) $v_e = v_p / 4$

23. The escape velocity of a sphere of mass *m* from earth having mass *M* and radius *R* is given by [NCERT 1981, 84; CBSE PMT 1999]

(a) $\sqrt{\frac{2GM}{R}}$ (b) $2\sqrt{\frac{GM}{R}}$

(c) $\sqrt{\frac{2GMm}{R}}$ (d) $\sqrt{\frac{GM}{R}}$

24. The escape velocity for a rocket from earth is 11.2 *km/sec*. Its value on a planet where acceleration due to gravity is double that on the earth and diameter of the planet is twice that of earth will be in *km/sec* [NCERT 1983; CPMT 1990; MP PMT 2000; UPSEAT 1999]

(a) 11.2 (b) 5.6

(c) 22.4 (d) 53.6

25. The escape velocity from the earth is about 11 *km/second*. The escape velocity from a planet having twice the radius and the same mean density as the earth, is [NCERT 1980; MP PMT 1987; MP PET 2001, 2003; AIIMS 2001; UPSEAT 1999]

(a) 22 *km/sec* (b) 11 *km/sec*

(c) 5.5 *km/sec* (d) 15.5 *km/sec*

26. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is [MNR 1986; MP PET 1995]

(a) Positive

(b) Negative

(c) Zero

(d) May be positive or negative depending upon its initial velocity

27. If *g* is the acceleration due to gravity at the earth's surface and *r* is the radius of the earth, the escape velocity for the body to escape out of earth's gravitational field is [NCERT 1975; RPET 2003]

(a) gr (b) $\sqrt{2gr}$

(c) g/r (d) r/g

28. The escape velocity of a projectile from the earth is approximately[DPMT 1982, 8

(a) 11.2 *m/sec* (b) 112 *km/sec*

(c) 11.2 *km/sec* (d) 11200 *km/sec*

29. The escape velocity of a particle of mass m varies as [CPMT 1978; RPMT 1999; AIEEE 2002]

(a) m^2 (b) m

(c) m^0 (d) m^{-1}

30. For the moon to cease to remain the earth's satellite, its orbital velocity has to increase by a factor of [MP PET 1994]

(a) 2 (b) $\sqrt{2}$

(c) $1/\sqrt{2}$ (d) $\sqrt{3}$

31. The escape velocity of an object from the earth depends upon the mass of the earth (M), its mean density (ρ), its radius (R) and the gravitational constant (G). Thus the formula for escape velocity is

(a) $v = R\sqrt{\frac{8\pi}{3}G\rho}$ (b) $v = M\sqrt{\frac{8\pi}{3}GR}$

(c) $v = \sqrt{2GMR}$ (d) $v = \sqrt{\frac{2GM}{R^2}}$

32. Escape velocity on a planet is v_e. If radius of the planet remains same and mass becomes 4 times, the escape velocity becomes [MP PMT 1996; DPMT 1999]

(a) $4v_e$ (b) $2v_e$

(c) v_e (d) $\frac{1}{2}v_e$

33. The mass of the earth is 81 times that of the moon and the radius of the earth is 3.5 times that of the moon. The ratio of the escape velocity on the surface of earth to that on the surface of moon will be [MP PMT/PET 1998; JIPMER 2000]

(a) 0.2 (b) 2.57

(c) 4.81 (d) 0.39

34. The escape velocity from the surface of earth is V_e. The escape velocity from the surface of a planet whose mass and radius are 3 times those of the earth will be

[MP PMT/PET 1998; JIPMER 2001, 02; Pb. PMT 2004]

(a) V_e (b) $3V_e$

(c) $9V_e$ (d) $27V_e$

35. How much energy will be necessary for making a body of 500 kg escape from the earth

$[g = 9.8\ m/s^2$, radius of earth $= 6.4\times10^6\ m]$

[MP PET 1999]

(a) About $9.8\times10^6\ J$ (b) About $6.4\times10^8\ J$

(c) About $3.1\times10^{10}\ J$ (d) About $27.4\times10^{12}\ J$

36. The escape velocity for the earth is 11.2 km/sec. The mass of another planet is 100 times that of the earth and its radius is 4 times that of the earth. The escape velocity for this planet will be [MP PMT 1999; Pb. PMT 2002]

(a) 112.0 km/s (b) 5.6 km/s

(c) 280.0 km/s (d) 56.0 km/s

37. The escape velocity of a planet having mass 6 times and radius 2 times as that of earth is

[CPMT 1999; MP PET 2003; Pb. PET 2002]

(a) $\sqrt{3}\ V_e$ (b) $3\ V_e$

(c) $\sqrt{2}\ V_e$ (d) $2\ V_e$

38. The escape velocity of an object on a planet whose g value is 9 times on earth and whose radius is 4 times that of earth in km/s is [EAMCET 1994]

(a) 67.2 (b) 33.6

(c) 16.8 (d) 25.2

39. The escape velocity on earth is 11.2 km/s. On another planet having twice radius and 8 times mass of the earth, the escape velocity will be [Bihar CMEET 1995]

(a) 3.7 km/s (b) 11.2 km/s

(c) 22.4 km/s (d) 43.2 km/s

40. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and the radius of the earth becomes half, the escape velocity would become [MP PMT 1995]

(a) 5.6 km/s

(b) 11.2 km/s (remain unchanged)

(c) 22.4 km/s

(d) 44.8 km/s

41. Given mass of the moon is 1/81 of the mass of the earth and corresponding radius is 1/4 of the earth. If escape velocity on the earth surface is 11.2 km/s, the value of same on the surface of the moon is

[CPMT 1997; AIIMS 2000; Pb. PMT 2001]

(a) 0.14 km/s (b) 0.5 km/s

(c) 2.5 km/s (d) 5 km/s

42. The angular velocity of rotation of star (of mass M and radius R) at which the matter start to escape from its equator will be

(a) $\sqrt{\frac{2GM^2}{R}}$ (b) $\sqrt{\frac{2GM}{g}}$

(c) $\sqrt{\frac{2GM}{R^3}}$ (d) $\sqrt{\frac{2GR}{M}}$

43. The least velocity required to throw a body away from the surface of a planet so that it may not return is (radius of the planet is $6.4\times10^6 m,\ g = 9.8\ m/sec^2$) [AMU (Engg.) 1999]

(a) $9.8\times10^{-3}\ m/sec$ (b) $12.8\times10^3\ m/sec$

(c) $9.8\times10^3\ m/sec$ (d) $11.2\times10^3\ m/sec$

44. How many times is escape velocity (V_e), of orbital velocity (V_0) for a satellite revolving near earth [RPMT 2000]

(a) $\sqrt{2}$ *times* (b) 2 *times*

(c) 3 *times* (d) 4 *times*

45. Escape velocity on earth is 11.2 km/s. What would be the escape velocity on a planet whose mass is 1000 times and radius is 10 times that of earth [DCE 2001; DPMT 2004]

(a) 112 km/s (b) 11.2 km/s

(c) 1.12 km/s (d) 3.7 km/s

46. If the radius of a planet is R and its density is ρ, the escape velocity from its surface will be [MP PMT 2001]

(a) $v_e \propto \rho R$ (b) $v_e \propto \sqrt{\rho}R$

(c) $v_e \propto \frac{\sqrt{\rho}}{R}$ (d) $v_e \propto \frac{1}{\sqrt{\rho}R}$

47. Escape velocity on the earth [BHU 2001]

(a) Is less than that on the moon

(b) Depends upon the mass of the body

(c) Depends upon the direction of projection

(d) Depends upon the height from which it is projected

48. If acceleration due to gravity on the surface of a planet is two times that on surface of earth and its radius is double that of earth. Then escape velocity from the surface of that planet in comparison to earth will be **[RPET 2001]**

(a) $2v_e$ (b) $3v_e$

(c) $4v_e$ (d) None of these

49. The escape velocity of a rocket launched from the surface of the earth **[UPSEAT 2001]**

(a) Does not depend on the mass of the rocket

(b) Does not depend on the mass of the earth

(c) Depends on the mass of the planet towards which it is moving

(d) Depends on the mass of the rocket

50. The ratio of the radii of planets A and B is k_1 and ratio of acceleration due to gravity on them is k_2. The ratio of escape velocities from them will be **[BHU 2002]**

(a) k_1k_2 (b) $\sqrt{k_1k_2}$

(c) $\sqrt{\frac{k_1}{k_2}}$ (d) $\sqrt{\frac{k_2}{k_1}}$

51. A mass of $6\times10^{24}\,kg$ is to be compressed in a sphere in such a way that the escape velocity from the sphere is $3\times10^8\,m/s$. Radius of the sphere should be $(G=6.67\times10^{-11}\,N-m^2/kg^2)$

(a) 9 *km* (b) 9 *m*

(c) 9 *cm* (d) 9 *mm*

52. The escape velocity of a body on an imaginary planet which is thrice the radius of the earth and double the mass of the earth is (v_e is the escape velocity of earth) **[Kerala (Med.) 2002]**

(a) $\sqrt{2/3}\,v_e$ (b) $\sqrt{3/2}\,v_e$

(c) $\sqrt{2}/3\,v_e$ (d) $2/\sqrt{3}\,v_e$

53. Escape velocity on the surface of earth is $11.2\,km/s$. Escape velocity from a planet whose mass is the same as that of earth and radius 1/4 that of earth is **[CBSE PMT 2000; JIPMER 2002; BHU 2004]**

(a) 2.8 *km/s* (b) 15.6 *km/s*

(c) 22.4 *km/s* (d) 44.8 *km/s*

54. The velocity with which a projectile must be fired so that it escapes earth's gravitation does not depend on **[AIIMS 2003]**

(a) Mass of the earth

(b) Mass of the projectile

(c) Radius of the projectile's orbit

(d) Gravitational constant

55. The radius of a planet is $\frac{1}{4}$ of earth's radius and its acceleration due to gravity is double that of earth's acceleration due to gravity. How many times will the escape velocity at the planet's surface be as compared to its value on earth's surface **[BCECE 2003; MH CET 2000]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $2\sqrt{2}$ (d) 2

56. The escape velocity for the earth is v_e. The escape velocity for a planet whose radius is four times and density is nine times that of the earth, is **[MP PET 2003]**

(a) $36v_e$ (b) $12v_e$

(c) $6v_e$ (d) $20v_e$

57. The escape velocity for a body projected vertically upwards from the surface of earth is 11 *km/s*. If the body is projected at an angle of 45° with the vertical, the escape velocity will be

(a) $\frac{11}{\sqrt{2}}\,km/s$ (b) $11\sqrt{2}\,km/s$

(c) 22 *km/s* (d) 11 *km/s*

58. If *V*, *R* and *g* denote respectively the escape velocity from the surface of the earth radius of the earth, and acceleration due to gravity, then the correct equation is **[MP PMT 2004]**

(a) $V=\sqrt{gR}$ (b) $V=\sqrt{\frac{4}{3}gR^3}$

(c) $V=R\sqrt{g}$ (d) $V=\sqrt{2gR}$

59. The escape velocity for a body of mass 1 *kg* from the earth surface is $11.2\,kms^{-1}$. The escape velocity for a body of mass 100 *kg* would be **[UPSEAT 2002]** **[DCE 2003]**

(a) $11.2\times10^2\,kms^{-1}$ (b) $11.2\,kms^{-1}$

(c) $11.2\times10^{-2}\,kms^{-1}$ (d) None of these

60. The acceleration due to gravity on a planet is same as that on earth and its radius is four times that of earth. What will be the value of escape velocity on that planet if it is v_e on earth

(a) v_e (b) $2v_e$

(c) $4v_e$ (d) $\frac{v_e}{2}$

61. If the radius of a planet is four times that of earth and the value of *g* is same for both, the escape velocity on the planet will be

(a) $11.2\,km/s$ (b) $5.6\,km/s$

(c) $22.4\,km/s$ (d) None

62. If the radius and acceleration due to gravity both are doubled, escape velocity of earth will become **[RPMT 2002]**

(a) 11.2 *km/s* (b) 22.4 *km/s*

(c) 5.6 *km/s* (d) 44.8 *km/s*

63. A planet has twice the radius but the mean density is $\frac{1}{4}th$ as compared to earth. What is the ratio of escape velocity from earth to that from the planet **[MH CET 2004]**

(a) 3 : 1 (b) 1 : 2

(c) 1 : 1 (d) 2 : 1

64. The escape velocity from earth is v_{es}. A body is projected with velocity $2v_{es}$ with what constant velocity will it move in the inter planetary space **[DCE 2002]**

(a) v_{es} (b) $3v_{es}$

(c) $\sqrt{3}v_{es}$ (d) $\sqrt{5}v_{es}$

65. A particle of mass 10 *g* is kept on the surface of a uniform sphere of mass 100 *kg* and radius 10 *cm*. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)

(a) 6.67×10^{-9} *J* (b) 6.67×10^{-10} *J*

(c) 13.34×10^{-10} *J* (d) 3.33×10^{-10} *J*

66. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is **[CBSE PMT 2005]**

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

67. 3 particles each of mass *m* are kept at vertices of an equilateral triangle of side *L*. The gravitational field at centre due to these particles is **[DCE 2005]**

(a) Zero (b) $\frac{3GM}{L^2}$

(c) $\frac{9GM}{L^2}$ (d) $\frac{12}{\sqrt{3}}\frac{GM}{L^2}$

68. The value of escape velocity on a certain planet is 2 *km/s*. Then the value of orbital speed for a satellite orbiting close to its surface is

(a) 12 *km/s* (b) 1 *km/s*

(c) $\sqrt{2}$ *km/s* (d) $2\sqrt{2}\ km/s$

69. Four particles each of mass *M*, are located at the vertices of a square with side *L*. The gravitational potential due to this at the centre of the square is **[Kerala PET 2005]**

(a) $-\sqrt{32}\frac{GM}{L}$ (b) $-\sqrt{64}\frac{GM}{L^2}$

(c) Zero (d) $\sqrt{32}\frac{GM}{L}$

70. There are two planets. The ratio of radius of the two planets is *K* but ratio of acceleration due to gravity of both planets is *g*. What will be the ratio of their escape velocity **[BHU 2005]**

(a) $(Kg)^{1/2}$ (b) $(Kg)^{-1/2}$

(c) $(Kg)^2$ (d) $(Kg)^{-2}$

Motion of Satellite

1. If v_e and v_o represent the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius *R*, then **[CPMT 1982; MP PMT 1997; KCET (Engg./Med.) 1999; AIIMS 2002]**

(a) $v_e = v_o$

(b) $\sqrt{2}v_o = v_e$

(c) $v_e = v_0/\sqrt{2}$

(d) v_e and v_o are not related

2. If *r* represents the radius of the orbit of a satellite of mass *m* moving around a planet of mass *M*, the velocity of the satellite is given by **[CPMT 1974; MP PMT 1987; RPMT 1999]**

(a) $v^2 = g\frac{M}{r}$ (b) $v^2 = \frac{GMm}{r}$

(c) $v = \frac{GM}{r}$ **[AIEEE 2005]** (d) $v^2 = \frac{GM}{r}$

3. Select the correct statement from the following **[MP PMT 1993]**

(a) The orbital velocity of a satellite increases with the radius of the orbit

(b) Escape velocity of a particle from the surface of the earth depends on the speed with which it is fired

(c) The time period of a satellite does not depend on the radius of the orbit

(d) The orbital velocity is inversely proportional to the square root of the radius of the orbit

4. An earth satellite of mass *m* revolves in a circular orbit at a height *h* from the surface of the earth. *R* is the radius of the earth and *g* is acceleration due to gravity at the surface of the earth. The velocity of the satellite in the orbit is given by **[NCERT 1983; AIEEE 2004]**

(a) $\frac{gR^2}{R+h}$ (b) gR

(c) $\frac{gR}{R+h}$ **[DCE 2005]** (d) $\sqrt{\frac{gR^2}{R+h}}$

5. Consider a satellite going round the earth in an orbit. Which of the following statements is wrong **[NCERT 1966]**

(a) It is a freely falling body

(b) It suffers no acceleration

(c) It is moving with a constant speed

(d) Its angular momentum remains constant

6. Two satellites of masses m_1 and $m_2 (m_1 > m_2)$ are revolving round the earth in circular orbits of radius r_1 and $r_2 (r_1 > r_2)$ respectively. Which of the following statements is true regarding their speeds v_1 and v_2 ? **[NCERT 1984; MNR 1995; BHU 1998]**

(a) $v_1 = v_2$ (b) $v_1 < v_2$

(c) $v_1 > v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

7. A satellite which is geostationary in a particular orbit is taken to another orbit. Its distance from the centre of earth in new orbit is 2 times that of the earlier orbit. The time period in the second orbit is**[NCERT 19**

(a) 4.8 *hours* (b) $48\sqrt{2}$ *hours*

(c) 24 *hours* (d) $24\sqrt{2}$ *hours*

8. The ratio of the K.E. required to be given to the satellite to escape earth's gravitational field to the K.E. required to be given so that the satellite moves in a circular orbit just above earth atmosphere is

(a) One (b) Two

(c) Half (d) Infinity

9. An astronaut orbiting the earth in a circular orbit 120 *km* above the surface of earth, gently drops a spoon out of space-ship. The spoon will [NCERT 1971]

(a) Fall vertically down to the earth
(b) Move towards the moon
(c) Will move along with space-ship
(d) Will move in an irregular way then fall down to earth

10. The period of a satellite in a circular orbit around a planet is independent of [NCERT 1974; AIEEE 2004]

(a) The mass of the planet
(b) The radius of the planet
(c) The mass of the satellite
(d) All the three parameters (a), (b) and (c)

11. If a satellite is orbiting the earth very close to its surface, then the orbital velocity mainly depends on [NCERT 1982]

(a) The mass of the satellite only
(b) The radius of the earth only
(c) The orbital radius only
(d) The mass of the earth only

12. The relay satellite transmits the T.V. programme continuously from one part of the world to another because its

(a) Period is greater than the period of rotation of the earth
(b) Period is less than the period of rotation of the earth about its axis
(c) Period has no relation with the period of the earth about its axis
(d) Period is equal to the period of rotation of the earth about its axis
(e) Mass is less than the mass of the earth

13. Two satellites *A* and *B* go round a planet *P* in circular orbits having radii $4R$ and R respectively. If the speed of the satellite *A* is $3V$, the speed of the satellite *B* will be.

[MNR 1991; AIIMS 1995; UPSEAT 2000]

(a) 12 V (b) 6 V
(c) $\frac{4}{3}V$ (d) $\frac{3}{2}V$

14. A geostationary satellite [CPMT 1990]

(a) Revolves about the polar axis
(b) Has a time period less than that of the near earth satellite
(c) Moves faster than a near earth satellite
(d) Is stationary in the space

15. A small satellite is revolving near earth's surface. Its orbital velocity will be nearly

[CPMT 1987; Orissa JEE 2002; JIPMER 2001, 02]

(a) 8 *km/sec* (b) 11.2 *km/sec*
(c) 4 *km/sec* (d) 6 *km/sec*

16. A satellite revolves around the earth in an elliptical orbit. Its speed [NCERT 1981; MP PET 2001]

(a) Is the same at all points in the orbit
(b) Is greatest when it is closest to the earth
(c) Is greatest when it is farthest from the earth
(d) Goes on increasing or decreasing continuously depending upon the mass of the satellite

17. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is *v*. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

[MNR 1994]

(a) $\frac{3}{2}v$ (b) $\sqrt{\frac{3}{2}}v$
(c) $\sqrt{\frac{2}{3}}v$ (d) $\frac{2}{3}v$

18. In a satellite if the time of revolution is *T*, then K.E. is proportional to [BHU 1995]

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$
(c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

19. If the height of a satellite from the earth is negligible in comparison to the radius of the earth *R*, the orbital velocity of the satellite is[MP PET 1995;

(a) gR (b) $gR/2$
(c) $\sqrt{g/R}$ (d) $\sqrt{gR}$

20. Choose the correct statement from the following : The radius of the orbit of a geostationary satellite depends upon [MNR 1984, 93]

[MP PMT 1995]

(a) Mass of the satellite, its time period and the gravitational constant
(b) Mass of the satellite, mass of the earth and the gravitational constant
(c) Mass of the earth, mass of the satellite, time period of the satellite and the gravitational constant
(d) Mass of the earth, time period of the satellite and the gravitational constant

21. Out of the following, the only incorrect statement about satellites is

(a) A satellite cannot move in a stable orbit in a plane passing through the earth's centre
(b) Geostationary satellites are launched in the equatorial plane
(c) We can use just one geostationary satellite for global communication around the globe
(d) The speed of a satellite increases with an increase in the radius of its orbit

22. A satellite is moving around the earth with speed v in a circular orbit of radius *r*. If the orbit radius is decreased by 1%, its speed will[MP PET 19

(a) Increase by 1% (b) Increase by 0.5%
(c) Decrease by 1% (d) Decrease by 0.5%

23. Orbital velocity of an artificial satellite does not depend upon

(a) Mass of the earth
(b) Mass of the satellite
(c) Radius of the earth
(d) Acceleration due to gravity

24. The time period of a geostationary satellite is

[EAMCET 1994; MP PMT 1999]

(a) 24 hours (b) 12 hours
(c) 365 days (d) One month

25. Orbital velocity of earth's satellite near the surface is 7 *km/s*. When the radius of the orbit is 4 times than that of earth's radius, then orbital velocity in that orbit is [EAMCET (Engg.) 1995]

(a) 3.5 *km/s* (b) 7 *km/s*
(c) 72 *km/s* (d) 14 *km/s*

26. Two identical satellites are at R and $7R$ away from earth surface, the wrong statement is (R = Radius of earth) [RPMT 1997]

(a) Ratio of total energy will be 4
(b) Ratio of kinetic energies will be 4
(c) Ratio of potential energies will be 4
(d) Ratio of total energy will be 4 but ratio of potential and kinetic energies will be 2

27. For a satellite escape velocity is 11 *km/s*. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be [CBSE PMT 1993; RPMT 1997]

(a) 11 *km/s* (b) $11\sqrt{3}$ *km/s*
(c) $\frac{11}{\sqrt{3}}$ *km/s* (d) 33 *km/s*

28. The mean radius of the earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. The cube of the radius of the orbit of a geostationary satellite will be

(a) R^2g/ω (b) $R^2\omega^2/g$
(c) Rg/ω^2 (d) R^2g/ω^2

29. Which one of the following statements regarding artificial satellite of the earth is incorrect [NDA 1995; MP PMT 2000]

(a) The orbital velocity depends on the mass of the satellite
(b) A minimum velocity of 8 *km/sec* is required by a satellite to orbit quite close to the earth
(c) The period of revolution is large if the radius of its orbit is large
(d) The height of a geostationary satellite is about 36000 *km* from earth

30. A ball is dropped from a spacecraft revolving around the earth at a height of 120 *km*. What will happen to the ball [CBSE PMT 1996; CPMT 2001; BHU 1999]

(a) It will continue to move with velocity *v* along the original orbit of spacecraft
(b) It will move with the same speed tangentially to the spacecraft
(c) It will fall down to the earth gradually
(d) It will go very far in the space

31. A satellite whose mass is M, is revolving in circular orbit of radius r around the earth. Time of revolution of satellite is [AMU 1999]

(a) $T \propto \frac{r^5}{GM}$ (b) $T \propto \sqrt{\frac{r^3}{GM}}$
(c) $T \propto \sqrt{\frac{r}{GM^2/3}}$ (d) $T \propto \sqrt{\frac{r^3}{GM^1/4}}$

32. An artificial satellite is placed into a circular orbit around earth at such a height that it always remains above a definite place on the surface of earth. Its height from the surface of earth is

(a) 6400 *km* (b) 4800 *km*
(c) 32000 *km* (d) 36000 *km*

33. The weight of an astronaut, in an artificial satellite revolving around the earth, is [BHU 1999]

(a) Zero
(b) Equal to that on the earth
(c) More than that on the earth
(d) Less than that on the earth

34. In the following four periods [AMU 2000]

(i) Time of revolution of a satellite just above the earth's surface (T_{st})
(ii) Period of oscillation of mass inside the tunnel bored along the diameter of the earth (T_{ma})
(iii) Period of simple pendulum having a length equal to the earth's radius in a uniform field of 9.8 *N/kg* (T_{sp})
(iv) Period of an infinite length simple pendulum in the earth's real gravitational field (T_{is})

(a) $T_{st} > T_{ma}$ (b) $T_{ma} > T_{st}$
(c) $T_{sp} < T_{is}$ [CBSE PMT 1992] (d) $T_{st} = T_{ma} = T_{sp} = T_{is}$

35. The periodic time of a communication satellite is [MP PMT 2000]

(a) 6 *hours* (b) 12 *hours*
(c) 18 *hours* (d) 24 *hours*

36. The orbital speed of an artificial satellite very close to the surface of the earth is V_o. Then the orbital speed of another artificial satellite at a height equal to three times the radius of the earth is

(a) $4\,V_o$ (b) $2\,V_o$
(c) $0.5\,V_o$ (d) $4\,V_o$

37. Which of the following statements is correct in respect of a geostationary satellite [MP PET 2001]

(a) It moves in a plane containing the Greenwich meridian
(b) It moves in a plane perpendicular to the celestial equatorial plane
(c) Its height above the earth's surface is about the same as the radius of the earth
(d) Its height above the earth's surface is about six times the radius of the earth

38. The distance of a geo-stationary satellite from the centre of the earth (Radius R = 6400 *km*) is nearest to [AFMC 2001]

(a) 5 R (b) 7 R
(c) 10 R (d) 18 R

39. If Gravitational constant is decreasing in time, what will remain unchanged in case of a satellite orbiting around earth

(a) Time period (b) Orbiting radius
(c) Tangential velocity (d) Angular velocity

40. Periodic time of a satellite revolving above Earth's surface at a height equal to R, radius of Earth, is

[g is acceleration due to gravity at Earth's surface]

[MP PMT 2002]

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $4\sqrt{2}\pi\sqrt{\frac{R}{g}}$

(c) $2\pi\sqrt{\frac{R}{g}}$ (d) $8\pi\sqrt{\frac{R}{g}}$

41. Given radius of Earth 'R' and length of a day 'T' the height of a geostationary satellite is [G–Gravitational Constant, M–Mass of Earth] [MP PMT 2002]

(a) $\left(\frac{4\pi^2 GM}{T^2}\right)^{1/3}$ (b) $\left(\frac{4\pi GM}{R^2}\right)^{1/3} - R$

(c) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$ (d) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} + R$

42. A geo-stationary satellite is orbiting the earth at a height of 6 R above the surface of earth, R being the radius of earth. The time period of another satellite at a height of 2.5 R from the surface of earth is

[UPSEAT 2002; AMU (Med.) 2002; Pb. PET 2003]

(a) 10 hr (b) $(6/\sqrt{2})hr$

(c) 6 hr (d) $6\sqrt{2}\,hr$

43. The distance between centre of the earth and moon is 384000 km. If the mass of the earth is $6\times10^{24}\,kg$ and $G = 6.66\times10^{-11}\,Nm^2/kg^2$. The speed of the moon is nearly

[MH CET 2002]

(a) 1 km/sec (b) 4 km/sec

(c) 8 km/sec (d) 11.2 km/sec

44. A satellite is launched into a circular orbit of radius 'R' around earth while a second satellite is launched into an orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is

(a) 0.7 (b) 1.0

(c) 1.5 (d) 3

45. Where can a geostationary satellite be installed

[MP PMT 2004]

(a) Over any city on the equator

(b) Over the north or south pole

(c) At height R above earth

(d) At the surface of earth

46. Distance of geostationary satellite from the surface of earth $radius(R_e = 6400\ km)$ in terms of R_e is [Pb. PET 2000]

(a) 13.76 R_e (b) 10.76 R_e

(c) 6.56 R_e (d) 2.56 R_e

47. A satellite is to revolve round the earth in a circle of radius 8000 km. The speed at which this satellite be projected into an orbit, will be [Pb. PET 2002]

(a) $3\ km/s$ (b) $16\ km/s$

(c) $7.15\ km/s$ (d) $8\ km/s$

48. Two satellite A and B, ratio of masses 3 : 1 are in circular orbits of radii r and $4r$. Then ratio of total mechanical energy of A to B is

(a) 1 : 3 (b) 3 : 1

(c) 3 : 4 (d) 12 : 1

49. The orbital velocity of a planet revolving close to earth's surface is

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$

(c) $\sqrt{\frac{2g}{R}}$ (d) $\sqrt{\frac{g}{R}}$

50. If the gravitational force between two objects were proportional to 1/R (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to [CBSE PMT 1994; JIPMER 2001, 02]

(a) $1/R^2$ (b) R^0

(c) R^1 (d) 1/R

51. A satellite moves around the earth in a circular orbit of radius r with speed v. If the mass of the satellite is M, its total energy is

(a) $-\frac{1}{2}Mv^2$ (b) $\frac{1}{2}Mv^2$

(c) $\frac{3}{2}Mv^2$ (d) Mv^2

52. A satellite with kinetic energy E_k is revolving round the earth in a circular orbit. How much more kinetic energy should be given to it so that it may just escape into outer space

(a) E_k (b) $2E_k$

(c) $\frac{1}{2}E_k$ (d) $3E_k$

53. Potential energy of a satellite having mass 'm' and rotating at a height of $6.4\times10^6\,m$ from the earth surface is

[AIIMS 2000; CBSE PMT 2001; BHU 2001] [EAMCET 2003]

(a) $-0.5\,mgR_e$ (b) $-mgR_e$

(c) $-2\,mgR_e$ (d) $4\,mgR_e$

54. When a satellite going round the earth in a circular orbit of radius r and speed v loses some of its energy, then r and v change as [JIPMER 2002; EAM

(a) r and v both with increase

(b) r and v both will decrease

(c) r will decrease and v will increase

(d) r will decrease and v will decrease

55. An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is

(a) 4 days (b) 8 days

(c) 16 days (d) 32 days

56. Which is constant for a satellite in orbit

[Bihar CMEET 1995]

(a) Velocity (b) Angular momentum

(c) Potential energy (d) Acceleration

(e) Kinetic energy

57. If satellite is shifted towards the earth. Then time period of satellite will be [RPMT 2000]

(a) Increase (b) Decrease

(c) Unchanged (d) Nothing can be said

58. Which of the following quantities does not depend upon the orbital radius of the satellite [DCE 2000,03]

(a) $\frac{T}{R}$ (b) $\frac{T^2}{R}$

(c) $\frac{T^2}{R^2}$ (d) $\frac{T^2}{R^3}$

59. The time period of a satellite of earth is 5 *hours*. If the separation between the earth and the satellite is increased to four times the previous value, the new time period will become

(a) 20 *hours* (b) 10 *hours*

(c) 80 *hours* (d) 40 *hours*

60. A satellite moves round the earth in a circular orbit of radius *R* making one revolution per day. A second satellite moving in a circular orbit, moves round the earth once in 8 days. The radius of the orbit of the second satellite is

[UPSEAT 2004]

(a) 8 *R* (b) 4*R*

(c) 2*R* (d) *R*

61. A person sitting in a chair in a satellite feels weightless because

(a) The earth does not attract the objects in a satellite

(b) The normal force by the chair on the person balances the earth's attraction

(c) The normal force is zero

(d) The person in satellite is not accelerated

62. Two satellites *A* and *B* go round a planet in circular orbits having radii 4*R* and *R*, respectively. If the speed of satellite *A* is 3*v*, then speed of satellite *B* is [Pb. PET 2004]

(a) $\frac{3v}{2}$ (b) $\frac{4v}{2}$

(c) $6v$ (d) $12v$

63. If $g \propto \frac{1}{R^3}$ (instead of $\frac{1}{R^2}$), then the relation between time period of a satellite near earth's surface and radius *R* will be

(a) $T^2 \propto R^3$ (b) $T \propto R^2$

(c) $T^2 \propto R$ (d) $T \propto R$

64. To an astronaut in a spaceship, the sky appears

[KCET 1994]

(a) Black (b) White

(c) Green (d) Blue

65. A geostationary satellite is revolving around the earth. To make it escape from gravitational field of earth, is velocity must be increased

(a) 100% (b) 41.4%

(c) 50% (d) 59.6%

66. A satellite moves in a circle around the earth. The radius of this circle is equal to one half of the radius of the moon's orbit. The satellite completes one revolution in

[J&K CET 2005]

(a) $\frac{1}{2}$ lunar month (b) $\frac{2}{3}$ lunar month

(c) $2^{-3/2}$ lunar month (d) $2^{3/2}$ lunar month

67. A satellite of mass *m* is placed at a distance *r* from the centre of earth (mass *M*). The mechanical energy of the satellite is

[J&K CET 2005]

(a) $-\frac{GMm}{r}$ (b) $\frac{GMm}{r}$

(c) $\frac{GMm}{2r}$ (d) $-\frac{GMm}{2r}$

Kepler's Laws of Planetary Motion

1. The distance of neptune and saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio [NCERT 1975; CBSE PMT 1994; MP

[AIIMS 1995; AIEEE 2003]

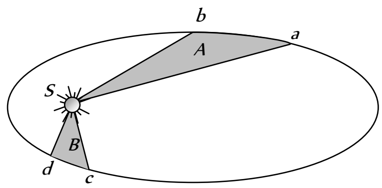
(a) $\sqrt{10}$ (b) 100

(c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

2. The figure shows the motion of a planet around the sun in an elliptical orbit with sun at the focus. The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time for the planet to move from *a* to *b* and *d* to *c* respectively, then

[CPMT 1986, 88]

[UPSEAT 2004]



(a) $t_1 < t_2$

(b) $t_1 > t_2$

(c) $t_1 = t_2$

(d) $t_1 \le t_2$

3. The period of a satellite in a circular orbit of radius *R* is *T*, the period of another satellite in a circular orbit of radius 4*R* is

AIIMS 2000; CBSE PMT 2002]

(a) 4*T* (b) *T*/4

(c) 8*T* (d) *T*/8

4. Orbit of a planet around a star is [CPMT 1982]

(a) A circle (b) An ellipse

(c) A parabola [RPMT 2002] (d) A straight line

5. If a body describes a circular motion under inverse square field, the time taken to complete one revolution T is related to the radius of the circular orbit as

[NCERT 1975; RPMT 2000]

(a) $T \propto r$ (b) $T \propto r^2$

(c) $T^2 \propto r^3$ (d) $T \propto r^4$

6. If the earth is at one-fourth of its present distance from the sun, the duration of the year will be [EAMCET 1987]

[J&K CET 2005]

(a) Half the present year

(b) One-eighth the present year

(c) One-fourth the present year

(d) One-sixth the present year

7. The earth revolves about the sun in an elliptical orbit with mean radius $9.3 \times 10^7\ m$ in a period of 1 year. Assuming that there are no outside influences

(a) The earth's kinetic energy remains constant

(b) The earth's angular momentum remains constant

(c) The earth's potential energy remains constant

(d) All are correct

8. Venus looks brighter than other planets because

[MNR 1985]

(a) It is heavier than other planets

(b) It has higher density than other planets

(c) It is closer to the earth than other planets

(d) It has no atmosphere

9. A planet moves around the sun. At a given point *P*, it is closest from the sun at a distance d_1 and has a speed v_1. At another point *Q*, when it is farthest from the sun at a distance d_2, its speed will be [MP PMT 1987; DCE 2002]

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$

(c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

10. The orbital speed of Jupiter is [MNR 1986; UPSEAT 2000]

(a) Greater than the orbital speed of earth

(b) Less than the orbital speed of earth

(c) Equal to the orbital speed of earth

(d) Zero

11. Two planets move around the sun. The periodic times and the mean radii of the orbits are T_1, T_2 and r_1, r_2 respectively. The ratio T_1 / T_2 is equal to [CPMT 1978]

(a) $(r_1 / r_2)^{1/2}$ (b) r_1 / r_2

(c) $(r_1 / r_2)^2$ (d) $(r_1 / r_2)^{3/2}$

12. Kepler's second law regarding constancy of aerial velocity of a planet is a consequence of the law of conservation of [CPMT 1990; AIIMS 2002]

(a) Energy (b) Angular momentum

(c) Linear momentum (d) None of these

13. The largest and the shortest distance of the earth from the sun are r_1 and r_2, its distance from the sun when it is at the perpendicular to the major axis of the orbit drawn from the sun

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 r_2}{r_1 + r_2}$

(c) $\frac{2 r_1 r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$

14. The rotation period of an earth satellite close to the surface of the earth is 83 minutes. The time period of another earth satellite in an orbit at a distance of three earth radii from its surface will be

(a) 83 minutes (b) $83 \times \sqrt{8}$ minutes

(c) 664 minutes (d) 249 minutes

15. A satellite of mass m is circulating around the earth with constant angular velocity. If radius of the orbit is R_0 and mass of the earth *M*, the angular momentum about the centre of the earth is [MP PMT 1996; RPMT 2000]

(a) $m\sqrt{GMR_0}$ (b) $M\sqrt{GmR_0}$

(c) $m\sqrt{\frac{GM}{R_0}}$ (d) $M\sqrt{\frac{GM}{R_0}}$

16. According to Kepler, the period of revolution of a planet (*T*) and its mean distance from the sun (*r*) are related by the equation [EAMCET (Med.) 1995; MH CET 2000; Pb. PET 2001]

(a) $T^3 r^3 =$ constant (b) $T^2 r^{-3} =$ constant

(c) $Tr^3 =$ constant (d) $T^2 r =$ constant

17. A planet revolves around sun whose mean distance is 1.588 times the mean distance between earth and sun. The revolution time of planet will be [RPET 1997]

(a) 1.25 years (b) 1.59 years

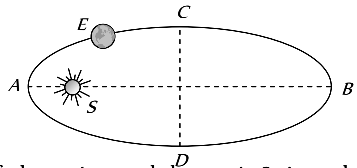
(c) 0.89 years (d) 2 years

18. A satellite *A* of mass m is at a distance of r from the centre of the earth. Another satellite *B* of mass $2m$ is at a distance of $2r$ from the earth's centre. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16

(c) 1 : 32 (d) $1 : 2\sqrt{2}$

19. The earth *E* moves in an elliptical orbit with the sun *S* at one of the foci as shown in figure. Its speed of motion will be maximum at the point [BHU 1994; CPMT 1997]



(a) *C*

(b) *A*

(c) *B*

(d) *D*

20. The period of revolution of planet *A* around the sun is 8 times that of *B*. The distance of *A* from the sun is how many times greater than that of *B* from the sun [CBSE PMT 1997; BHU 2001]

(a) 2 (b) 3

(c) 4 (d) 5

21. If the radius of earth's orbit is made 1/4, the duration of an year will become [BHU 1998; JIPMER 2001, 2002]

(a) 8 times (b) 4 times

(c) 1/8 times (d) 1/4 times

22. Planetary system in the solar system describes [DCE 1999]

(a) Conservation of energy

(b) Conservation of linear momentum

(c) Conservation of angular momentum [CBSE PMT 1991]

(d) None of these

23. If mass of a satellite is doubled and time period remain constant the ratio of orbit in the two cases will be [RPET 2000]

(a) 1 : 2 (b) 1 : 1

(c) 1 : 3 (d) None of these

24. The earth revolves round the sun in one year. If the distance between them becomes double, the new period of revolution will be [MP PMT 1994]

(a) 1/2 *year* (b) $2\sqrt{2}$ *years*

(c) 4 *years* (d) 8 *years*

25. Kepler discovered [DCE 2000]

(a) Laws of motion

(b) Laws of rotational motion

(c) Laws of planetory motion

(d) Laws of curvilinear motion

26. In the solar system, which is conserved [DCE 2001]

(a) Total Energy (b) K.E.

(c) Angular Velocity (d) Linear Momentum

27. The maximum and minimum distances of a comet from the sun are $8 \times 10^{12}\ m$ and $1.6 \times 10^{12}\ m$. If its velocity when nearest to the sun is 60 *m/s*, what will be its velocity in *m/s* when it is farthest

(a) 12 (b) 60

(c) 112 (d) 6

28. A body revolved around the sun 27 times faster then the earth what is the ratio of their radii [DPMT 2002]

(a) 1/3 (b) 1/9
(c) 1/27 (d) 1/4

29. The period of moon's rotation around the earth is nearly 29 *days*. If moon's mass were 2 fold, its present value and all other things remained unchanged, the period of moon's rotation would be nearly [Kerala (Engg.) 2002]

(a) $29\sqrt{2}$ *days* (b) $29/\sqrt{2}$ *days*
(c) 29×2 *days* (d) 29 *days*

30. Two planets at mean distance d_1 and d_2 from the sun and their frequencies are n_1 and n_2 respectively then [Kerala (Med.) 2002]

(a) $n_1^2 d_1^2 = n_2 d_2^2$ (b) $n_2^2 d_2^3 = n_1^2 d_1^3$
(c) $n_1 d_1^2 = n_2 d_2^2$ (d) $n_1^2 d_1 = n_2^2 d_2$

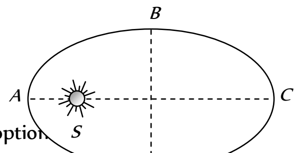
31. Which of the following astronomer first proposed that sun is static and earth rounds sun [AFMC 2002]

(a) Copernicus (b) Kepler
(c) Galileo (d) None

32. The distance of a planet from the sun is 5 times the distance between the earth and the sun. The time period of the planet is

(a) $5^{3/2}$ *years* (b) $5^{2/3}$ *years*
(c) $5^{1/3}$ *years* (d) $5^{1/2}$ *years*

33. A planet is revolving around the sun as shown in elliptical path



The correct option is

(a) The time taken in travelling DAB is less than that for BCD
(b) The time taken in travelling DAB is greater than that for BCD
(c) The time taken in travelling CDA is less than that for ABC
(d) The time taken in travelling CDA is greater than that for ABC

34. In the previous question the orbital velocity of the planet will be minimum at [UPSEAT 2003; RPET 2002]

(a) A (b) B
(c) C (d) D

35. The radius of orbit of a planet is two times that of the earth. The time period of planet is [BHU 2003; CPMT 2004]

(a) 4.2 *years* (b) 2.8 *years*
(c) 5.6 *years* (d) 8.4 *years*

36. The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then the new angular momentum will be [MP PET 2003]

(a) 16 L (b) 64 L
(c) $\frac{L}{4}$ (d) 4 L

37. According to Kepler's law the time period of a satellite varies with its radius as [Orissa JEE 2003]

(a) $T^2 \propto R^3$ (b) $T^3 \propto R^2$
(c) $T^2 \propto (1/R^3)$ (d) $T^3 \propto (1/R^2)$

38. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (r). If so the correct relation for areal velocity is [EAMCET 2003]

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$
(c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

39. The ratio of the distances of two planets from the sun is 1.38. The ratio of their period of revolution around the sun is

(a) 1.38 (b) $1.38^{3/2}$
(c) $1.38^{1/2}$ (d) 1.38^3
(e) 1.38^2.

40. Kepler's second law (law of areas) is nothing but a statement of [UPSEAT 2003]

(a) Work energy theorem
(b) Conservation of linear momentum
(c) Conservation of angular momentum
(d) Conservation of energy

41. In an elliptical orbit under gravitational force, in general [UPSEAT 2003] [UPSEAT 2004]

(a) Tangential velocity is constant
(b) Angular velocity is constant
(c) Radial velocity is constant
(d) Areal velocity is constant

42. If a new planet is discovered rotating around Sun with the orbital radius double that of earth, then what will be its time period (in earth's days) [DCE 2004]

(a) 1032 (b) 1023
(c) 1024 (d) 1043

43. Suppose the law of gravitational attraction suddenly changes and becomes an inverse cube law i.e. $F \propto 1/r^3$, but still remaining a central force. Then [UPSEAT 2002]

(a) Keplers law of areas still holds
(b) Keplers law of period still holds
(c) Keplers law of areas and period still hold
(d) Neither the law of areas, nor the law of period still holds

44. What does not change in the field of central force [MP PMT 2004]

(a) Potential energy (b) Kinetic energy
(c) Linear momentum (d) Angular momentum

45. The eccentricity of earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is [NCERT 1973]

(a) 2.507 (b) 1.033
(c) 8.324 (d) 1.000

46. The mass of a planet that has a moon whose time period and orbital radius are T and R respectively can be written as [AMU 1995]

(a) $4\pi^2 R^3 G^{-1} T^{-2}$ (b) $8\pi^2 R^3 G^{-1} T^{-2}$
(c) $12\pi^2 R^3 G^{-1} T^{-2}$ (d) $16\pi^2 R^3 G^{-1} T^{-2}$

47. If orbital velocity of planet is given by $v = G^a M^b R^c$, then [EAMCET 1994]

(a) $a = 1/3, b = 1/3, c = -1/3$

(b) $a = 1/2, b = 1/2, c = -1/2$

(c) $a = 1/2, b = -1/2, c = 1/2$

(d) $a = 1/2, b = -1/2, c = -1/2$

48. Hubble's law states that the velocity with which milky way is moving away from the earth is proportional to

[Kerala PMT 2004]

(a) Square of the distance of the milky way from the earth

(b) Distance of milky way from the earth

(c) Mass of the milky way

(d) Product of the mass of the milky way and its distance from the earth

(e) Mass of the earth

49. Two satellite are revolving around the earth with velocities v_1 and v_2 and in radii r_1 and $r_2 (r_1 > r_2)$ respectively. Then

[BHU 2005]

(a) $v_1 = v_2$ (b) $v_1 > v_2$

(c) $v_1 < v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

50. The condition for a uniform spherical mass m of radius r to be a black hole is [G= gravitational constant and g= acceleration due to gravity] [AIIMS 2005]

(a) $(2Gm/r)^{1/2} \le c$ (b) $(2Gm/r)^{1/2} = c$

(c) $(2Gm/r)^{1/2} \ge c$ (d) $(gm/r)^{1/2} \ge c$

51. Earth is revolving around the sun if the distance of the Earth from the Sun is reduced to 1/4 of the present distance then the present day length reduced by [BHU 2005]

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) $\frac{1}{6}$

Critical Thinking

Objective Questions

1. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-\frac{5}{2}}$, then T^2 is proportional to

[IIT 1989; RPMT 1997]

(a) R^3 (b) $R^{7/2}$

(c) $R^{5/2}$ (d) $R^{3/2}$

2. The magnitudes of the gravitational force at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then [IIT 1994]

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 < R$ and $r_2 < R$

3. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of earth

(a) The acceleration of S is always directed towards the centre of the earth

(b) The angular momentum of S about the centre of the earth changes in direction but its magnitude remains constant

(c) The total mechanical energy of S varies periodically with time

(d) The linear momentum of S remains constant in magnitude

4. A mass M is split into two parts, m and $(M-m)$, which are then separated by a certain distance. What ratio of m/M maximizes the gravitational force between the two parts

[AMU 2000]

(a) 1/3 (b) 1/2

(c) 1/4 (d) 1/5

5. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$

(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

6. If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would

[IIT 1981; CPMT 1981; MP PMT 1996, 97; Roorkee 1992; MP PET 1999; Kerala PMT 2004]

(a) Decrease by 2% (b) Remain unchanged

(c) Increase by 2% (d) Increase by 1%

7. The radius and mass of earth are increased by 0.5%. Which of the following statements are true at the surface of the earth

(a) g will increase

(b) g will decrease

(c) Escape velocity will remain unchanged

(d) Potential energy will remain unchanged

8. In order to make the effective acceleration due to gravity equal to zero at the equator, the angular velocity of rotation of the earth about its axis should be ($g = 10\,ms^{-2}$ and radius of earth is 6400 *kms*) [Roorkee 2000]

(a) $0\ rad sec^{-1}$ (b) $\frac{1}{800} rad sec^{-1}$

(c) $\frac{1}{80} rad sec^{-1}$ (d) $\frac{1}{8} rad sec^{-1}$

9. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$

(c) 4 (d) 2

10. A body of mass m is taken from earth surface to the height h equal to radius of earth, the increase in potential energy will be
CBSE PMT 1991; Kurukshetra CEE 1996; CMEET Bihar 1995; MNR 1998; AIEEE 2004]

(a) mgR (b) $\frac{1}{2}mgR$

(c) $2\ mgR$ (d) $\frac{1}{4}mgR$

11. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is[IIT 1997 Cancelled; MH CET 2002; MP PMT 2000]

(a) $-E_0$ (b) $1.5\ E_0$

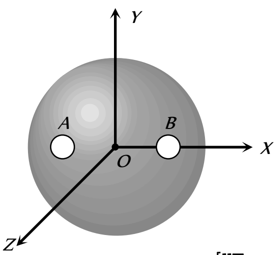
(c) $2\ E_0$ (d) E_0

12. A rocket of mass M is launched vertically from the surface of the earth with an initial speed V. Assuming the radius of the earth to be R and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

(a) $R/\left(\frac{gR}{2V^2}-1\right)$ (b) $R\left(\frac{gR}{2V^2}-1\right)$

(c) $R/\left(\frac{2gR}{V^2}-1\right)$ (d) $R\left(\frac{2gR}{V^2}-1\right)$

13. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively are taken out of the solid leaving behind spherical cavities as shown in figure [IIT 1993]



(a) The gravitational force due to this object at the origin is zero

(b) The gravitational force at the point B (2, 0, 0) is zero

(c) The gravitational potential is the same at all points of the circle $y^2+z^2=36$

(d) The gravitational potential is the same at all points on the circle $y^2+z^2=4$

14. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[2G\frac{(m_1-m_2)}{r}\right]^{1/2}$ (b) $\left[\frac{2G}{r}(m_1+m_2)\right]^{1/2}$

(c) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ (d) $\left[\frac{2G}{r}m_1m_2\right]^{1/2}$

15. A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k<1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be : (R = radius of earth)
[Roorkee 1999; RPET 1999]

(a) $\frac{R}{k^2+1}$ (b) $\frac{R}{k^2-1}$

(c) $\frac{R}{1-k^2}$ (d) $\frac{R}{k+1}$

[NCERT 1971; CPMT 1971, 97; IIT 1983;

16. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius $(1.01)R$. The period of the second satellite is larger than that of the first one by approximately [IIT 1995]

(a) 0.5% (b) 1.0%

(c) 1.5% (d) 3.0%

17. If the distance between the earth and the sun becomes half its present value, the number of days in a year would have been

(a) 64.5 (b) 129

(c) 182.5 (d) 730

18. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a satellite orbiting a few hundred kilometres above the earth's surface $(R_{\text{Earth}}=6400\,km)$ will approximately be
[IIT-JEE (Screening) 2002]

(a) 1/2 h (b) 1 h

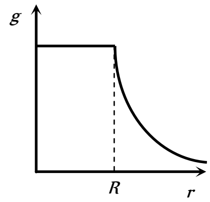
(c) 2 h (d) 4 h

[AMU 1995]

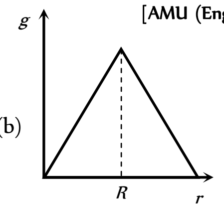
Graphical Questions

1. Assuming the earth to have a constant density, point out which of the following curves show the variation of acceleration due to gravity from the centre of earth to the points far away from the surface of earth
[AMU (Engg.) 2000]

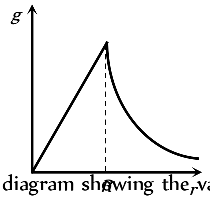
(a)



(b)



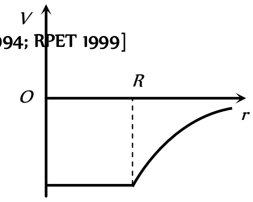
(c)



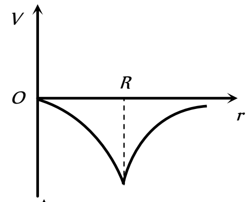
(d) None of these

2. The diagram showing the variation of gravitational potential of earth with distance from the centre of earth is
[BHU 1994; RPET 1999]

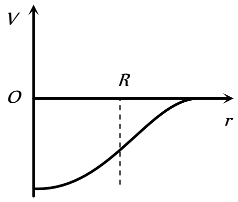
(a)



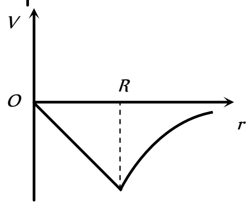
(b)



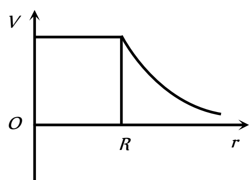
(c)

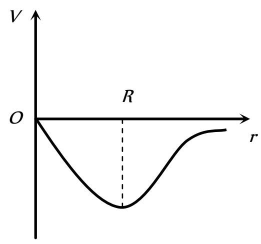


(d)



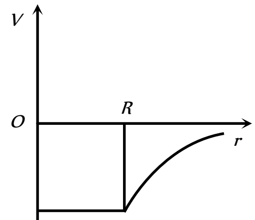
3. By which curve will the variation of gravitational potential of a hollow sphere of radius R with distance be depicted



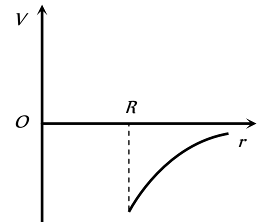


(a) (b)

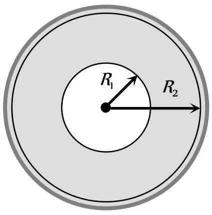
(c)



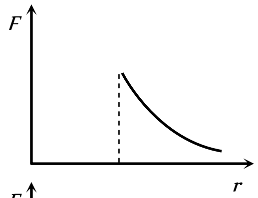
(d)



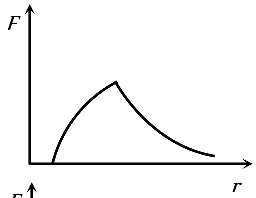
4. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$



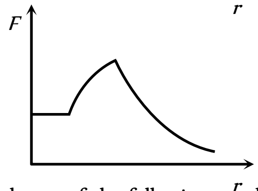
(a)



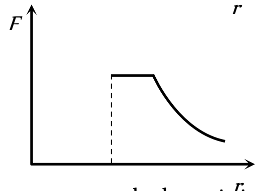
(b)



(c)

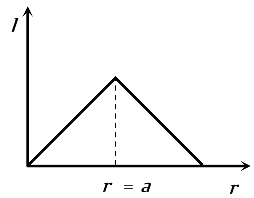


(d)

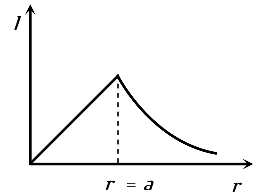


5. Which one of the following graphs represents correctly the variation of the gravitational field (F) with the distance (r) from the centre of a spherical shell of mass M and radius a

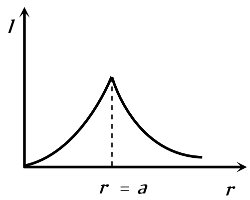
(a)



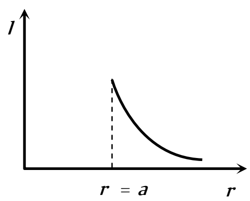
(b)



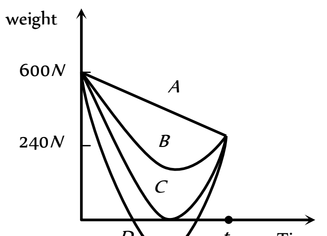
(c)



(d)



6. Suppose, the acceleration due to gravity at the earth's surface is 10 m/s and at the surface of Mars it is 4.0 m/s. A 60 kg passenger goes from the earth to the Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represents the weight (net gravitational force)of the passenger as a function of time.



(a) A

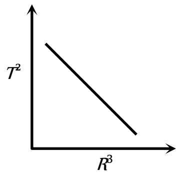
(b) B

(c) C

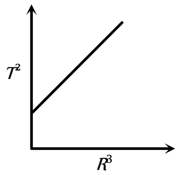
(d) D

7. Which of the following graphs represents the motion of a planet moving about the sun [NCERT 1983]

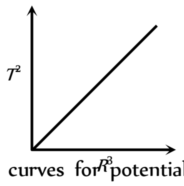
(a)



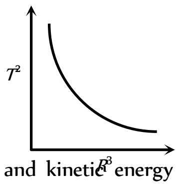
(b)



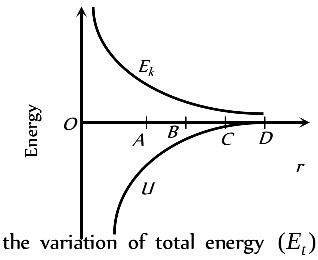
(c)



(d)



8. The curves for potential energy (U) and kinetic energy (E_k) of a two particle system are shown in figure. At what points the system will be bound?



(a) Only at point D

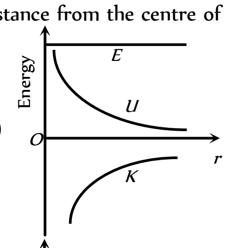
(b) Only at point A

(c) At point D and A

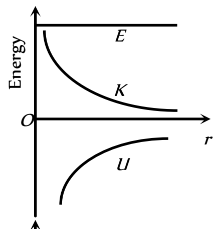
(d) At points A, B and C

9. The correct graph representing the variation of total energy (E_t) kinetic energy (E_k) and potential energy (U) of a satellite with its distance from the centre of earth is

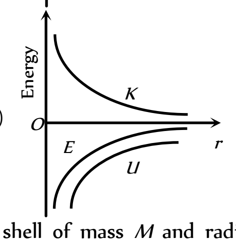
(a)



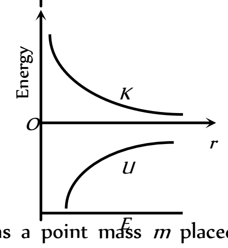
(b)



(c)

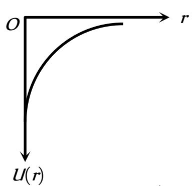


(d)

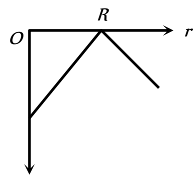


10. A shell of mass M and radius R has a point mass m placed at a distance r from its centre. The gravitational potential energy $U(r)$ vs r will be

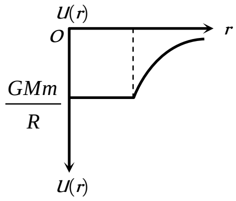
(a)



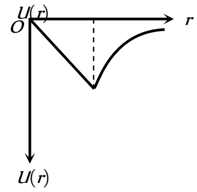
(b)



(c)



(d)



Assertion & Reason

For AIIMS Aspirants

Read the assertion and reason carefully to mark the correct option out of the options given below:

(a) *If both assertion and reason are true and the reason is the correct explanation of the assertion.*
(b) *If both assertion and reason are true but reason is not the correct explanation of the assertion.*
(c) *If assertion is true but reason is false.*
(d) *If the assertion and reason both are false.*
(e) *If assertion is false but reason is true.*

1. Assertion : Smaller the orbit of the planet around the sun, shorter is the time it takes to complete one revolution.
Reason : According to Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from sun.

2. Assertion : Gravitational force between two particles is negligibly small compared to the electrical force.
Reason : The electrical force is experienced by charged particles only.

3. Assertion : The universal gravitational constant is same as acceleration due to gravity.
Reason : Gravitational constant and acceleration due to gravity have same dimensional formula.

4. Assertion : The value of acceleration due to gravity does not depend upon mass of the body on which force is applied.
Reason : Acceleration due to gravity is a constant quantity.

5. Assertion : If a pendulum is suspended in a lift and lift is falling freely, then its time period becomes infinite.
Reason : Free falling body has acceleration equal to acceleration due to gravity.

6. Assertion : If earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.
Reason : The value of acceleration due to gravity is independent of rotation of earth.

7. Assertion : The difference in the value of acceleration due to gravity at pole and equator is proportional to square of angular velocity of earth.
Reason : The value of acceleration due to gravity is minimum at the equator and maximum at the pole.

8. Assertion : There is no effect of rotation of earth on acceleration due to gravity at poles.
Reason : Rotation of earth is about polar axis.

9. Assertion : A force act upon the earth revolving in a circular orbit about the sun. Hence work should be done on the earth.
Reason : The necessary centripetal force for circular motion of earth comes from the gravitational force between earth and sun.

10. Assertion : The ratio of inertial mass to gravitational mass is equal to one.
Reason : The inertial mass and gravitational mass of a body are equivalent.

11. Assertion : Gravitational potential of earth at every place on it is negative.
Reason : Every body on earth is bound by the attraction of earth.

12. Assertion : Even when orbit of a satellite is elliptical, its plane of rotation passes through the centre of earth.
Reason : According to law of conservation of angular momentum plane of rotation of satellite always remain same.

13. Assertion : A planet moves faster, when it is closer to the sun in its orbit and vice versa.
Reason : Orbital velocity in orbital of planet is constant.

14. Assertion : Orbital velocity of a satellite is greater than its escape velocity.
Reason : Orbit of a satellite is within the gravitational field of earth whereas escaping is beyond the gravitational field of earth.

15. Assertion : If an earth satellite moves to a lower orbit, there is some dissipation of energy but the satellite speed increases.
Reason : The speed of satellite is a constant quantity.

16. Assertion : Earth has an atmosphere but the moon does not.
Reason : Moon is very small in comparison to earth.

17. Assertion : The time period of geostationary satellite is 24 hours.
Reason : Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis.

18. Assertion : The principle of superposition is not valid for gravitational force.
Reason : Gravitational force is a conservative force.

19. Assertion : Two different planets have same escape velocity.
Reason : Value of escape velocity is a universal constant.

20. Assertion : The time period of revolution of a satellite close to surface of earth is smaller than that revolving away from surface of earth.
Reason : The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius.

21. Assertion : When distance between two bodies is doubled and also mass of each body is also doubled, gravitational force between them remains the same.
Reason : According to Newton's law of gravitation, force is directly proportional to mass of bodies and inversely proportional to square of distance between them.

22. Assertion : Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very large height.
Reason : The path of a projectile is independent of the gravitational force of earth.

23. Assertion : A body becomes weightless at the centre of earth.
Reason : As the distance from centre of earth decreases, acceleration due to gravity increases.

24. Assertion : Space rockets are usually launched in the equatorial line from west to east.

Reason : The acceleration due to gravity is minimum at the equator.

25. Assertion : The binding energy of a satellite does not depend upon the mass of the satellite.

Reason : Binding energy is the negative value of total energy of satellite.

26. Assertion : We can not move even a finger without disturbing all the stars.

Reason : Every body in this universe attracts every other body with a force which is inversely proportional to the square of distance between them.

27. Assertion : If earth were a hollow sphere, gravitational field intensity at any point inside the earth would be zero.

Reason : Net force on a body inside the sphere is zero.

28. Assertion : For a satellite revolving very near to earth's surface the time period of revolution is given by 1 hour 24 minutes.

Reason : The period of revolution of a satellite depends only upon its height above the earth's surface.

29. Assertion : A person sitting in an artificial satellite revolving around the earth feels weightless.

Reason : There is no gravitational force on the satellite.

30. Assertion : The speed of satellite always remains constant in an orbit.

Reason : The speed of a satellite depends on its path.

31. Assertion : The speed of revolution of an artificial satellite revolving very near the earth is $8\,kms^{-1}$.

Reason : Orbital velocity of a satellite, become independent of height of near satellite.

32. Assertion : Gravitational field is zero both at centre and infinity.

Reason : The dimensions of gravitational field is $[LT^{-2}]$.

33. Assertion : For the planets orbiting around the sun, angular speed, linear speed, *K.E.* changes with time, but angular momentum remains constant.

Reason : No torque is acting on the rotating planet. So its angular momentum is constant.

Answers

Newton's Law of Gravitation

1	a	2	b	3	b	4	b	5	a
6	d	7	b	8	c	9	c	10	d
11	d	12	a	13	d	14	a	15	d
16	b	17	c	18	a	19	c	20	a
21	a	22	a	23	e	24	c		

Acceleration Due to Gravity

1	d	2	b	3	d	4	a	5	b
6	a	7	b	8	d	9	b	10	b
11	c	12	a	13	b	14	c	15	a
16	c	17	c	18	c	19	a	20	d
21	a	22	c	23	d	24	b	25	a
26	d	27	b	28	b	29	b	30	a
31	b	32	a	33	c	34	b	35	c
36	a	37	d	38	b	39	c	40	b
41	c	42	b	43	c	44	b	45	c
46	b	47	a	48	a	49	b	50	a
51	a	52	a	53	c	54	a	55	c
56	d	57	a	58	d	59	b	60	c
61	b	62	a	63	c	64	a	65	c
66	a	67	d	68	d	69	a	70	a
71	a	72	b	73	b	74	b	75	d
76	a	77	d	78	a	79	a	80	b
81	a	82	a	83	d	84	a	85	b
86	b	87	b						

Gravitation Potential, Energy and Escape Velocity

1	c	2	a	3	d	4	a	5	d
6	a	7	b	8	c	9	c	10	c
11	d	12	a	13	b	14	b	15	a
16	d	17	c	18	a	19	b	20	b
21	c	22	b	23	a	24	c	25	a
26	b	27	b	28	c	29	c	30	b
31	a	32	b	33	c	34	a	35	c
36	d	37	a	38	a	39	c	40	c
41	c	42	c	43	d	44	a	45	a
46	b	47	d	48	a	49	a	50	b
51	d	52	a	53	c	54	b	55	a
56	b	57	d	58	d	59	b	60	b
61	c	62	b	63	c	64	c	65	b
66	b	67	a	68	c	69	a	70	a

Motion of Satellite

1	b	2	d	3	d	4	d	5	b
6	b	7	b	8	b	9	c	10	c
11	b	12	d	13	b	14	a	15	a
16	b	17	c	18	d	19	d	20	d
21	d	22	b	23	b	24	a	25	a
26	d	27	a	28	d	29	a	30	a
31	b	32	d	33	a	34	c	35	d
36	c	37	d	38	b	39	c	40	b
41	c	42	d	43	a	44	d	45	a
46	c	47	c	48	d	49	b	50	b
51	a	52	a	53	a	54	c	55	b

56	b	57	b	58	d	59	d	60	b
61	c	62	c	63	b	64	a	65	b
66	c	67	d						

Kepler's laws of Planetary Motion

1	c	2	c	3	c	4	b	5	c
6	b	7	b	8	c	9	c	10	b
11	d	12	b	13	c	14	c	15	a
16	b	17	d	18	d	19	b	20	c
21	c	22	c	23	b	24	b	25	c
26	a	27	a	28	b	29	d	30	b
31	a	32	a	33	a	34	c	35	b
36	d	37	a	38	c	39	b	40	c
41	d	42	a	43	d	44	d	45	b
46	a	47	b	48	b	49	c	50	c
51	c								

Critical Thinking Questions

1	b	2	ab	3	a	4	b	5	a
6	c	7	bcd	8	b	9	d	10	b
11	c	12	c	13	acd	14	b	15	c
16	c	17	b	18	c				

Graphical Questions

1	c	2	c	3	c	4	b	5	d
6	c	7	c	8	d	9	c	10	c

Assertion and Reason

1	a	2	b	3	d	4	c	5	a
6	c	7	b	8	a	9	e	10	a
11	a	12	a	13	c	14	e	15	c
16	b	17	b	18	e	19	d	20	a
21	a	22	c	23	c	24	b	25	e
26	a	27	a	28	a	29	c	30	e
31	a	32	b	33	a				

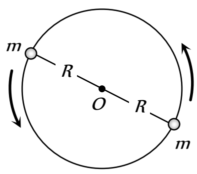
AS Answers and Solutions

Newton's Law of Gravitation

1. (a)
2. (b) As it depends on the weight of the body.
3. (b) Due to inertia of direction.
4. (b)
5. (a)
6. (d) $F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$
7. (b)
8. (c) $F = G\frac{m_1 m_2}{r^2} = 6.675 \times \frac{1 \times 1}{1^2} \times 10^{-11} = 6.675 \times 10^{-11}\, N$
9. (c) Centripetal force provided by the gravitational force of attraction between two particles

   i.e. $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$

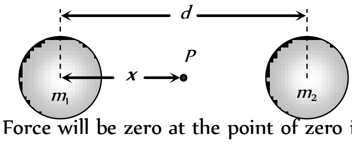
   $\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$



10. (d) $m = 6 \times 10^{24}\, kg,\;\; \omega = 2 \times 10^{-7}\, rad/s,\;\; R = 1.5 \times 10^{11}\, m$

    The force exerted by the sun on the earth $F = m\omega^2 R$

    By substituting the value we can get, $F = 36 \times 10^{21}\, N$
11. (d)
12. (a) k represents gravitational constant which depends only on the system of units.
13. (d)



    Force will be zero at the point of zero intensity

    $x = \frac{\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}} d = \frac{\sqrt{81M}}{\sqrt{81M} + \sqrt{M}} D = \frac{9}{10} D.$
14. (a)
15. (d) $g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{(1.74 \times 10^6)^2} = 1.62\, N/kg$
16. (b) Actually gravitational force provides the centripetal force.
17. (c)
18. (a) $F \propto xm \times (1-x)m = xm^2(1-x)$

    For maximum force $\frac{dF}{dx} = 0$

    $\Rightarrow \frac{dF}{dx} = m^2 - 2xm^2 = 0 \Rightarrow x = 1/2$
19. (c)
20. (a)
21. (a) Gravitational force does not depend on the medium.
22. (a)
23. (e)
24. (c) $F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2} = \frac{4}{9}\pi^2 \rho^2 R^4$

    $\therefore\; F \propto R^4$

Acceleration Due to Gravity

1. (d)
2. (b) The value of g at the height h from the surface of earth

   $g' = g\left(1 - \frac{2h}{R}\right)$

   The value of g at depth x below the surface of earth

   $g' = g\left(1 - \frac{x}{R}\right)$

   These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

   On solving, we get $x = 2h$
3. (d) Time period of simple pendulum $T = 2\pi\sqrt{\frac{l}{g'}}$

   In artificial satellite $g' = 0 \;\therefore\; T =$ infinite.
4. (a) $g = \frac{4}{3}\pi\rho G R$. If ρ = constant then $\frac{g_1}{g_2} = \frac{R_1}{R_2}$
5. (b) Time of decent $t = \sqrt{\frac{2h}{g}}$. In vacuum no other force works except gravity so time period will be exactly equal.
6. (a)

7. (b) Because acceleration due to gravity increases

8. (d) Because acceleration due to gravity decreases

9. (b) We know that $g = \frac{GM}{R^2}$

On the planet $g_p = \frac{GM/7}{R^2/4} = \frac{4g}{7} = \frac{4}{7}g$

Hence weight on the planet $= 700 \times \frac{4}{7} = 400\ gm\,wt$

10. (b) In pendulum clock the time period depends on the value of g, while in spring watch, the time period is independent of the value of g.

11. (c) $g = \frac{GM}{R^2} = \frac{GM_0}{(D_0/2)^2} = \frac{4GM_0}{D_0^2}$

12. (a)

13. (b) $\frac{g'}{g} = \frac{M'}{M}\left(\frac{R}{R'}\right)^2 = \left(\frac{2M}{M}\right)\left(\frac{R}{2R}\right)^2 = \frac{1}{2}$

$\Rightarrow g' = \frac{g}{2} = \frac{9.8}{2} = 4.9\ m/s^2$

14. (c)

15. (a)

16. (c) For the condition of weightlessness at equator

$\omega = \sqrt{\frac{g}{R}}\ \therefore\ \omega = \sqrt{\frac{1}{640 \times 10^3}} = \frac{1}{800} rad/s$

17. (c) $g = \frac{GM}{r^2}$. Since M and r are constant, so $g = 9.8\ m/s^2$

18. (c) $g = \frac{GM}{R^2}$ and $M = \frac{4}{3}\pi R^3 \times \rho$

$\therefore g = \frac{4}{3}\frac{\pi R^3 \times G\rho}{R^2} \Rightarrow \rho = \frac{3g.}{4\pi RG}$

19. (a) Because value of g decreases when we move either in coal mine or at the top of mountain.

20. (d) $g = \frac{4}{3}\pi\rho GR\ \therefore\ \frac{g_1}{g_2} = \frac{R_1\rho_1}{R_2\rho_2}$

21. (a) $g = \frac{GM}{R^2}$ (Given $M_e = 81M_m,\ R_e = 3.5R_m$)

Substituting the above values, $\frac{g_m}{g_e} = 0.15$

22. (c) Value of g decreases when we go from poles to equator.

23. (d)

24. (b) Because value of g decreases with increasing height.

25. (a) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{6400}{6400+64}\right)^2 \Rightarrow g' = 960.40\ cm/s^2$

26. (d)

27. (b) $g' = g - \omega^2 R\cos^2\lambda$

Rotation of the earth results in the decreased weight apparently. This decrease in weight is not felt at the poles as the angle of latitude is 90°.

28. (b) $g = \frac{GM}{R^2}$. If radius shrinks to half of its present value then g will becomes four times.

29. (b) Using $g = \frac{GM}{R^2}$ we get $g_m = g/5$

30. (a) $g = g_p - R\omega^2\cos^2\lambda = g_p - \omega^2 R\cos^2 60° = g_p - \frac{1}{4}R\omega^2$

31. (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \left(\frac{n-1}{n}\right)R$

32. (a) $g \propto \frac{GM}{r^2}\ \therefore\ g \propto \frac{1}{r^2}$ or $r \propto \frac{1}{\sqrt{g}}$

If g decrease by one percent then r should be increase by $\frac{1}{2}$% *i.e.* $R = \frac{1}{2 \times 100} \times 6400 = 32\ km$

33. (c) $g = \frac{4}{3}G\pi R\rho \Rightarrow \frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

34. (b) $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{2} = \frac{R}{R+h}$

$\Rightarrow R + h = 2R\ \therefore\ h = R$

35. (c) Acceleration due to gravity at poles is independent of the angular spe

36. (a) Mass of the ball always remain constant. It does not depend upon the acceleration due to gravity

37. (d) $g_m = \frac{GM_m}{R_m^2}$ and $g_m = \frac{g_e}{6} = \frac{9.8}{6}\ m/s^2 = 1.63\ m/s^2$

Substituting $R_m = 1.768 \times 10^6 m,\ g_m = 1.63\ m/s^2$

and $G = 6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$ We get

$M_m = 7.65 \times 10^{22} kg$

38. (b) $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow$ when $h = R$ then $g' = \frac{g}{4}$

So the weight of the body at this height will become one-fourth.

39. (c) $g = \frac{GM}{R^2}$ and $K = \frac{L^2}{2I}$

If mass of the earth and its angular momentum remains constant then $g \propto \frac{1}{R^2}$ and $K \propto \frac{1}{R^2}$

i.e. if radius of earth decreases by 2% then g and K both increases by 4%.

40. (b) Weight is least at the equator.

41. (c) $g \propto \frac{1}{R^2}$

Percentage change in g = 2(percentage change in R)

$= 2 \times 1.5 = -3\%$

42. (b) $g \propto \frac{1}{R^2}$. If radius of earth decreases by 2% then g will increase by 4% *i.e.* weight of the body at earth surface will increase by 4%

43. (c) Mass does not vary from place to place.

44. (b) $g = \frac{GM}{R^2} \Rightarrow R = \sqrt{\frac{GM}{g}}$

Substituting the above values we get $R = 1.87 \times 10^6 m$.

45. (c) Weight of the body at equator = $\frac{3}{5}$ of initial weight

$\therefore g' = \frac{3}{5} g$ (because mass remains constant)

$g' = g - \omega^2 R \cos^2 \lambda \Rightarrow \frac{3}{5} g = g - \omega^2 R \cos^2 (0°)$

$\Rightarrow \omega^2 = \frac{2g}{5R} \Rightarrow \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2 \times 10}{5 \times 6400 \times 10^3}}$

$= 7.8 \times 10^{-4} \frac{rad}{\sec}$

46. (b) $h = 32\,km$, $R = 6400\,km$, so $h << R$

$g' = g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{2 \times 32}{6400}\right) \Rightarrow g' = \frac{99}{100} g = 0.99\,g$

47. (a) Same change in the value of g can be observed at a depth x and height $2x$

given $d = x = 10\,km \quad \therefore h = 2x = 20\ km$

48. (a)

49. (b) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{100} = \left(\frac{R}{R+h}\right)^2 \Rightarrow h = 9R$

50. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R + \frac{R}{2}}\right)^2 = \frac{4}{9} g$

$\therefore W' = \frac{4}{9} \times W = \frac{4}{9} \times 72 = 32N$

51. (a) $g' = g - \omega^2 R \cos^2 \lambda \Rightarrow 0 = g - \omega^2 R \cos^2 60^o$

$0 = g - \frac{\omega^2 R}{4} \Rightarrow \omega = 2\sqrt{\frac{g}{R}} = \frac{1}{400} \frac{rad}{\sec} = 2.5 \times 10^{-3} \frac{rad}{sec}$

52. (a) $g' = g\left(1 - \frac{d}{R}\right) = 9.8\left(1 - \frac{100}{6400}\right) = 9.66\,m/s^2$

53. (c) $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$. By solving $h = R$

54. (a) $g = \frac{4}{3}\pi\rho G R \therefore g \propto r\rho \therefore \frac{g_e}{g_m} = \frac{R}{r} \times \frac{\rho_e}{\rho_m}$

55. (c) $g_p = g_e \left(\frac{M_p}{M_e}\right)\left(\frac{R_e}{R_p}\right)^2 = 9.8\left(\frac{1}{80}\right)(2)^2$

$= 9.8/20 = 0.49\ m/s^2$

56. (d) Range of projectile $R = \frac{u^2 \sin 2\theta}{g}$

if u and θ are constant then $R \propto \frac{1}{g}$

$\frac{R_m}{R_e} = \frac{g_e}{g_m} \Rightarrow \frac{R_m}{R_e} = \frac{1}{0.2} \Rightarrow R_m = \frac{R_e}{0.2} \Rightarrow R_m = 5R_e$

57. (a) For condition of weightlessness of equator

$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800} = 1.25 \times 10^{-3} \frac{rad}{s}$

58. (d) $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$

$\Rightarrow R + h = \sqrt{2}\,R \Rightarrow h = (\sqrt{2} - 1)R = 0.414\,R$

59. (b) $g \propto \rho R$

60. (c) $H = \frac{u^2}{2g} \Rightarrow H \propto \frac{1}{g} \Rightarrow \frac{H_B}{H_A} = \frac{g_A}{g_B}$

Now $g_B = \frac{g_A}{12}$ as $g \propto \rho R$

$\therefore \frac{H_B}{H_A} = \frac{g_A}{g_B} = 12 \Rightarrow H_B = 12 \times H_A = 12 \times 1.5 = 18m$

61. (b)

62. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$

63. (c) $g = \frac{4}{3}\pi\rho G R \Rightarrow g \propto dR$ $\quad (\rho = d$ given in the problem$)$

64. (a) Inside the earth $g' = \frac{4}{3}\pi\rho G r \ \therefore g' \propto r$

65. (c) $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{4}{9} g \quad \therefore W' = \frac{4}{9} W$

66. (a) $g \propto \rho$

67. (d)

68. (d)

69. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{3R/2}\right)^2 = \frac{4}{9} g$

$\therefore W' = \frac{4}{9} \times mg = \frac{4 \times 200 \times 9.8}{9} = 880\ N$

70. (a) $g = \frac{4}{3}\pi G\rho R \Rightarrow g \propto \rho R \Rightarrow \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m} \times \frac{R_e}{R_m}$

$\Rightarrow \frac{6}{1} = \frac{5}{3} \times \frac{R_e}{R_m} \Rightarrow R_m = \frac{5}{18} R_e$

71. (a) $g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+2R}\right)^2 = \frac{g}{9}$

72. (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3}{4} R$

73. (b) For height $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} = 1\%$;

For depth $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} = \frac{h}{R} = \frac{1}{2}\% = 0.5\%$

74. (b) As $g = \frac{GM}{R^2}$ therefore 1% decrease in mass will decreases the value of g by 1%.

But 1% decrease in radius will increase the value of g by 2%.

As a whole value of g increase by 1%.

75. (d) $g = \frac{4}{3}\pi\rho GR \Rightarrow \frac{R_p}{R_e} = \left(\frac{g_p}{g_e}\right)\left(\frac{\rho_e}{\rho_p}\right) = (1) \times \left(\frac{1}{2}\right)$

$\Rightarrow R_p = \frac{R_e}{2} = \frac{R}{2}$

76. (a) $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} \times \frac{R_1}{R_2} = \frac{3}{2} \times \frac{2}{3} = 1$

77. (d) Because the body weighs zero in satellite

78. (a) Radius of earth R = 6400 km $\therefore$ $h = \frac{R}{4}$

Acceleration due to gravity at a height h

$$g_h = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+\frac{R}{4}}\right)^2 = \frac{16}{25}g$$

At depth 'd value of acceleration due to gravity

$g_d = \frac{1}{2}g_h$ (According to problem)

$$\Rightarrow g_d = \frac{1}{2}\left(\frac{16}{25}\right)g \Rightarrow g\left(1-\frac{d}{R}\right) = \frac{1}{2}\left(\frac{16}{25}\right)g$$

By solving we get $d = 4.3 \times 10^6\, m$

79. (a) $g' = g - \omega^2 R\cos^2\lambda$

For weightlessness at equator $\lambda = 0°$ and $g' = 0$

$\therefore\ 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800}\frac{rad}{\sec}$

80. (b) Weight on surface of earth, $mg = 500\, N$

and weight below the surface of earth at $d = \frac{R}{2}$

$$mg' = mg\left(1-\frac{d}{R}\right) = mg\left(1-\frac{1}{2}\right) = \frac{mg}{2} = 250\, N$$

81. (a) $g = \frac{4}{3}\pi GR\rho$ and $g' = \frac{4}{3}\pi GR'\rho$

$\therefore\ \frac{g'}{g} = \frac{R'}{R} = 0.2 \Rightarrow g' = 0.2\, g$

82. (a)

83. (d) $\frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2 = \left(\frac{1}{9}\right)\left(\frac{2}{1}\right)^2 = \frac{4}{9} \Rightarrow g_m = \frac{4}{9}g_e$

$\therefore\ W_m = \frac{4}{9} \times W_e = \frac{4}{9} \times 90 = 40\, kg$

84. (a) $g' = g - \omega^2 R$, when ω increases g' decreases.

85. (b) $\frac{g'}{g} = \frac{M'}{M} \times \frac{R^2}{R'^2} = \frac{1}{2} \times \frac{4}{1} = \frac{2}{1}$

86. (b) Acceleration due to gravity at latitude λ is given by

$g' = g - R\omega^2\cos^2\lambda$

At 30^o, $g_{30°} = g - R\omega^2\cos^2 30^o = g - \frac{3}{4}R\omega^2$

$\therefore\ g - g_{30} = \frac{3}{4}\omega^2 R.$

87. (b) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore$ $\frac{g}{G} = \frac{M}{R^2}$

Gravitational Potential, Energy and Escape Velocity

1. (c) $\Delta U = \frac{mgh}{1+h/R}$

Substituting $R = 5h$ we get $\Delta U = \frac{mgh}{1+1/5} = \frac{5}{6}mgh$

2. (a) $I = \frac{-dV}{dx}$

If V = 0 then gravitational field is necessarily zero.

3. (d) Gravitational potential $= \int I\, dx = \int_x^\infty \frac{K}{x^3}dx$

$$= K\left(\frac{x^{-3+1}}{-3+1}\right)_x^\infty = \left|\frac{-K}{2x^2}\right|_x^\infty = \frac{K}{2x^2}$$

4. (a) $U = -\frac{GMm}{r}$

$$\Rightarrow 7.79 \times 10^{28} = \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{r}$$

$\Rightarrow r = 3.8 \times 10^8\, m$

5. (d) $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mg\,nR}{1+\frac{nR}{R}} = \frac{nm\,gR}{n+1}$

6. (a) Gravitational potential at mid point

$V = \frac{-GM_1}{d/2} + \frac{-GM_2}{d/2}$

Now, $PE = m \times V = \frac{-2Gm}{d}(M_1 + M_2)$

[m = mass of particle]

So, for projecting particle from mid point to infinity

$KE = |PE|$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{d}(M_1+M_2) \Rightarrow v = 2\sqrt{\frac{G(M_1+M_2)}{d}}$$

7. (b) Potential energy of the 1 kg mass which is placed at the earth surface = $-\frac{GM}{R}$

its potential energy at infinite = 0

$\therefore$ Work done = change in potential energy = $\frac{GM}{R}$

8. (c)

9. (c) $\frac{G \times 100}{x^2} = \frac{G \times 10000}{(1-x)^2} \Rightarrow \frac{10}{x} = \frac{100}{1-x} \Rightarrow x = \frac{1}{11}m$

10. (c)

11. (d) $\Delta U = U_2 - U_1 = \dfrac{mgh}{1+\frac{h}{R_e}} = \dfrac{mgR_e}{1+\frac{R_e}{R_e}} = \dfrac{mgR_e}{2}$

$\Rightarrow U_2 - (-mgR_e) = \dfrac{mgR_e}{2} \Rightarrow U_2 = -\dfrac{1}{2}mgR_e$

12. (a) $v_e = \sqrt{\dfrac{2GM}{R}} = 100 \Rightarrow \dfrac{GM}{R} = 5000$

Potential energy $U = -\dfrac{GMm}{R} = -5000\ J$

13. (b) $\Delta U = \dfrac{mgh}{1+\frac{h}{R}} = \dfrac{mg\times 3R}{1+\frac{3R}{R}} = \dfrac{3}{4}mgR$

14. (b) Potential energy $U = \dfrac{-GMm}{r} = -\dfrac{GMm}{R+h}$

$U_{initial} = -\dfrac{GMm}{3R}$ and $U_{final} = -\dfrac{-GMm}{2R}$

Loss in PE = gain in $KE = \dfrac{GMm}{2R} - \dfrac{GMm}{3R} = \dfrac{GMm}{6R}$

15. (a) If body is projected with velocity v $(v < v_e)$ then

height up to which it will rise, $h = \dfrac{R}{\frac{v_e^2}{v^2} - 1}$

$v = \dfrac{v_e}{2}$ (given) $\therefore\ h = \dfrac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \dfrac{R}{4-1} = \dfrac{R}{3}$

16. (d) Change in potential energy in displacing a body from r_1 to r_2 is given by

$\Delta U = GMm\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right] = GMm\left(\dfrac{1}{2R} - \dfrac{1}{3R}\right) = \dfrac{GMm}{6R}$

17. (c) $\dfrac{1}{2}mv_e^2 = \dfrac{1}{2}m\ 2gR = mgR$

18. (a) $K.E. = \dfrac{GMm}{2R}$

19. (b) $I = \dfrac{-dV}{dr}$. If I = 0 then V = constant

20. (b) This should be equal to escape velocity *i.e.* $\sqrt{2gR}$

21. (c) $v_e = \sqrt{\dfrac{2GM}{R}}$ *i.e.* escape velocity depends upon the mass and radius of the planet.

22. (b) $v_e = \sqrt{\dfrac{2GM}{R}} = R\sqrt{\dfrac{8}{3}\pi G\rho}$

If mean density is constant then $v_e \propto R$

$\dfrac{v_e}{v_p} = \dfrac{R_e}{R_p} = \dfrac{1}{2} \Rightarrow v_e = \dfrac{v_p}{2}$

23. (a) Escape velocity does not depend on the mass of the projectile

24. (c) $\dfrac{v_p}{v_e} = \sqrt{\dfrac{g_p}{g_e}\times\dfrac{R_p}{R_e}} = \sqrt{2\times 2} = 2$

$\Rightarrow v_p = 2\times v_e = 2\times 11.2 = 22.4\ km/s$

25. (a) $v_e = \sqrt{\dfrac{2GM}{R}} = R\sqrt{\dfrac{8}{3}\pi G\rho}\ \therefore v_e \propto R$ if ρ = constant

Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice *i.e.* 22 *km/s*.

26. (b) If missile launched with escape velocity then it will escape from the gravitational field and at infinity its total energy becomes zero.

But if the velocity of projection is less than escape velocity then sum of energies will be negative. This shows that attractive force is working on the satellite.

27. (b)

28. (c)

29. (c) Because it does not depend on the mass of projectile

30. (b) $v_e = \sqrt{2}\,v_0$, *i.e.* if the orbital velocity of moon is increased by factor of $\sqrt{2}$ then it will escape out from the gravitational field of earth.

31. (a)

32. (b) $v_e = \sqrt{\dfrac{2GM}{R}} \Rightarrow v_e \propto \sqrt{M}$ if $R = \text{constant}$

If the mass of the planet becomes four times then escape velocity will become 2 times.

33. (c) Escape velocity $v_e = \sqrt{\dfrac{2GM}{R}}$

$\therefore\ \dfrac{v_e}{v_m} = \sqrt{\dfrac{M_eR_m}{M_mR_e}} = \sqrt{\dfrac{81}{3.5}} = 4.81$

34. (a) $v_e = \sqrt{\dfrac{2GM}{R}}\ \therefore\ v_e \propto \sqrt{\dfrac{M}{R}}$

If mass and radius of the planet are three times than that of earth then escape velocity will be same.

35. (c) Potential energy of a body at the surface of earth

$PE = -\dfrac{GMm}{R} = -\dfrac{gR^2m}{R} = -mgR$

$= -500\times 9.8\times 6.4\times 10^6 = -3.1\times 10^{10}\ J$

So if we give this amount of energy in the form of kinetic energy then body escape from the earth.

36. (d) Escape velocity $v = \sqrt{\dfrac{2GM}{R}} \Rightarrow \dfrac{v_p}{v_e} = \sqrt{\dfrac{M_p}{M_e}\times\dfrac{R_e}{R_p}}$

$\Rightarrow v_p = 5v_e = 5\times 11.2 = 56\ km/s$

37. (a) $\dfrac{v_p}{v_e} = \sqrt{\dfrac{M_p}{M_e}\times\dfrac{R_e}{R_p}} = \sqrt{6\times\dfrac{1}{2}} = \sqrt{3}\ \therefore\ v_p = \sqrt{3}\,v_e$

38. (a) $\frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{9 \times 4} = 6 \therefore v_p = 6 \times v_e = 67.2 km/s$

39. (c) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_p}{R_e}} = \sqrt{8 \times \frac{1}{2}} = 2 \therefore v_p = 2 \times v_e = 22.4 km/s$

40. (c) $v_e = \sqrt{\frac{2GM}{R}} \therefore v_e \propto \sqrt{\frac{M}{R}}$

If M becomes double and R becomes half then escape velocity becomes two times.

41. (c) On earth $v_e = \sqrt{\frac{2GM}{R}} = 11.2\ km/s$

On moon $v_m = \sqrt{\frac{2GM \times 4}{81 \times R}} = \frac{2}{9}\sqrt{\frac{2GM}{R}}$

$= \frac{2}{9} \times 11.2 = 2.5\ km/s$

42. (c) Escape velocity $v = \sqrt{\frac{2GM}{R}}$

If star rotates with angular velocity ω

then $\omega = \frac{v}{R} = \frac{1}{R}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^3}}$

43. (d) Escape velocity from surface of earth $v_e = \sqrt{2gR}$

$= \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 \times 10^3\ m/s$

44. (a)

45. (a) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \frac{R_e}{R_p}} = \sqrt{(1000) \times \left(\frac{1}{10}\right)} = 10$

$v_p = 10 \times 11.2 = 112\ km/s$

46. (b) $v_e = R\sqrt{\frac{8}{3}G\pi\rho} \therefore v_e \propto R\sqrt{\rho}$

47. (d) $v_e = \sqrt{\frac{2GM}{(R+h)}}$

48. (a) $v = \sqrt{2gR}$. If acceleration due to gravity and radius of the planet, both are double that of earth then escape velocity will be two times. *i.e.* $v_p = 2v_e$

49. (a)

50. (b) $v = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{k_1 \times k_2} = \sqrt{k_1 k_2}$

51. (d) $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{R}} = 3 \times 10^8$

By solving $R = 9\ mm$

52. (a) $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{3}} = \sqrt{\frac{2}{3}} \therefore v_p = \sqrt{\frac{2}{3}} v_e$

53. (c) $v_e \propto \frac{1}{\sqrt{R}}$. If R becomes $\frac{1}{4}$ then v_e will be 2 times.

54. (b)

55. (a) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{4}} = \frac{1}{\sqrt{2}}$

$\therefore v_p = \frac{v_e}{\sqrt{2}}$

56. (b) $v \propto R\sqrt{\rho} \therefore \frac{v_p}{v_e} = \frac{R_p}{R_e} \times \sqrt{\frac{\rho_p}{\rho_e}} = 4 \times \sqrt{9} = 12$

$\Rightarrow v_p = 12 v_e$

57. (d) Escape velocity does not depends upon the angle of projection.

58. (d)

59. (b) Escape velocity is independent of mass of object.

60. (b) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{1 \times 4} = 2$

$\therefore v_p = 2v_e$

61. (c) $v = \sqrt{2gR} \Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \times \frac{R_p}{R_e}} = \sqrt{1 \times 4} = 2$

$\Rightarrow v_p = 2 \times v_e = 2 \times 11.2 = 22.4\ km/s$

62. (b) $v = \sqrt{2gR}$. If g and R both are doubled then v will becomes two times *i.e.* 11.2 × 2 = 22.4 *km/s*

63. (c) $v = R\sqrt{\frac{8}{3}\pi\rho G} \Rightarrow \frac{v_p}{v_e} = \frac{R_p}{R_e}\sqrt{\frac{\rho_p}{\rho_e}} = 2\sqrt{\frac{1}{4}} = 1$

64. (c) Velocity of body in inter planetary space $v' = \sqrt{v^2 - v_{es}^2}$

where v_{es} = escape velocity and

v = velocity of projection

$\therefore \quad v' = \sqrt{(2v_{es})^2 - v_{es}^2} = \sqrt{3v_{es}^2} \Rightarrow v' = \sqrt{3}\ v_{es}$

65. (b) Potential energy of system of two mass

$U = \frac{-GMm}{R} = \frac{-6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{-2}}$

$U = -6.67 \times 10^{-10}\ J$

So, the amount of work done to take the particle up to infinite will be $6.67 \times 10^{-10}\ J$

66. (b) For a moving satellite kinetic energy $= \frac{GMm}{2r}$

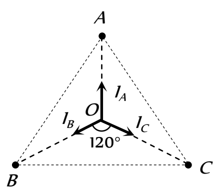
Potential energy $= \frac{-GMm}{r}$

$\therefore \frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{2}$

67. (a) Due to three particles net intensity at the centre

$I = \vec{I}_A + \vec{I}_B + \vec{I}_C = 0$

because out of these three intensities one equal in magnitude and the angle between each other is 120°.



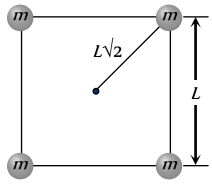
68. (c) $v_0 = \frac{v_e}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ km/s$

69. (a) Potential at the centre due to single mass = $\dfrac{-GM}{L/\sqrt{2}}$

Potential at the centre due to all four masses

$= -4\dfrac{GM}{L/\sqrt{2}} \quad -4\sqrt{2}\dfrac{GM}{L}$

$= -\sqrt{32} \times \dfrac{GM}{L}$.



70. (a) $v = \sqrt{2gR} \therefore \dfrac{v_1}{v_2} = \sqrt{\dfrac{g_1}{g_2} \times \dfrac{R_1}{R_2}} = \sqrt{g \times K} = (Kg)^{1/2}$

Motion of Satellite

1. (b) $v_e = \sqrt{2gR}$ and $v_0 = \sqrt{gR} \therefore \sqrt{2}\, v_0 = v_e$

2. (d)

3. (d) $v_0 = \sqrt{\dfrac{GM}{r}}$

4. (d) $v_0 = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{gR^2}{R+h}}$

5. (b) Centripetal acceleration works on it.

6. (b) $v = \sqrt{\dfrac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depends upon the mass of the satellite

7. (b) $T \propto r^{3/2}$. If r becomes double then time period will becomes $(2)^{3/2}$ times.

So new time period will be $24 \times 2\sqrt{2}\ hr$ *i.e.* $T = 48\sqrt{2}$

8. (b) *K.E.* required for satellite to escape from earth's gravitational field $\dfrac{1}{2}mv_e^2 = \dfrac{1}{2}m\left(\sqrt{\dfrac{2GM}{R}}\right)^2 = \dfrac{GMm}{R}$

K.E. required for satellite to move in circular orbit

$\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}m\left(\sqrt{\dfrac{GM}{R}}\right)^2 = \dfrac{GMm}{2R}$

The ratio between these two energies = 2

9. (c) The velocity of the spoon will be equal to the orbital velocity when dropped out of the space-ship.

10. (c)

11. (b) $v_0 = \sqrt{gR}$

12. (d) Telecommunication satellites are geostationary satellite

13. (b) $v = \sqrt{\dfrac{GM}{R}} \Rightarrow \dfrac{v_A}{v_B} = \sqrt{\dfrac{R_B}{R_A}} = \sqrt{\dfrac{R}{4R}} = \dfrac{1}{2}$

$\therefore \dfrac{v_A}{v_B} = \dfrac{3V}{v_B} = \dfrac{1}{2} \therefore v_B = 6\,V$

14. (a)

15. (a)

16. (b)

17. (c) $v = \sqrt{\dfrac{GM}{R+h}}$

For first satellite $h = 0$, $v_1 = \sqrt{\dfrac{GM}{R}}$

For second satellite $h = \dfrac{R}{2}$, $v_2 = \sqrt{\dfrac{2GM}{3R}}$

$v_2 = \sqrt{\dfrac{2}{3}}v_1 = \sqrt{\dfrac{2}{3}}v$

18. (d) $v = \sqrt{\dfrac{GM}{r}} \quad \therefore\ K.E. \propto v^2 \propto \dfrac{1}{r}$ and $T^2 \propto r^3$

$\therefore\ K.E. \propto T^{-2/3}$

19. (d)

20. (d) $T = 2\pi\sqrt{\dfrac{r^3}{GM}} \Rightarrow r^3 = \dfrac{GMT^2}{4\pi^2} \Rightarrow r = \left[\dfrac{GMT^2}{4\pi^2}\right]^{1/3}$

21. (d) $v \propto \dfrac{1}{\sqrt{r}}$. The speed of satellite decreases with an increase in the radius of its orbit.

22. (b) $v \propto \dfrac{1}{\sqrt{r}}$.

% increase in speed = $\dfrac{1}{2}$ (% decrease in radius)

$= \dfrac{1}{2}(1\%) = 0.5\%$

i.e. speed will increase by 0.5%

23. (b) $v = \sqrt{\dfrac{GM}{r}}$

24. (a)

25. (a) $v \propto \dfrac{1}{\sqrt{r}}$. If orbital radius becomes 4 times then orbital velocity will become half. *i.e.* $\dfrac{7}{2} = 3.5\ km/s$

26. (d) Orbital radius of satellites $r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$U_1 = \dfrac{-GMm}{r_1}$ and $U_2 = \dfrac{-GMm}{r_2}$

$K_1 = \dfrac{GMm}{2r_1}$ and $K_2 = \dfrac{GMm}{2r_2}$

$E_1 = \dfrac{GMm}{2r_1}$ and $E_2 = \dfrac{GMm}{2r_2}$

$\therefore \dfrac{U_1}{U_2} = \dfrac{K_1}{K_2} = \dfrac{E_1}{E_2} = 4$

27. (a) Escape velocity is same for all angles of projection.

28. (d) Orbital velocity $v_0 = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{gR^2}{r}}$ and $v_0 = r\omega$

This gives $r^3 = \frac{R^2 g}{\omega^2}$

29. (a)
30. (a) Due to inertia it will continue to move along the original path of the space craft.
31. (b)
32. (d)
33. (a)
34. (c) (i) $T_{st} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

[As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\frac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$

[As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\frac{R}{g}}$ [As $l = \infty$]

35. (d)
36. (c) $v \propto \frac{1}{\sqrt{r}}$, If $r = R$ then $v = V_0$

If $r = R + h = R + 3R = 4R$ then $v = \frac{V_0}{2} = 0.5V_0$

37. (d)
38. (b) $6R$ from the surface of earth and $7R$ from the centre.
39. (c) $T^2 = \frac{4\pi^2}{GM}r^3$. If G is variable then time period, angular velocity and orbital radius also changes accordingly.
40. (b) $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{(2R)^3}{gR^2}} = 4\sqrt{2}\pi\sqrt{\frac{R}{g}}$
41. (c) $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM}(R+h)^3$

$\Rightarrow R + h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} \Rightarrow h = \left[\frac{GMT^2}{4\pi^2}\right]^{\frac{1}{3}} - R$

42. (d) Distances of the satellite from the centre are $7R$ and $3.5R$ respectively.

$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\frac{3.5R}{7R}\right)^{3/2} = 6\sqrt{2}\,hr$

43. (a) $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{384000\times10^3}} = 1\,km/s$
44. (d) % change in $T = \frac{3}{2}$ (% change in R) $= \frac{3}{2}\times(2)\% = 3\%$
45. (a)
46. (c)
47. (c) $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} = \sqrt{\frac{10\times(64\times10^5)^2}{8000\times10^3}}$

$= 71.5\times10^2\,m/s = 7.15\,km/s$

48. (d) Total mechanical energy of satellite

$E = \frac{-GMm}{2r} \Rightarrow \frac{E_A}{E_B} = \frac{m_A}{m_B}\times\frac{r_B}{r_A} \Rightarrow \frac{3}{1}\times\frac{4r}{r} = \frac{12}{1}$

49. (b)
50. (b) Gravitational force provides the required centripetal force for orbiting the satellite

$\frac{mv^2}{R} = \frac{K}{R}$ because $\left(F \propto \frac{1}{R}\right)$

$\therefore\ v \propto R^{\circ}$

51. (a) Total energy = – (kinetic energy) = $-\frac{1}{2}Mv^2$
52. (a) Binding energy = – kinetic energy

And if this amount of energy (E_k) given to satellite then it will escape into outer space

53. (a) Potential energy = $\frac{-GMm}{r} = \frac{GMm}{R_e + h} = \frac{-GMm}{2R_e}$

$= -\frac{gR_e^2 m}{2R_e} = -\frac{1}{2}mgR_e = -0.5mgR_e$

54. (c) $B.E. = -\frac{GMm}{r}$. If $B.E.$ decreases then r also decreases and v increases as $v \propto \frac{1}{\sqrt{r}}$
55. (b) Time period of communication satellite $T_c = 1$ day

Time period of another satellite = T_s

$\frac{T_s}{T_c} = \left(\frac{r_s}{r_c}\right)^{3/2} = (4)^{3/2} \Rightarrow T_s = T_c\times(4)^{3/2} = 8$ days.

56. (b) Angular momentum is conserved in central field.
57. (b) $T^2 \propto r^3$
58. (d) $T^2 \propto R^3 \therefore \frac{T^2}{R^3} =$ constant
59. (d) $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = T_1(4)^{3/2} = 8T_1 = 40hr$
60. (b) Given that, $T_1 = 1$ day and $T_2 = 8$ days

$\therefore\ \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{8}{1}\right)^{2/3} = 4$

$\Rightarrow r_2 = 4r_1 = 4R$

61. (c)
62. (c) $\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2$

$\Rightarrow v_B = 2\times v_A = 2\times3v = 6v$

63. (b) Gravitational force provides the required centripetal force

$m\omega^2 R = \frac{GMm}{R^3} \Rightarrow \frac{4\pi^2}{T^2} = \frac{GM}{R^4} \Rightarrow T \propto R^2$

64. (a)

65. (b) $v_e = \sqrt{2}v_0 = 1.414\, v_0$

Fractional increase in orbital velocity $\left(\frac{\Delta v}{v}\right)$

$$= \frac{v_e - v_0}{v_0} = 0.414$$

$\therefore$ Percentage increase = 41.4%

66. (c) Time period of revolution of moon around the earth = 1 lunar month.

$$\frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2} \Rightarrow T_s = 2^{-3/2} \text{ lunar month.}$$

67. (d)

Kepler's Laws of Planetary Motion

1. (c) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = (1000)^{1/2} = 10\sqrt{10}$

2. (c) Areal velocity of the planet remains constant. If the areas *A* and *B* are equal then $t_1 = t_2$.

3. (c) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} \Rightarrow T_2 = 8T_1$

4. (b)

5. (c)

6. (b) Since $T^2 \propto r^3$ $\therefore \left(\frac{T'}{T}\right)^2 = \left(\frac{1}{4}\right)^3 \Rightarrow T' = \frac{1}{8}T$

7. (b) Kinetic and potential energies varies with position of earth *w.r.t.* sun. Angular momentum remains constant every where.

8. (c)

9. (c) Angular momentum remains constant

$$mv_1 d_1 = mv_2 d_2 \Rightarrow v_2 = \frac{v_1 d_1}{d_2}$$

10. (b) Orbital radius of Jupiter > Orbital radius of Earth

$\frac{v_J}{v_e} = \frac{r_e}{r_J}$. As $r_J > r_e$ therefore $v_J < v_e$

11. (d) $T^2 \propto r^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

12. (b) $\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$

13. (c) The earth moves around the sun is elliptical path. so by using the properties of ellipse

$r_1 = (1+e)a$ and $r_2 = (1-e)a$

$\Rightarrow a = \frac{r_1 + r_2}{2}$ and $r_1 r_2 = (1-e^2)a^2$

where a = semi major axis

b = semi minor axis

e = eccentricity

Now required distance = semi latusrectum $= \frac{b^2}{a}$

$$= \frac{a^2(1-e^2)}{a} = \frac{(r_1 r_2)}{(r_1 + r_2)/2} = \frac{2r_1 r_2}{r_1 + r_2}$$

14. (c) For first satellite $r_1 = R$ and $T_1 = 83\ minute$

For second satellite $r_2 = 4R$

$$T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = T_1(4)^{3/2} = 8T_1 = 8 \times 83 = 664 \textit{ minutes}$$

15. (a) Angular momentum = Mass ×Orbital velocity ×Radius

$$= m \times \left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = m\sqrt{GMR_0}$$

16. (b) $\frac{T^2}{r^3}$ = constant $\Rightarrow T^2 r^{-3}$ = constant

17. (d) $\frac{T_{\text{plant}}}{T_{\text{earth}}} = \left(\frac{r_{\text{plant}}}{r_{\text{earth}}}\right)^{3/2} = (1.588)^{3/2} = 2 \;\therefore\; T_{\text{planet}} = 2\, year$

18. (d) Mass of the satellite does not effects on time period

$$\frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \left(\frac{1}{8}\right)^{1/2} = \frac{1}{2\sqrt{2}}$$

19. (b) Speed of the earth will be maximum when its distance from the sun is minimum because mvr = constant

20. (c) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2} \Rightarrow 8 = \left(\frac{r_A}{r_B}\right)^{3/2} \Rightarrow r_A = (8)^{2/3} r_B = 4r_B$.

21. (c) $T^2 \propto r^3$. If *r* made half then *T* will become $\frac{T}{8}$.

22. (c)

23. (b) Mass of satellite does not affects on orbital radius.

24. (b) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2} \Rightarrow T_2 = 2\sqrt{2}$ *years.*

25. (c)

26. (a)

27. (a) By conservation of angular momentum *mvr* = constant

$v_{\min} \times r_{\max} = v_{\max} \times r_{\min}$

$$\therefore v_{\min} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12\, m/s$$

28. (b) $\omega_{\text{body}} = 27\omega_{\text{earth}}$

$$T^2 \propto r^3 \Rightarrow \omega^2 \propto \frac{1}{r^3} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \;\therefore r \propto \frac{1}{\omega^{2/3}}$$

$$\Rightarrow \frac{r_{\text{body}}}{r_{\text{earth}}} = \left(\frac{\omega_{\text{earth}}}{\omega_{\text{body}}}\right)^{2/3} = \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}$$

29. (d) Time period does not depends upon the mass of satellite.

30. (b) $\frac{T^2}{R^3} = \frac{T^2}{d^3} = \frac{1}{n^2 d^3} = \text{constant}$

$\therefore n_1^2 d_1^3 = n_2^2 d_2^3$ [where n = frequency]

31. (a)

32. (a)

33. (a) During path DAB planet is nearer to sun as comparison with path BCD. So time taken in travelling DAB is less than that for BCD because velocity of planet will be more in region DAB.

34. (c) Because distance of point C is maximum from the sun.

35. (b) $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 1 \times (2)^{3/2} = 2.8\ year$

36. (d) $L = mvr = m\sqrt{\frac{GM}{r}}r = m\sqrt{GMr}\ \therefore\ L \propto \sqrt{r}$

37. (a)

38. (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{dA}{dt} \propto vr \propto \omega r^2$

39. (b)

40. (c)

41. (d)

42. (a) $T^2 \propto R^3 \Rightarrow \left(\frac{T_P}{T_E}\right)^2 = \left(\frac{R_P}{R_E}\right)^3 = \left(\frac{2R_E}{R_E}\right)^3$

$\Rightarrow \frac{T_P}{T_E} = (2)^{3/2} = 2\sqrt{2}$

$\Rightarrow T_P = 2\sqrt{2} \times 365 = 1032.37 = 1032$ days

43. (d)

44. (d) For central force, torque is zero.

$\because \tau = \frac{dL}{dt} = 0 \Rightarrow L$ = constant

i.e. Angular momentum is constant.

45. (b) $\frac{v_{max}}{v_{min}} = \frac{1+e}{1-e} = \frac{1+0.0167}{1-0.0167} = 1.033$

46. (a) $m\omega^2 R = \frac{GMm}{R^2} \Rightarrow \left(\frac{2\pi}{T}\right)^2 R = \frac{GM}{R^2} \Rightarrow M = \frac{4\pi^2 R^3}{GT^2}$

47. (b) $v = \sqrt{\frac{GM}{R}} = G^{1/2}M^{1/2}R^{-1/2}$

48. (b)

49. (c) $v = \sqrt{\frac{GM}{R}}$ if $r_1 > r_2$ then $v_1 < v_2$

50. (c) Escape velocity for that body $v_e = \sqrt{\frac{2Gm}{r}}$

v_e should be more than or equal to speed of light

i.e. $\sqrt{\frac{2Gm}{r}} \geq c$

51. (c) $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{1}{4}\right)^{3/2} = \frac{1}{8}$

Critical Thinking Questions

1. (b) For revolution of planet centripetal force is provided by gravitational force of attraction

$m\omega^2 R \propto R^{-5/2} \Rightarrow \frac{1}{T^2} \propto R^{-7/2} \Rightarrow T^2 \propto R^{7/2}$

2. (a, b) $g = \frac{4}{3}\pi\rho G r\ \therefore\ g \propto r$ if $r < R$

$g = \frac{GM}{r^2}\quad \therefore\ g \propto \frac{1}{r^2}$ if $r > R$

If $r_1 < R$ and $r_2 < R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

If $r_1 > R$ and $r_2 > R$ then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

3. (a)

4. (b) $F = \frac{Gm(M-m)}{r^2}$

For maximum force $\frac{dF}{dm} = 0$

$\Rightarrow \frac{d}{dm}\left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2}\right) = 0$

$\Rightarrow M - 2m = 0 \Rightarrow \frac{m}{M} = \frac{1}{2}$

5. (a) $m\omega^2 R \propto \frac{1}{R^n} \Rightarrow m\left(\frac{4\pi^2}{T^2}\right)R \propto \frac{1}{R^n} \Rightarrow T^2 \propto R^{n+1}$

$\therefore T \propto R^{\left(\frac{n+1}{2}\right)}$

6. (c) $g = \frac{GM}{R^2}$. If mass remains constant then $g \propto \frac{1}{R^2}$

% increase in g = 2(% decrease in R) = 2 × 1% = 2%.

7. (b, c, d) $g = \frac{GM}{R^2}, v_e = \sqrt{\frac{2GM}{R}}$ and $U = \frac{-GMm}{R}$

$\therefore\ g \propto \frac{M}{R^2}, v_e \propto \sqrt{\frac{M}{R}}$ and $U \propto \frac{M}{R}$

If both mass and radius are increased by 0.5% then v_e and U remains unchanged where as g decrease by 0.5%.

8. (b) $g' = g - \omega^2 R\cos^2\lambda$

For weightlessness at equator $\lambda = 0$ and $g' = 0$

$\therefore\ 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{1}{800}\frac{rad}{s}$

9. (d) If acceleration due to gravity is g at the surface of earth then at height R it value becomes $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{4}$

$T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{g/4}}\ \therefore\ \frac{T_2}{T_1} = 2$

10. (b) $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{1}{2}mgR\ (\because\ h = R)$

11. (c) Potential energy = 2 × (Total energy) = $2E_0$

Because we know = $U = \frac{-GMm}{r}$ and $E_0 = \frac{-GMm}{2r}$

12. (c) $\Delta K.E. = \Delta U$

$$\Rightarrow \frac{1}{2}MV^2 = GM_e M\left(\frac{1}{R} - \frac{1}{R+h}\right) \quad \text{...(i)}$$

Also $g = \dfrac{GM_e}{R^2}$ ...(ii)

On solving (i) and (ii) $h = \dfrac{R}{\left(\dfrac{2gR}{V^2} - 1\right)}$

13. (a, c, d) Since cavities are symmetrical *w.r.t. O*. So the gravitational force at the centre is zero.

The radius of the circle $z^2 + y^2 = 36$ is 6.

For all points for $r \geq 6$, the body behaves as if whole of the mass is concentrated at the centre. So the gravitational potential is same.

Above is true for $z^2 + y^2 = 4$ as well.

14. (b) Let velocities of these masses at *r* distance from each other be v_1 and v_2 respectively.

By conservation of momentum

$m_1v_1 - m_2v_2 = 0$

$\Rightarrow m_1v_1 = m_2v_2$ ... (i)

By conservation of energy

change in *P.E.*=change in *K.E.*

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2Gm_1m_2}{r} \quad \text{...(ii)}$$

On solving equation (i) and (ii)

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}} \text{ and } v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}$$

$$\therefore v_{app} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}(m_1+m_2)}$$

15. (c) Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}} \Rightarrow \frac{1}{2}mk^2 2gR = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow h = \frac{Rk^2}{1-k^2}$$

Height of Projectile from the earth's surface = *h*

Height from the centre $r = R + h = R + \dfrac{Rk^2}{1-k^2}$

By solving $r = \dfrac{R}{1-k^2}$

16. (c) In the problem orbital radius is increased by 1%.

Time period of satellite $T \propto r^{3/2}$

Percentage change in time period

$= \dfrac{3}{2}$ (% change in orbital radius) $= \dfrac{3}{2}(1\%) = 1.5\%$

17. (b) According to Kepler's third law, the ratio of the squares of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun *i.e.*

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left[\frac{r_1}{\frac{1}{2}r_1}\right]^3 = 8 \Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$$

$$\therefore T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \ days}{2\sqrt{2}} = 129 \ days$$

18. (c) $\dfrac{T_2}{T_1} = \left(\dfrac{r_2}{r_1}\right)^{3/2} \Rightarrow T_2 = 24\left(\dfrac{6400}{36000}\right)^{3/2} \cong 2\,hour$

Graphical Questions

1. (c) $g \propto r$ (if $r < R$) and $g \propto \dfrac{1}{r^2}$ (if $r > R$)

2. (c) $V_{in} = \dfrac{-Gm}{2R}\left[3 - \left(\dfrac{r}{R}\right)^2\right]$, $V_{\text{surface}} = \dfrac{-GM}{R}$, $V_{out} = \dfrac{-GM}{r}$

3. (c) For hollow sphere

$V_{in} = \dfrac{-GM}{R}$, $V_{\text{surface}} = \dfrac{-GM}{R}$, $V_{out} = \dfrac{-GM}{r}$

i.e. potential remain constant inside the sphere and it is equal to potential at the surface and increase when the point moves away from the surface of sphere.

4. (b) $F = 0$ when $0 \leq r \leq R_1$

because intensity is zero inside the cavity.

F increase when $R_1 \leq r \leq R_2$

$F \propto \dfrac{1}{r^2}$ when $r > R_2$

5. (d) Intensity will be zero inside the spherical shell.

I = 0 upto *r* = *a* and $I \propto \dfrac{1}{r^2}$ when $r > a$

6. (c) Initially the weight of the passenger = 60 × 10 = 600 *N*

Finally the weight of the passenger = 60 × 4 = 240 *N*

and during the flight in between some where its weight will be zero because at that point gravitational pull of earth and mars will be equal.

7. (c) Kepler's law $T^2 \propto R^3$

8. (d) The system will be bound at points where total energy is negative. In the given curve at point A, B and C the P.E. is more than K.E.

9. (c) $U = \dfrac{-GMm}{r}$, $K = \dfrac{GMm}{2r}$ and $E = \dfrac{-GMm}{2r}$

For a satellite *U,K* and *E* varies with *r* and also *U* and *E* remains negative whereas *K* remain always positive.

10. (c) Gravitational P.E. = *m* × gravitational potential

$U = mV$ So the graph of *U* will be same as that of *V* for a spherical shell.

Assertion and Reason

1. (a) According to Kepler's third law $T^2 \propto r^3$

If r is small then T will also be small.

2. (b) For two electron $\frac{F_g}{F_e} = 10^{-43}$ *i.e.* gravitational force is negligible in comparison to electrostatic force of attraction.

3. (d) The universal gravitational constant G is totally different from g.

$$G = \frac{FR^2}{Mm}$$

The constant G is scalar and posses the dimensions $[M^{-1}L^3T^{-2}]$.

$$g = \frac{GM}{R^2}$$

g is a vector and has got the dimensions $[M^0LT^{-2}]$.

It is not a universal constant.

4. (c) Acceleration due to gravity is given by $g = \frac{GM}{R^2}$. Thus it does not depend on mass of body on which it is acting. Also it is not a constant quantity it changes with change in value of both M and R (distance between two bodies).

5. (a) If a pendulum is suspended in a lift and lift is moving downward with some acceleration a, then time period of pendulum is given by, $T = 2\pi\sqrt{\frac{l}{g-a}}$.

In the case of free fall, $a = g$ then $T = \infty$

i.e., the time period of pendulum becomes infinite.

6. (c) The value of g at any place is given by the relation, $g' = g - \omega^2 R_e \cos^2 \lambda$

When λ is angle of latitude and ω is the angular velocity of earth. If earth suddenly stops rotating, then $\omega = 0$

$\therefore$ $g' = g$ *i.e.*, the value of g will be same at all places.

7. (b) Acceleration due to gravity,

$g' = g - R\omega^2 \cos^2 \lambda$

At equator, $\lambda = 0°$ *i.e.* $\cos 0° = 1$ $\therefore$ $g_e = g = R\omega^2$

At poles, $\lambda = 90°$ *i.e.* $\cos 90° = 0$ $\therefore$ $g_p = g$

Thus, $g_p - g_e = g - g + R\omega^2 = R\omega^2$

Also, the value of g is maximum at poles and minimum at equators.

8. (a) As the rotation of earth takes place about polar axis therefore body placed at poles will not feel any centrifugal force and its weight or acceleration due to gravity remains unaffected.

9. (e) Earth revolves around the sun in circular path and required centripetal force is provided by gravitational force between earth and sun but the work done by this centripetal force is zero.

10. (a) Inertial mass and gravitational mass are equivalent. Both are scalar quantities and measured in the same unit. They are quite different in the method of their measurement. Also gravitational mass of a body is affected by the presence of other bodies near it where as internal mass remain unaffected.

11. (a) Because gravitational force is always attractive in nature and every body is bound by this gravitational force of attraction of earth.

12. (a) As no torque is acting on the planet, its angular momentum must remain constant in magnitude as well as direction. Therefore, plane of rotation must pass through the centre of earth.

13. (c) According to Kepler's law of planetary motion, a planet revolves around the sun in such a way that its areal velocity is constant. *i.e.*, it move faster, when it is closer the sun and vice-versa.

14. (e) Escape velocity = $\sqrt{2} \times$ orbital velocity.

15. (c) Due to resistance force of atmosphere, the satellite revolving around the earth losses kinetic energy. Therefore in a particular orbit the gravitational attraction of earth on satellite becomes greater than that required for circular orbit there. Therefore satellite moves down to a lower orbit. In the lower orbit as the potential energy $(U = -GMm/r)$ becomes more negative,

Hence kinetic energy $\left(E_K = GMm/2r\right)$ increases, and hence speed of satellite increases.

16. (b) If root mean square velocity of the gas molecules is less than escape velocity from that planet (or satellite) then atmosphere will remain attached with that planet and if $v_{rms} > v_{\text{escape}}$ then there will be no atmosphere on the planet. This is the reason for no atmosphere at moon.

17. (b) As the geostationary satellite is established in an orbit in the plane of the equator at a particular place, so it move in the same sense as the earth and hence its time period of revolution is equal to 24 hours, which is equal to time period of revolution of earth about its axis.

18. (e) The total gravitational force on one particle due to number of particles is the resultant force of attraction (or gravitational force) exerted on the given particle due to individual particles. *i.e.*, $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + ...$ It means the principle of superposition is valid.

19. (d) As, escape velocity $= \sqrt{\frac{2GM}{R}}$, so its value depends on mass of planet and radius of the planet. The two different planets have same escape velocity, when these quantities (mass and radius) are equal.

20. (a) According to kepler's law $T^2 \propto r^3 \propto (R+h)^3$

i.e. if distance of satellite is more then its time period will be more.

21. (a) According to Newton's law of gravitation,

$F = \frac{Gm_1m_2}{r^2}$. When m_1, m_2 and r_2 all are doubled,

$F = \frac{G(2m_1)(2m_2)}{(2r)^2} = \frac{Gm_1m_2}{r^2}$, *i.e.* F remains the same.

22. (c) Upto ordinary heights, the change in the distance of a projectile from the centre of earth is negligible compared to the radius of earth. Hence the projectile moves under a nearly uniform gravitational force and the path is parabolic. But for the projectiles moving to a large height, the gravitational force decreases quite decreasing variable force, the path of the projectile becomes elliptical.

23. (c) As the distance from centre of earth decreases, acceleration due to gravity decreases and at the centre of earth it becomes zero.

$g' = g\left(1 - \frac{d}{R}\right)$. If $d = R$ then $g' = 0$

24. (b) We know that earth revolves from west to east about its polar axis. Therefore, all the particles on the earth have velocity from west to east.

This velocity is maximum in the equatorial line, as $v = R\omega$, where R is the radius of earth and ω is the angular velocity of revolution of earth about its polar axis.

When a rocket is launched from west to east in equatorial plane, the maximum linear velocity is added to the launching velocity of the rocket, due to which launching becomes easier.

25. (e) Binding energy $= \frac{GMm}{R} = -$ [Total energy of satellite]

and it is clear that it depends upon the mass of the satellite.

26. (a) According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the objects with respect to finger changes, hence the force of attraction changes, disturbing the entire universe, including stars.

27. (a) Intensity inside a hollow sphere is zero, so force is also equal to zero. $\vec{F} = m\vec{E}$

28. (a) The time period of satellite which is very near to earth is given by

$$T = 2\pi\sqrt{\frac{R}{g}} \cong 84\ min = 1hr.\ 24\ min$$

29. (c) A person feels his weight only when the surface on which he is standing exerts a reactionary force on him. Because the acceleration of the person and that of the satellite revolving round the earth are equal (= g), hence acceleration of the person with respect to the satellite is zero.

Therefore person feels weightless on satellite, although the gravitational force is acting on a satellite.

30. (e) If the orbital path of a satellite is circular, then its speed is constant and if the orbital path of a satellite is elliptical, then its speed in its orbit is not constant. In that case its areal velocity is constant.

31. (a) $v_0 = R_e\sqrt{\frac{g}{R_e + h}}$

For satellite revolving very near to earth $R_e + h = R_e$

As (h<<R)

$v_o = \sqrt{R_e g} \simeq \sqrt{64 \times 10^5 \times 10} = 8 \times 10^3\ m/s = 8\ kms^{-1}$

Which is independent of height of a satellite.

32. (b)

33. (a) The torque on a body is given by $\vec{\tau} = \frac{d\vec{L}}{dt}$

In case of planet orbiting around sun no torque is acting on it.

$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ = constant.

Gravitation

SET Self Evaluation Test -8

1. Two identical spheres are placed in contact with each other. The force of gravitation between the spheres will be proportional to (R = radius of each sphere)

(a) R (b) R^2

(c) R^4 (d) None of these

2. Suppose that the force of earth's gravity suddenly disappears, choose the correct answer out of the following statements

(a) The weight of the body will become zero but mass remains the same

(b) The mass of the body will become zero but the weight remains the same

(c) Both the mass and weight will be the same

(d) Mass and weight will remain the same

3. An earth satellite is moved from one stable circular orbit to a further stable circular orbit, which one of the following quantities increase

(a) Gravitational force (b) Gravitational P.E.

(c) Linear orbital speed (d) Centripetal acceleration

4. Two planets revolve round the sun with frequencies N_1 and N_2 revolutions per year. If their average orbital radii be R_1 and R_2 respectively, then R_1 / R_2 is equal to

(a) $(N_1 / N_2)^{3/2}$ (b) $(N_2 / N_1)^{3/2}$

(c) $(N_1 / N_2)^{2/3}$ (d) $(N_2 / N_1)^{2/3}$

5. There is no atmosphere on the moon because

(a) It is closer to the earth

(b) It revolves round the earth

(c) It gets light from the sun

(d) The escape velocity of gas molecules is lesser than their root mean square velocity here

6. Two heavenly bodies S_1 and S_2, not far off from each other are seen to revolve in orbits

(a) Around their common centre of mass

(b) Which are arbitrary

(c) With S_1 fixed and S_2 moving round S_1

(d) With S_2 fixed and S_1 moving round S_2

7. The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth

(a) Is the same (b) Is smaller

(c) Is greater (d) Varies with its phase

8. A clock S is based on oscillation of a spring and a clock P is based on pendulum motion. Both clocks run at the same rate on earth. On a planet having the same density as earth but twice the radius

(a) S will run faster than P

(b) P will run faster than S

(c) They will both run at the same rate as on the earth

(d) None of these

9. Consider earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a balloon. The value of g measured by

(a) A goes on decreasing and that by B goes on increasing

(b) B goes on decreasing and that by A goes on increasing

(c) Each decreases at the same rate

(d) Each decreases at different rates

10. The mass of the moon is $\frac{1}{81}$ of the earth but the gravitational pull is $\frac{1}{6}$ of the earth. It is due to the fact that

(a) The radius of the moon is $\frac{81}{6}$ of the earth

(b) The radius of the earth is $\frac{9}{\sqrt{6}}$ of the moon

(c) Moon is the satellite of the earth

(d) None of the above

11. A weight is suspended from the ceiling of a lift by a spring balance. When the lift is stationary the spring balance reads W. If the lift suddenly falls freely under gravity, the reading on the spring balance will be

(a) W (b) 2 W

(c) $W/2$ (d) 0

12. If a planet consists of a satellite whose mass and radius were both half that of the earth, the acceleration due to gravity at its surface would be (g on earth = 9.8 m/sec^2)

(a) $4.9\, m/sec^2$ (b) $8.9\, m/sec^2$

(c) $19.6\, m/sec^2$ (d) $29.4\, m/sec^2$

13. At a given place where acceleration due to gravity is 'g' m/sec^2, a sphere of lead of density 'd' kg/m^3 is gently released in a column of liquid of density 'ρ' kg/m^3. If $d > \rho$, the sphere will

(a) Fall vertically with an acceleration 'g' m/sec^2

(b) Fall vertically with no acceleration

(c) Fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$

(d) Fall vertically with an acceleration $g\left(\frac{\rho}{d}\right)$

14. g_e and g_p denote the acceleration due to gravity on the surface of the earth and another planet whose mass and radius are twice as that of earth. Then

(a) $g_p = g_e$ (b) $g_p = g_e / 2$

(c) $g_p = 2g_e$ (d) $g_p = g_e / 4$

15. If the value of g at the surface of the earth is 9.8 m/sec^2, then the value of g at a place 480 km above the surface of the earth will be (Radius of the earth is 6400 km)

(a) $8.4\, m/\sec^2$ (b) $9.8\, m/\sec^2$

(c) $7.2\, m/\sec^2$ (d) $4.2\, m/\sec^2$

16. The acceleration due to gravity about the earth's surface would be half of its value on the surface of the earth at an altitude of (R = 4000 *mile*)

(a) 1200 *mile* (b) 2000 *mile*

(c) 1600 *mile* (d) 4000 *mile*

17. A pendulum clock is set to give correct time at the sea level. This clock is moved to hill station at an altitude of 2500 *m* above the sea level. In order to keep correct time of the hill station, the length of the pendulum

(a) Has to be reduced

(b) Has to be increased

(a) Needs no adjustment

(d) Needs no adjustment but its mass has to be increased

18. At some point the gravitational potential and also the gravitational field due to earth is zero. The point is

(a) On earth's surface

(b) Below earth's surface

(c) At a height R_e from earth's surface (R_e = radius of the earth)

(d) At infinity

19. A body falls freely under gravity. Its speed is v when it has lost an amount U of the gravitational energy. Then its mass is

(a) $\frac{Ug}{v^2}$ (b) $\frac{U^2}{g}$

(c) $\frac{2U}{v^2}$ (d) $2\,Ugv^2$

20. The ratio of the radius of the earth to that of the moon is 10. The ratio of acceleration due to gravity on the earth and on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is

(a) 10 (b) 6

(c) Nearly 8 (d) 1.66

21. Escape velocity from the moon surface is less than that on the earth surface, because

(a) Moon has no atmosphere while the earth has

(b) Radius of moon is less than that of the earth

(c) Moon is nearer to the sun

(d) Moon is attracted by other planets

22. The ratio of the radius of a planet '*A*' to that of planet '*B*' is '*r*'. The ratio of acceleration due to gravity on the planets is '*x*'. The ratio of the escape velocities from the two planets is

(a) xr (b) $\sqrt{\frac{r}{x}}$

(c) $\sqrt{rx}$ (d) $\sqrt{\frac{x}{r}}$

23. Time period of revolution of a nearest satellite around a planet of radius R is T. Period of revolution around another planet, whose radius is 3R but having same density is

(a) T (b) $3T$

(c) $9T$ (d) $3\sqrt{3}T$

24. The maximum possible velocity of a satellite orbiting round the earth in a stable orbit is

(a) $\sqrt{2R_e g}$ (b) $\sqrt{R_e g}$

(c) $\sqrt{\frac{R_e g}{2}}$ (d) Infinite

25. A man inside an artificial satellite feels weightlessness because the force of attraction due to earth is

(a) Zero at that place

(b) Is balanced by the force of attraction due to moon

(c) Equal to the centripetal force

(d) Non-effective due to particular design of the satellite

26. Two identical satellites A and B are circulating round the earth at the height of R and $2R$ respectively, (where R is radius of the earth). The ratio of kinetic energy of A to that of B is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 2 (d) $\frac{3}{2}$

27. The mean radius of the earth's orbit round the sun is 1.5×10^{11}. The mean radius of the orbit of mercury round the sun is $6\times10^{10}\, m$. The mercury will rotate around the sun in

(a) A year (b) Nearly 4 years

(c) Nearly $\frac{1}{4}$ year (d) 2.5 years

Answers and Solutions

(SET - 8)

1. (c) $F = G\frac{M\times M}{R^2} = \frac{G\times\left(\frac{4}{3}\pi R^3\rho\right)^2}{(2R)^2} \Rightarrow F \propto \frac{R^6}{R^2} \propto R^4$

2. (a) In the absence of gravity weight of the bodies will become zero but mass will not change.

3. (b) $U = \frac{-GMm}{r}$. If r increases then U also increases.

4. (d) According to Kepler's law $T^2 \propto R^3$

If N is the frequencs then $N^2 \propto (R)^{-3}$

or $\frac{N_2}{N_1} = \left(\frac{R_2}{R_1}\right)^{-3/2} \Rightarrow \frac{R_1}{R_2} = \left(\frac{N_2}{N_1}\right)^{2/3}$

5. (d)

6. (a)

7. (a) Force between earth and moon $F = \frac{Gm_m m_e}{r^2}$

This amount of force, both earth and moon will exert on each other *i.e.* they exert same force on each other.

8. (b) $g = \frac{4}{3}\pi\rho GR$. If density is same then $g \propto R$

According to problem $R_p = 2R_e \;\therefore\; g_p = 2g_e$

For clock P (based on pendulum motion) $T = 2\pi\sqrt{\frac{l}{g}}$

Time period decreases on planet so it will run faster because $g_p > g_e$

For clock S (based on oscillation of spring) $T = 2\pi\sqrt{\frac{m}{k}}$

So it does not change.

9. (d) For scientist A which goes down in a mine $g' = g\left(1 - \frac{d}{R}\right)$

For scientist B, which goes up in a air $g' = g\left(1 - \frac{2h}{R}\right)$

So it is clear that value of g measured by each will decreases at different rates.

10. (b) Gravitational pull depends upon the acceleration due to gravity on that planet.

$M_m = \frac{1}{81}M_e,\ g_m = \frac{1}{6}g_e$

$g = \frac{GM}{R^2} \Rightarrow \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2}$

$\therefore\ R_e = \frac{9}{\sqrt{6}}R_m$

11. (d) Reading of spring balance $R = m(g - a)$

If the lift falls freely then $a = g \;\therefore\; R = 0$

12. (c) $g = \frac{GM}{R^2} \;\therefore\; g \propto \frac{M}{R^2}$

According to problem $M_p = \frac{M_e}{2}$ and $R_p = \frac{R_e}{2}$

$\therefore\ \frac{g_p}{g_e} = \left(\frac{M_p}{M_e}\right)\left(\frac{R_e}{R_p}\right)^2 = \left(\frac{1}{2}\right) \times (2)^2 = 2$

$\Rightarrow g_p = 2g_e = 2 \times 9.8 = 19.6\ m/s^2$

13. (c) Apparent weight = actual weight – upthrust force

$$Vdg' = Vdg - V\rho g$$

$\Rightarrow g' = \left(\frac{d - \rho}{d}\right)g$

14. (b) $g \propto \frac{M}{R^2}$. If mass and radius of the planet are twice then g_ρ will be half that of g_e *i.e.* $g_p = \frac{g_e}{2}$

15. (a) The value of g on the surface of the earth $g \propto \frac{1}{R^2}$

At height h from the surface of the earth $g' \propto \frac{1}{(R + h)^2}$

$\therefore\ g' = g\frac{R^2}{(R + h)^2} = \frac{9.8 \times (6400)^2}{(6400 + 480)^2} = 8.4\ m/s^2$

16. (c) $\frac{g'}{g} = \left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{1}{2} = \left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{1}{2} = \left(\frac{4000}{4000 + h}\right)^2$

By solving we get $h = 1656.85\ mile \approx 1600\ mile$

17. (a) $T = 2\pi\sqrt{\frac{l}{g}}$. At the hill g will decrease so to keep the time period same the length of pendulum has to be reduced.

18. (d) $V = \frac{-GM}{r}$ and $I = \frac{GM}{r^2}$

$V = 0$ and $I = 0$ at $r = \infty$

19. (c) U = Loss in gravitational energy = gain in K.E.

So, $U = \frac{1}{2}mv^2 \Rightarrow m = \frac{2U}{v^2}$

20. (c) $\frac{v_e}{v_m} = \sqrt{\frac{g_e}{g_m}\frac{R_e}{R_m}} = \sqrt{6 \times 10} = \sqrt{60} \cong 8$ (nearly)

21. (b)

22. (c) $v_e = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{x \times r} \;\therefore\; \frac{v_A}{v_B} = \sqrt{rx}$

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23. (a) Time period of satellite which is very near to planet

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\frac{4}{3}\pi R^3\rho}} \;\therefore\; T \propto \sqrt{\frac{1}{\rho}}$$

i.e. time period of nearest satellite does not depends upon the radius of planet, it only depends upon the density of the planet.

In the problem, density is same so time period will be same.

24. (b) Otherwise centrifugal force exceeds the force of attraction or we can say that gravitational force won't be able to keep the satellite in circular motion.

25. (c)

26. (d) $\frac{K_A}{K_B} = \frac{r_B}{r_A} = \left(\frac{R+h_B}{R+h_A}\right) = \left(\frac{R+2R}{R+R}\right) = \frac{3}{2}$

27. (c) $\frac{T_{\text{mercury}}}{T_{\text{earth}}} = \left(\frac{r_{\text{mercury}}}{r_{\text{earth}}}\right)^{3/2} = \left(\frac{6\times 10^{10}}{1.5\times 10^{11}}\right)^{3/2} = \frac{1}{4}$ (approx.)

$\therefore T_{\text{mercury}} = \frac{1}{4}\text{ year}$