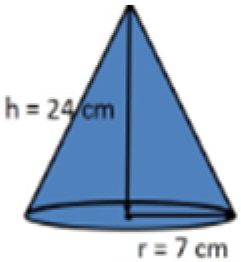
Mensuration

Exercise 10.1

Q. 1. A joker's cap is in the form of right circular cone whose base radius is 7 cm and height is 24 cm. Find the area of the sheet required to make 10 such caps.

Answer :



Given that, the radius of cone(r) is 7 cm and height(h) is

24 cm.

And, Surface area of cone is - πrl

Also, slant height(l) = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{7^2 + 24^2}$

$\Rightarrow l = \sqrt{49 + 576}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow l = 25$

⇒ Surface area of joker's cap

$$= \frac{22}{7} \times 7 \times 25$$

= $550 cm^2$

⇒ ∴ the area of the sheet required to make 10 such caps

= $550 \times 10\ cm^2$

$= 5500\ cm^2$

Q. 2. A sports company was ordered 100 paper cylinders for packing shuttle cocks. The required dimensions of the cylinder are 35 cm length/height and its radius is 7 cm. Find the required area of thick paper sheet needed to make 100 cylinders?

Answer : Given that, the radius of cylinder(r) required is 7 cm and height (h) is 35 cm.

And, Surface area of cylinder is - $2\pi rh$

$\Rightarrow$ Surface area $= 2 \times \frac{22}{7} \times 7 \times 35$

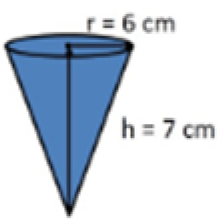
$= 1540 cm^2$

$\Rightarrow$The required area of thick paper sheet needed to make 100 cylinders = $1540 \times 100\ cm^2$

$= 154000\ cm^2$

Q. 3. Find the volume of right circular cone with radius 6 cm. and height 7 cm.

Answer :



Given that, the radius of cone(r) is 6 cm and height(h) is 7 cm.

And, volume of the cone $= \frac{1}{3}\pi r^2$

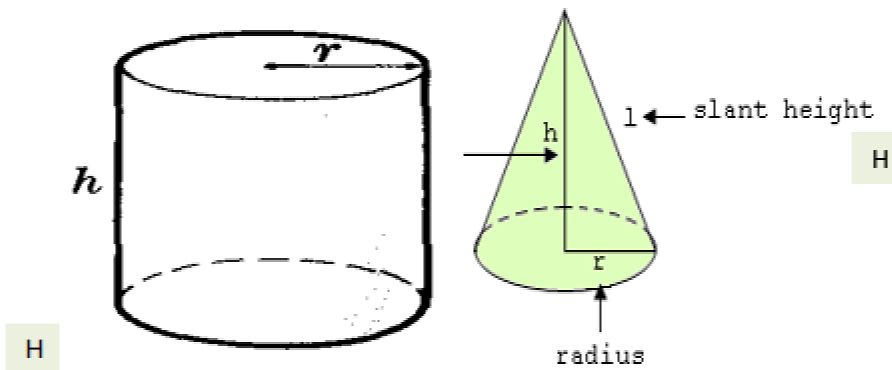
$\Rightarrow$ Volume of the right circular cone $= \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 24$

$$= \frac{1}{3} \times \frac{22}{7} \times 36 \times 24$$

$= 264\ cm^3$

Q. 4. The lateral surface area of a cylinder is equal to the curved surface area of a cone. If their bases be the same, find the ratio of the height of the cylinder to the slant height of the cone.

Answer :



Given that, the lateral surface area of a cylinder is equal to the curved surface area of a cone and their bases are same.

⇒Let r = radius of cylinder = radius of cone, h = height of cylinder and l = slant height of the cone.

And, lateral surface area of cylinder = 2πrh

Also, curved surface area of cone = πrl

⇒2πrh = πrl

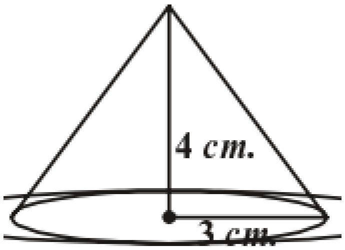
$$\Rightarrow \frac{h}{l} = \frac{\pi \times r}{2 \times \pi \times r}$$

$$\Rightarrow \frac{h}{l} = \frac{1}{2}$$

⇒ The ratio of the height of the cylinder to the slant height of the cone = 1:2

Q. 5. A self-help group wants to manufacture joker's caps of 3 cm. radius and 4 cm height. If the available paper sheet is 1000 cm^2, then how many caps can be manufactured from that paper sheet?

Answer :



Given that, the radius(r) of cone is 3 cm and height (h) is

4 cm.

And, the available paper sheet is- 1000 cm^2

And, Surface area of cone is - πrl

Also, slant height(l) = $\sqrt{r^2 + h^2}$

⇒ l = $\sqrt{3^2 + 4^2}$

⇒ l = $\sqrt{9 + 16}$

⇒ l = $\sqrt{25}$

⇒ l = 5

⇒ Surface area of joker's cap = $\frac{22}{7} \times 3 \times 5$

$$= \frac{330}{7}\ \text{cm}^2$$

And, the available paper sheet is- 1000 cm^2

⇒ No. of caps that can be manufactured from that paper sheet

$$= \frac{1000}{\frac{330}{7}}\ \text{cm}^2$$

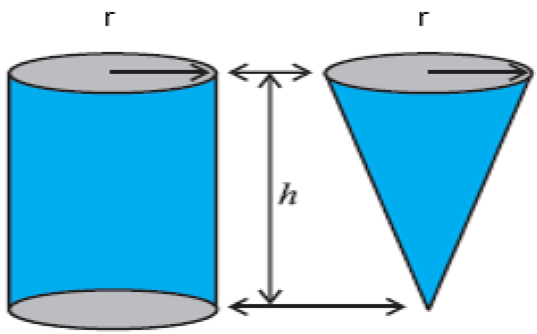
$$= 1000 \times \frac{7}{330}\ \text{cm}^2$$

= 21.21 cm^2

$= 21\ cm^2$

Q. 6. A cylinder and cone have base of equal radii and are of equal heights. Show that their volumes are in the ratio of 3 : 1.

Answer :



Given that, A cylinder and cone have base of equal radii and are of equal heights and their volumes are in the ratio of 3 : 1.

⇒ Let r = radius of cylinder = radius of cone,

and h = height of cylinder = height of cone.

⇒ And, volume of cylinder = $\pi r^2 h$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

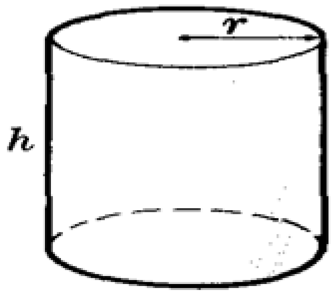
$$\Rightarrow \frac{\text{volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h}$$

$$\Rightarrow \frac{\text{volume of cylinder}}{\text{Volume of cone}} = \frac{3}{1}$$

⇒ Their volumes are in the ratio of 3 : 1.

Q. 7. The shape of solid iron rod is a cylindrical. Its height is 11 cm. and base diameter is 7 cm. Then find the total volume of 50 such rods?

Answer :



Given that, height(h) of cylinder is 11 cm and diameter(d) is 7 cm.

$\Rightarrow$ radius(r) of cylinder $= \frac{7}{2}$ $(\because r = \frac{d}{2})$

And, the volume of cylinder $= \pi r^2 h$

$\Rightarrow$ volume of one rod $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11$

$\Rightarrow$ volume of one rod $= \frac{11858}{28}$

= 423.5

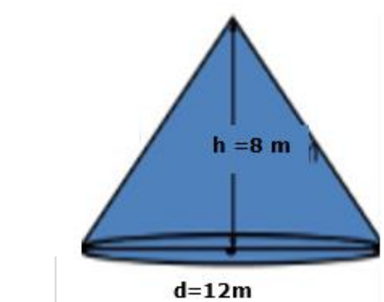
$\Rightarrow$ the total volume of 50 such rods = 423.5 × 50

= 21,175cm^3

Q. 8. A heap of rice is in the form of a cone of diameter 12 m. and height 8 m. Find its volume? How much canvas cloth is required to cover the heap?

(use π = 3.14)

Answer :



Given that, the diameter(d) of cone is 12 cm and heigh(h)t is 8 cm.

$\Rightarrow$ radius(r) $= \frac{12}{2} = 6\text{ cm} \left(\because\ r = \frac{d}{2}\right)$

And, Surface area of cone is - πrl

Also, volume of cone $= \frac{1}{3}\pi r^2 h$

Also, slant height(l) = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{6^2 + 8^2}$

$\Rightarrow l = \sqrt{36 + 64}$

$\Rightarrow l = \sqrt{100}$

$\Rightarrow l = 10$

$\Rightarrow$ volume of heap $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times 3.14 \times 6^2 \times 8$$

= 301.44 cm^3

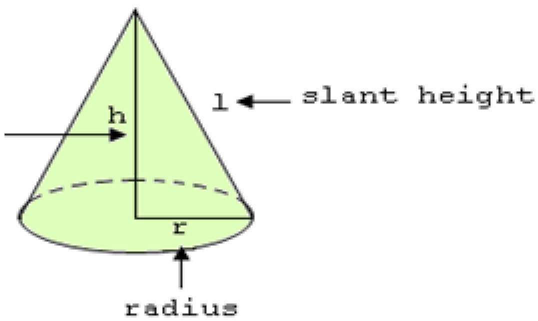
$\Rightarrow$ Surface area of cone = 3.14 × 6 × 10

= 188.4 cm^2

$\Rightarrow \therefore$ canvas cloth required to cover the heap = 188.4 cm^2

Q. 9. The curved surface area of a cone is 4070cm^2 and its diameter is 70 cm. What is its slant height?

Answer :



Given that, curved surface area of a cone is 4070 cm^2 and diameter(d) is 70 cm.

$$\Rightarrow \text{radius}(r) = \frac{70}{2} = 35 \left(\because r = \frac{d}{2}\right)$$

Let the slant height be l.

And, curved surface area of a cone = πrl

⇒ πrl = 4070

$$\Rightarrow \frac{22}{7} \times 35 \times l = 4070$$

$$\Rightarrow l = \frac{4070 \times 7}{22 \times 35}$$

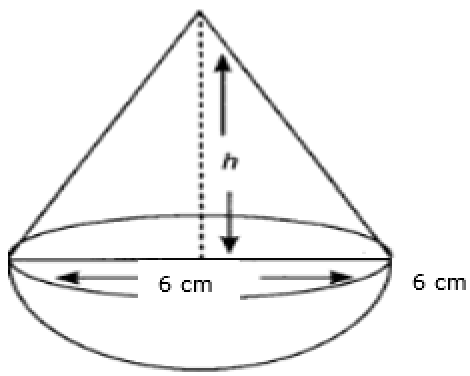
⇒ l = 37 cm

Exercise 10.2

Q. 1. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm respectively. Determine the surface area of the toy.

(use π = 3.14)

Answer :



Given that, diameter(d) is 6 cm and height(h) is 4 cm.

⇒ radius(r) $= \frac{6}{2} = 3\text{ cm} (\because r = d/2)$

Also, slant height(l) = $\sqrt{r^2 + h^2}$

⇒ $l = \sqrt{3^2 + 4^2}$

⇒ $l = \sqrt{9 + 16}$

⇒ $l = \sqrt{25}$

⇒ $l = 5$

And, surface area of the toy = surface area of the cone + surface area of hemisphere

⇒ Surface area of toy = $\pi rl + 2\pi r^2$

$= \pi r(l + 2r)$

$$= \frac{22}{7} \times 3(5 + 2 \times 3)$$

$$= \frac{22}{7} \times 33$$

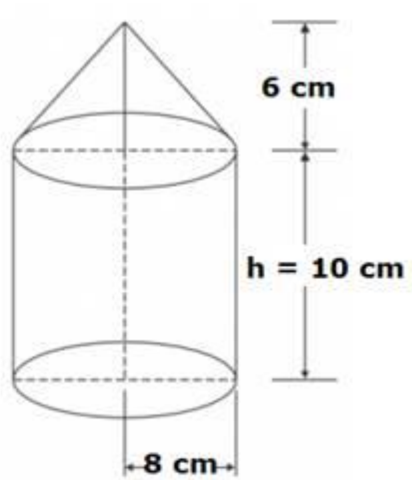
$= 103.71\text{ cm}^2$

Q. 2. A solid is in the form of a right circular cylinder with a hemisphere one end and a cone at the other end. The radius of the common base is 8 cm. and the heights of the cylindrical and conical portion are 10 cm and 6 cm respectively. Find the total surface area of the solid.

(use π = 3.14)

Answer :

The figure is shown below:



Given that, radius(r) is 8 cm, height of cylinder(H) is 10 cm and height of cone(h) is 6cm.

Also, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{8^2 + 6^2}$

$\Rightarrow l = \sqrt{64 + 36}$

$\Rightarrow l = \sqrt{100}$

$\Rightarrow l = 10$

Now, total surface area of solid = surface area of cone + surface area of cylinder + surface area of sphere

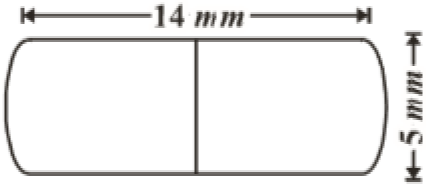
⇒ total surface area of solid = $\pi rl + 2\pi rH + 2\pi r^2 = \pi r(l + 2H + 2r)$

$= 3.14 \times 8(10 + 2 \times 10 + 2 \times 8)$

$= 3.14 \times 8 \times 46$

$= 1155.55\ cm^2$

Q. 3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the capsule is 14 mm. and the width is 5 mm. Find its surface area.



Answer : Given that, the length of the capsule is 14 mm and the width is 5 mm.

$$\Rightarrow \text{Radius}(r) = \frac{5}{2} = 2.5 \text{ mm } (\because \text{ width } = d = 2r)$$

⇒ height of cylinder(h) = total height-2 × radius of hemisphere

= 14-5 = 9mm

⇒ surface area of capsule = surface area of cylinder + 2 × surface

Area of one hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

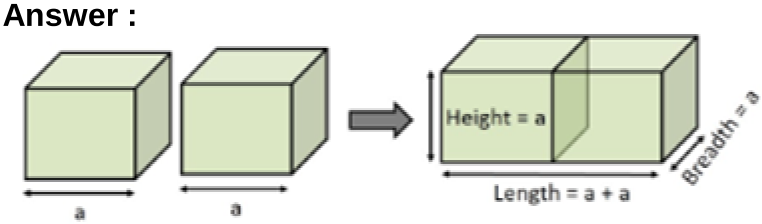
$$= 2 \times \frac{22}{7} \times 2.5(9 + 2 \times 2.5)$$

$$= \frac{1540}{7}$$

$= 220 \text{ mm}^2$

Q. 4. Two cubes each of volume 64cm^2 are joined end to end together. Find the surface area of the resulting cuboid.

Answer :



Given that, volume of cube is 64 cm^3.

Also, volume of cube = a^3 (where, a is side)

$\Rightarrow a^3 = 64$

$\Rightarrow a = 4$

$\Rightarrow$ Length(l) of cuboid = $2 \times a = 8$ cm,

Breadth(b) = height(h) = a = 4 cm

$\Rightarrow$ the total surface area of the resulting cuboid = 2(lb + bh + hl)

$\Rightarrow 2 \times (8 \times 4 + 4 \times 4 + 4 \times 8)$

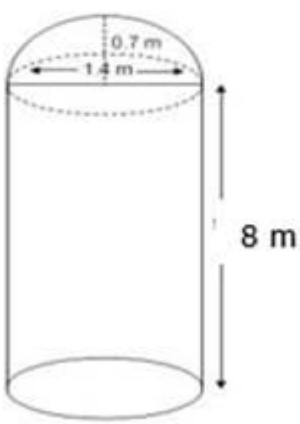
$= 2 \times 80$

$= 160$ cm^2

Q. 5. A storage tank consists of a circular cylinder with a hemisphere stuck on either end. If the external diameter of the cylinder be 1.4 m. and its length be 8 m. find the cost of painting it on the outside at rate of D20 per m^2.

Answer :

The figure is shown below:



Given that, diameter(d) of cylinder is 1.4 m and length(h) is 8 m.

$\Rightarrow$ the radius of sphere(r) = radius of cylinder $= \frac{\text{diameter}}{2} = \frac{1.4}{2} = 0.7$

$(\because r = \frac{d}{2})$

⇒ surface area of tank = surface area of cylinder + 2(surface

Area of hemisphere)

= 2πrh + 2(2πr$^{2)}$

= 2πr(h + 2r)

$= 2 \times \frac{22}{7} \times 0.7(8 + 2 \times 0.7)$

$= 2 \times \frac{22}{7} \times 0.7 \times 9.4$

= 41.36 m^2

⇒ ∴ the cost of painting it on the outside at rate of D20 per m^2

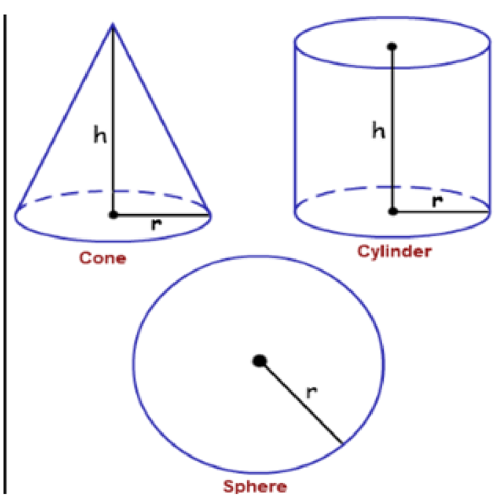
= 41.36 × 20

= D827.20

Q. 6. A sphere, a cylinder and a cone have the same radius and same height. Find the ratio of their volumes.

[Hint: Diameter of the sphere is equal to the heights of the cylinder and the cone.]

Answer :



Given that, a sphere, a cylinder and a cone have the same radius(say r) and same height(say h).

⇒ ∵ they have same height,

⇒ diameter of sphere = height of cylinder

⇒ radius of sphere $= \mathrm{r} = \frac{\mathrm{h}}{2}$ $(\because \mathrm{r} = \frac{\mathrm{d}}{2})$

⇒ h = 2r

Ratio of their volumes = vol. of sphere : vol. of cylinder : vol. of cone

$$= \frac{4}{3}\pi r^3 : \pi r^2 h : \frac{1}{3}\pi r^2 h$$

$$= \frac{4}{3}\pi r^3 : \pi r^2 (2r) : \frac{1}{3}\pi r^2 (2r)$$

$$= \frac{4}{3}\pi r^3 : 2\,\pi r^3 : \frac{2}{3}\pi r^3$$

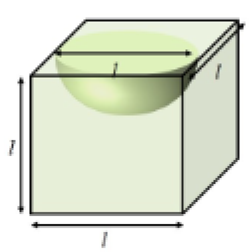
$$= \frac{4}{3} : 2 : \frac{2}{3}$$

Multiplying the whole by 3, we get-

⇒ Ratio of their volumes = 4:6:2

Q. 7. A hemisphere is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the length of the cube. Determine the surface area of the remaining solid.

Answer :



Give that, A hemisphere is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the length of the cube.

Now, let the side of cube be ‘l’

⇒ radius of hemisphere(r) $= \frac{l}{2}$ $(\because r = \frac{d}{2})$

⇒ surface area of remaining solid = surface area of cube- surface area of hemisphere

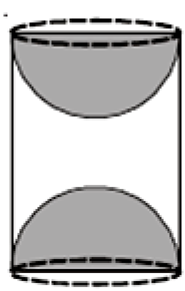
$= 6l^2 - 2\pi r^2$

$$= 6l^2 - 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 - \pi\frac{l^2}{2}$$

$= l^2(6 - \frac{\pi}{2})$ sq units

Q. 8. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm. and its radius of the base is of 3.5 cm, find the total surface area of the article.



Answer : Given that, radius(r) of base is 3.5 cm and height(h) of cylinder is 10 cm.

⇒ the total surface area of the article = surface area of cylinder +

2(Surface area of hemisphere)

$= 2\pi rh + 2(2\pi r^2)$

$= 2\pi r(h + 2r)$

$$= 2 \times \frac{22}{7} \times 3.5(10 + 2 \times 3.5)$$

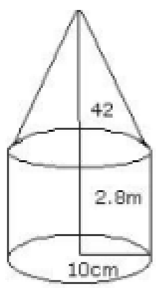
$$= 2 \times \frac{22}{7} \times 3.5 \times 17$$

= 374 cm^2

Exercise 10.3

Q. 1. An iron pillar consists of a cylindrical portion of 2.8 m. height and 20 cm. in diameter and a cone of 42 cm. height surmounting it. Find the weight of the pillar if 1cm^3 of iron weighs 7.5 g.

Answer :



Given that, height of cylinder(H) = 2.8m = 280 cm, diameter(d) is 20 cm and height of cone(h) is 42 cm.

$$\Rightarrow \text{radius}(r) = \frac{20}{2} = 10 \text{ cm} \left(\because r = \frac{d}{2}\right)$$

⇒ vol. of pole = vol. of cylinder + vol. of cone

$$= \pi r^2 H + \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 (H + \frac{1}{3}h)$$

$$= \frac{22}{7} \times 10^2 \left(280 + \frac{1}{3} \times 42\right)$$

$$= \frac{22}{7} \times 100 \times 294$$

$= 92400\ cm^3$

If $1\ cm^3$ of iron weighs 7.5 g.

⇒ the weight of the pillar = 92400 × 7.5

= 693,000 g

= 693 kg (∵ 1kg = 1000g)

Q. 2. A toy is made in the form of hemisphere surmounted by a right cone whose circular base is joined with the plane surface of the hemisphere. The radius of the base of the cone is 7 cm. and its volume is $\frac{3}{2}$ of the hemisphere. Calculate the height of the cone and the surface area of the toy correct to 2 places of decimal

$\left(\text{Take } \pi = 3\frac{1}{7}\right).$

Answer : Given that, The radius of the base(r) of the cone is 7 cm and its volume is $\frac{3}{2}$ of the hemisphere.

∵ circular base of cone is joined with the plane surface of the hemisphere,

⇒ radius of hemisphere = radius of base of cone = 7 cm

Also,vol. of cone $=\frac{3}{2} \times$ vol. of hemisphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{3}{2} \times \frac{2}{3}\pi r^3$$

⇒ h = 3r

⇒h = 3 × 7

⇒h = 21 cm

Also, slant height(l) = $\sqrt{r^2 + h^2}$

⇒ l = $\sqrt{7^2 + 21^2}$

⇒ l = $\sqrt{490}$

⇒ l = 22.13

⇒ Slant Height of cone (l) = 22.13 cm

Now, surface area of toy = surface area of cone + surface area of hemisphere

⇒ surface area of toy = $\pi rl + 2\pi r^2$

= $\pi r(l + 2r)$

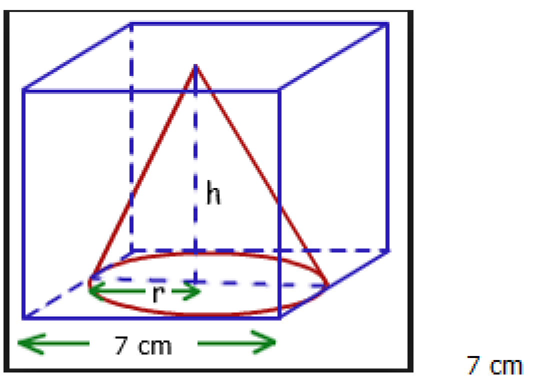
$$= \frac{22}{7} \times 7(22.13 + 2 \times 7)$$

$$= \frac{22}{7} \times 7 \times 36.13$$

= 794.86 cm^2

Q. 3. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm.

Answer : The figure is shown below:



Give that, edge of cube is 7 cm

⇒ largest possible height of cone(h) is 7 cm and diameter of base(d) is 7 cm

⇒ radius(r) = $\frac{7}{2}$ ($\because$ $r = \frac{d}{2}$)

⇒ the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7$$

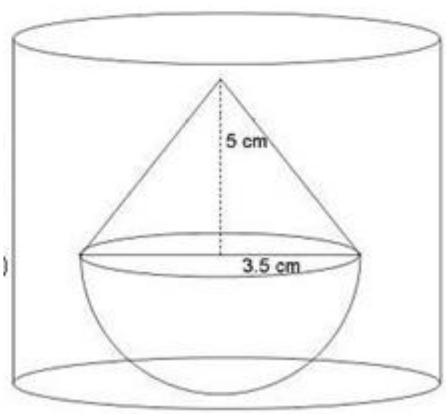
= 89.83 cm^2

Q. 4. A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of right circular cone mounted on a hemisphere is immersed into the tub. The radius of the hemisphere is 3.5 cm and height of cone outside the hemisphere is 5 cm.

Find the volume of water left in the tub

$$\left(\text{Take } \pi = 3\frac{1}{7}\right).$$

Answer :



Given that, radius of cylinder(r) is 5 cm and height(h) is 9.8 cm. Also, radius of hemisphere(R) is 3.5 cm and height(H) of cone outside the hemisphere is 5 cm.

⇒ vol. of solid = vol. of hemisphere + vol. of cone

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5^2(2 \times 3.5 + 5)$$

= 154 cm^3

⇒ vol. of tub = $\pi r^2 h$

$$= \frac{22}{7} \times 5^2 \times 9.8$$

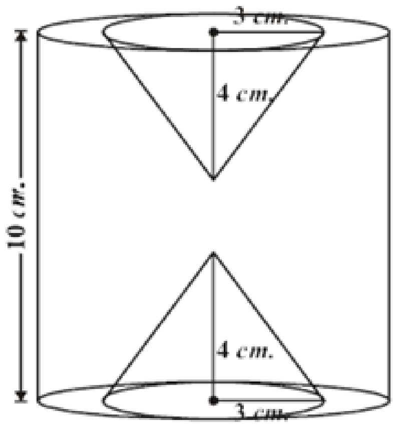
= 770 cm^3

⇒ the volume of water left in the tub = vol.of tub-vol. of solid

= 770-154

= 616 cm^3

Q. 5. In the adjacent figure, the height of a solid cylinder is 10 cm and diameter is 7 cm. Two equal conical holes of radius 3 cm and height 4 cm are cut off as shown the figure. Find the volume of the remaining solid.



Answer : Given that, height of cylinder is 10 cm and diameter is 7 cm.Also, radius of cone is 3 cm and height is 4 cm.

⇒ radius of cylinder $= \frac{7}{2}$ $(\because r = \frac{d}{2})$

⇒ the volume of the remaining solid = vol. of cylinder-2(vol. Of cone

$$= \pi r^2 h - 2\left(\frac{1}{3}\pi r^2 h\right)$$

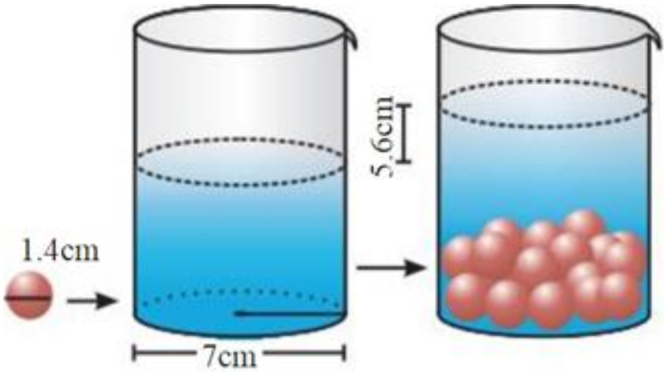
$$= \frac{22}{7}(\frac{7}{2} \times \frac{7}{2} \times 10 - \frac{2}{3} \times 3 \times 3 \times 4)$$

$$= \frac{22}{7}\left(\frac{245}{2} - 24\right)$$

= 309.57 cm^3

Q. 6. Spherical Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm., which contains some water. Find the number of marbles that should be dropped in to the beaker, so that water level rises by 5.6 cm.

Answer :



Given that, diameter(d) of sphere is 1.4 cm and diameter(D) of cylindrical beaker is 7 cm and height(h) required is 5.6 cm.

$\Rightarrow$ vol. of one spherical marble $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{1.4}{2} \times \frac{1.4}{2} \left(\because r = \frac{d}{2}\right)$

$= 1.437\ cm^3$

$\Rightarrow$ vol.of water reqired(cylindrical) $= \pi R^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 5.6 \left(\because R = \frac{D}{2}\right)$

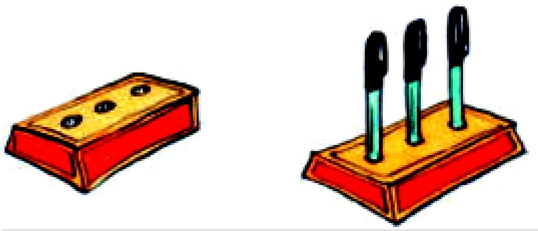
$= 215.7\ cm^3$

$\therefore$ the number of marbles that should be dropped in to the beaker, so that water level rises by 5.6 cm $= \frac{\text{vol.of water}}{\text{vol.of marble}}$

$$= \frac{215.7}{1.437}$$

= 150

Q. 7. A pen stand is made of wood in the shape of cuboid with three conical depressions to hold the pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depression is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



Answer : Given that, the dimensions of the cuboid are l = 15 cm by b = 10 cm by h = 3.5 cm and the radius(r) of each of the depression is 0.5 cm and the depth(h) is 1.4 cm.

⇒volume of wood in entire stand = vol.of cuboid-3(vol. of cones)

$$= \text{lbh} - 3 \times \frac{1}{3}\pi r^2 h$$

$$= 15 \times 10 \times 3.5 - \frac{22}{7} \times .5 \times .5 \times 1.4$$

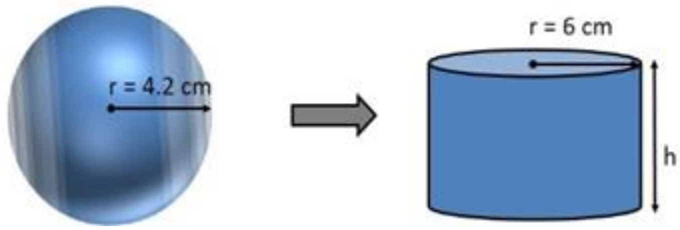
= 525-1.1

= 523.9 cm^3

Exercise 10.4

Q. 1. A metallic sphere of radius 4.2 cm. is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer :



Given that, radius of sphere(r) is 4.2 cm and radius of cylinder(R) is 6 cm.

Let the height of cylinder be h.

∵ the sphere is melted and recast into cylinder

⇒ vol. of sphere = vol. of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h$$

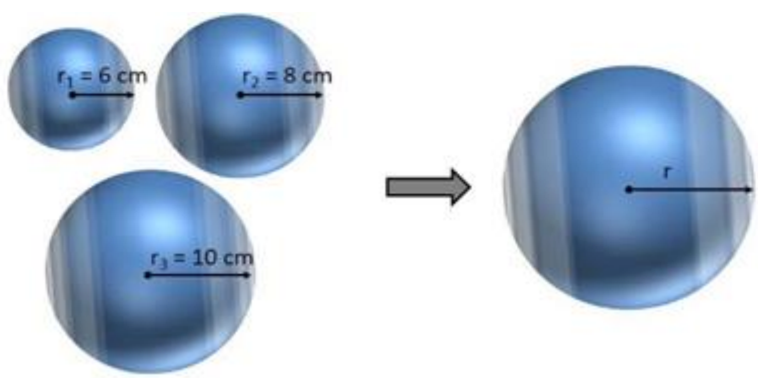
$$\Rightarrow h = \frac{4}{3} \times \frac{r^3}{R^2}$$

$$\Rightarrow h = \frac{4}{3} \times \frac{4.2^3}{6^2}$$

⇒ h = 2.74 cm

Q. 2. Three metallic spheres of radii 6 cm., 8 cm. and 10 cm. respectively are melted together to form a single solid sphere. Find the radius of the resulting sphere.

Answer :



Given that, radii of spheres melted are 6 cm, 8 cm and 10 cm.

Let the radius of solid sphere be R

Also, volume of sphere $= \frac{4}{3}\pi r^3$

∵ these are melted to make a single sphere

⇒ vol. of solid sphere = sum of vol. of all melted sphere

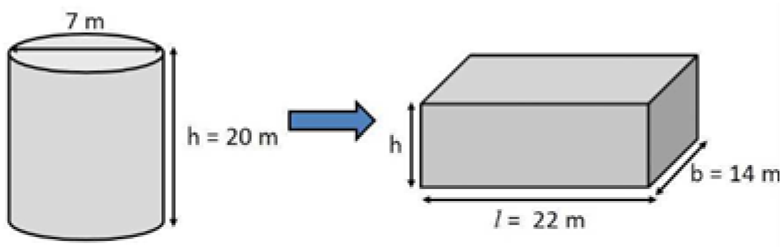
$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times R^3 = \frac{4}{3} \times \frac{22}{7}(6^3 + 8^3 + 10^3)$$

⇒ $R^3 = 1728$

⇒ R = 12 cm

Q. 3. A 20 m deep well of diameter 7 m. is dug and the earth got by digging is evenly spread out to form a rectangular platform of base 22m × 14m. Find the height of the platform.

Answer :



Given that, height of cylindrical well(H) is 20 m and diameter(d) is 7 m. And length(l) and breadth(b) of rectangular platform are 22m and 14 m respectively.

Now, let the height of platform be h.

$$\Rightarrow \text{radius of cylinder(r)} = \frac{7}{2} \left(\because r = \frac{d}{2}\right)$$

⇒ vol. Of well = vol. of platform

⇒ $\pi r^2 H = lbh$

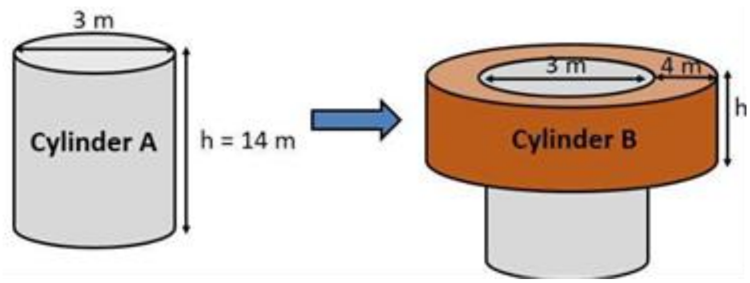
$$\Rightarrow h = \frac{\pi r^2 H}{lb}$$

$$\Rightarrow h = \frac{\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20}{22 \times 14}$$

⇒ h = 2.5 cm

Q. 4. A well of diameter 14 m. is dug 15 m. deep. The earth taken out of it has been spread evenly to form circular embankment of width 7 m. Find the height of the embankment.

Answer :



Given that, height of cylindrical well(h) is 15 m and diameter(d) is 14 m. And width of circular embankment is 7 m.

$\Rightarrow$ radius(r) of well $= \frac{14}{2} = 7\text{ m} \left(\because r = \frac{d}{2}\right)$

$\Rightarrow$ outer radius of Embankment(R1) = (7 + 7) = 14 m

$\Rightarrow$ inner radius(r1) = 7 m

Let the height of the embankment be H.

$\Rightarrow$ vol. of well = vol. of embankment

$\Rightarrow \pi r^2 h = \pi(R1^2 - r1^2)H$

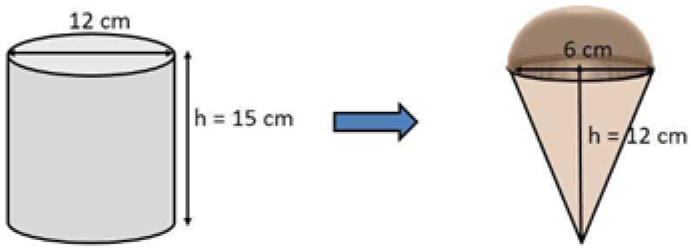
$$\Rightarrow H = \frac{r^2 h}{R1^2 - r1^2}$$

$$\Rightarrow H = \frac{7 \times 7 \times 15}{14 \times 14 - 7 \times 7}$$

$\Rightarrow$ H = 5 cm

Q. 5. A container shaped like a right circular cylinder having diameter 12 cm. and height 15 cm. is full of ice cream. The ice-cream is to be filled into cones of height 12 cm. and diameter 6 cm., having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer :



Given that, diameter(d) of cylindrical container is 12 cm and height(h) is 15 cm. and, diameter(D) of cone is 6cm and height(H) is 12 cm

We know that, $\text{radius} = \frac{\text{diameter}}{2}$

$\Rightarrow$ radius of cylinder(r) $= \frac{12}{2}$

And, radius of cone(R) $= \frac{6}{2}$

$\Rightarrow$ volume of container = $\pi r^2 h$

$$= \pi \times \left(\frac{12}{2}\right)^2 \times 15$$

= 540π

$\Rightarrow$ vol. of 1 such cone = vol.of cone + vol. of hemisphere

$$= \frac{1}{3}\pi R^2 H + \frac{2}{3}\pi R^3$$

$$= \frac{1}{3} \times \pi \times \left(\frac{6}{2}\right)^2 \times 12 + \frac{2}{3} \times \pi \times \left(\frac{6}{2}\right)^2$$

= 54π

$\Rightarrow$ the number of such cones which can be filled with ice cream

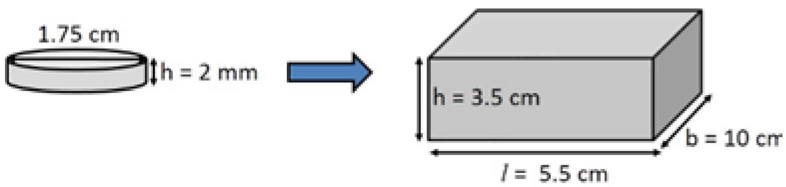
$$= \frac{\text{vol. of container}}{\text{vol. of 1 cone}}$$

$$= \frac{540\pi}{54\pi}$$

= 10

Q. 6. How many silver coins, 1.75 cm. in diameter and thickness 2 mm, need to be melted to form a cuboid of dimensions 5.5cm × 10cm × 3.5cm.?

Answer :



Given, that dimensions of cuboid formed is 5.5 × 10 × 3.5 and the diameter(d) of cylindrical coin is 1.75 cm and height(h) is 2mm = 0.2cm.

⇒ l = 5.5, b = 10 and h = 3.5

⇒ radius(r) of coin $= \frac{1.75}{2} = 0.875 \text{ cm} \left(\because r = \frac{d}{2}\right)$

⇒ vol. of silver coin = $\pi r^2 h$

= 3.14 × 0.878 × 0.875 × 0.2

= 0.48

⇒ vol. of cuboid = lbh

= 5.5 × 10 × 3.5

= 192.5

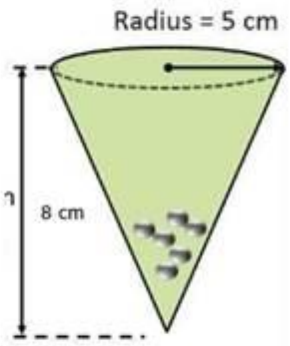
⇒ ∴ no. of silver coins $= \frac{\text{vol.of silver coin}}{\text{vol.of cuboid}}$

$$= \frac{192.5}{0.48}$$

= 400

Q. 7. A vessel is in the form of an inverted cone. Its height is 8 cm. and the radius of its top is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, $\frac{1}{4}$ of the water flows out. Find the number of lead shots dropped into the vessel.

Answer :



Given that, radius(r) of cone is 5 cm and height(h) is 8 cm.And, radius(R) of spheres is 0.5 cm.

Also, When lead shots are dropped into the vessel, $\frac{1}{4}$of the water flows out.

$\Rightarrow$ vol. of water in cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 8$$

$$= \frac{200\pi}{3}$$

$\Rightarrow$ vol. of 1 spherical lead shot $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi \times 0.5^3$$

$$= \frac{0.5\pi}{3}$$

$\Rightarrow \frac{1}{4}$ (vol. of water in cone) = n × vol. of 1 lead shot

(where, n is no. of lead shots)

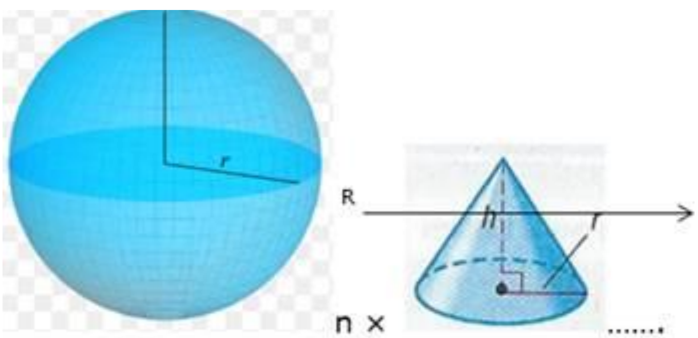
$$\Rightarrow n = \frac{\text{vol. of water in cone}}{4 \times \text{vol. of 1 spherical lead shot}}$$

$$\Rightarrow n = \frac{220\pi}{4 \times 0.5\pi}$$

⇒ n = 100

Q. 8. A solid metallic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

Answer :



Given that, diameter(d) of sphere is 28 cm and diameter(D) of cone is $\frac{14}{3}$ cm and height(h) is 3 cm.

⇒ radius(r) of sphere $= \frac{28}{2} = 14\ \text{cm}$

And, radius(R) of cone $= \frac{\frac{14}{3}}{2} = \frac{7}{3}\text{cm}\ (\because\ r = \frac{d}{2})$

⇒vol. of spheres $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times (14)^3$$

⇒ vol. of cone $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3}\pi \times \left(\frac{7}{3}\right)^2 \times 3$$

⇒ vol. of sphere = n × vol. of 1 cone

(where, n is no. of cones)

$$\Rightarrow n = \frac{\text{vol. of sphere}}{\text{vol. of 1 cone}}$$

$$\Rightarrow n = \frac{\frac{4}{3}\pi \times (14)^3}{\frac{1}{3}\pi \times \left(\frac{7}{3}\right)^2 \times 3}$$

⇒ n = 672