JEE Advanced

Single Correct Answer Type

1. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is

   a. $f/2$ b. f

   c. $2f$ d. $4f$ **(IIT-JEE 1987)**

2. Two bodies M and N, of equal masses, are suspended from two separate massless springs of spring constants k_1 and k_2, respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is

   a. $\frac{k_1}{k_2}$ b. $\sqrt{\frac{k_1}{k_2}}$

   c. $\frac{k_2}{k_1}$ d. $\sqrt{\frac{k_2}{k_1}}$ **(IIT-JEE 1988)**

3. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with

small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

a. $\frac{1}{2\pi}\left(\frac{k-A\rho g}{M}\right)^{1/2}$ b. $\frac{1}{2\pi}\left(\frac{k+A\rho g}{M}\right)^{1/2}$

c. $\frac{1}{2\pi}\left(\frac{k+\rho-gL}{M}\right)^{1/2}$ d. $\frac{1}{2\pi}\left(\frac{k+A-\rho g}{A\rho g}\right)^{1/2}$

(IIT-JEE 1990)

4. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force is applied perpendicular to the side faces of A. After the force is withdrawn, block A executes small oscillations the time period of which is given by

a. $2\pi\sqrt{M\eta L}$ b. $2\pi\sqrt{\frac{M-\eta}{L}}$

c. $2\pi\sqrt{\frac{M-L}{\eta}}$ d. $2\pi\sqrt{\frac{M-N}{\eta L}}$

(IIT-JEE 1992)

5. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

This expression may be considered to be a result of the superposition of

a. two   b. three
c. four   d. five   **(IIT-JEE 1992)**

6. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modules of the wire are A and Y, respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to:

a. $2\pi\sqrt{\frac{m}{K}}$ b. $2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$

c. $2\pi\sqrt{\frac{mYA}{KL}}$ d. $2\pi\sqrt{\frac{mL}{YA}}$

(IIT-JEE 1993)

7. A particle of mass m is executing oscillations about the origin on the axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

a. proportional to $1/\sqrt{a}$
b. independent of a
c. proportional to $\sqrt{a}$
d. proportional to $a^{3/2}$ **(IIT-JEE 1998)**

8. A particle free to move along the x-axis has potential energy given by $U(x) = k\,[1-\exp(-x^2)]$ for $-\infty \le x \le -\infty$ where k is a positive constant of appropriate dimensions. Then

a. at points away from the origin, the particle is in unstable equilibrium
b. for any finite non-zero value of x, there is a force directed away from the origin
c. if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
d. for small displacements from $x = 0$, the motion is simple harmonic   **(IIT-JEE 1999)**

9. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by

a. $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ b. $2\pi\sqrt{\frac{L}{g\sin\alpha}}$

c. $2\pi\sqrt{\frac{L}{g}}$ d. $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

(IIT-JEE 2000)

10. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

a. $T_1 < T_2$ b. $T_1 > T_2$
c. $T_1 = T_2$ d. $T_1 = 2T_2$

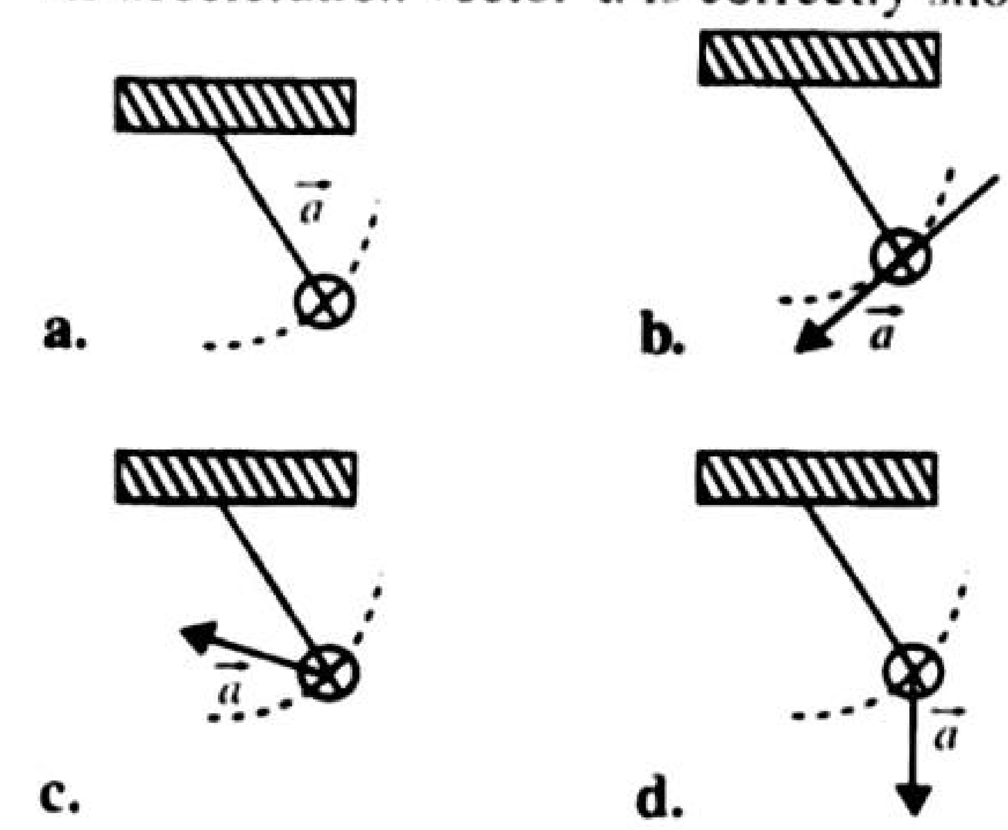
(IIT-JEE 2001)

11. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth suface, where R is the radius of the earth. The value of T_2/T_1 is

a. 1   b. $\sqrt{2}$ c. 4   d. 2

(IIT-JEE 2001)

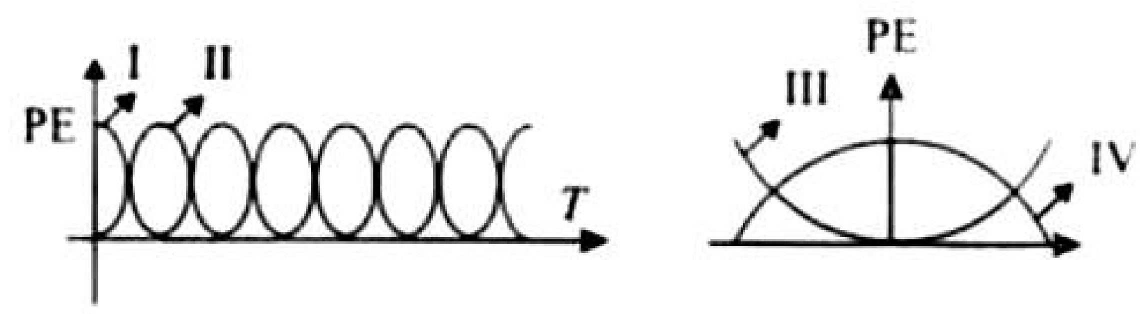
12. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in:



(IIT-JEE 2002)

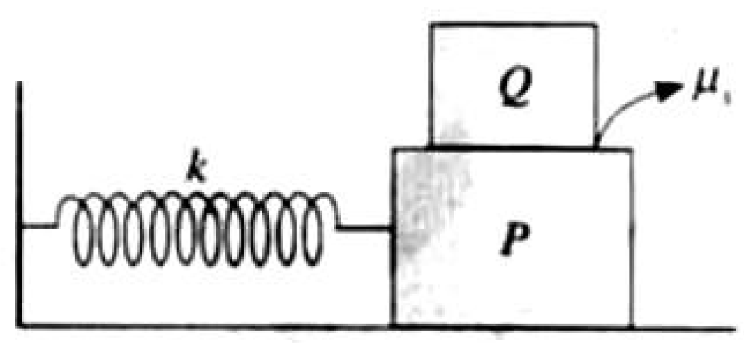
13. For a particle executing SHM, the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the

variation of potential (PE) as a function of time t and displacement x.



a. I, III b. II, IV
c. II, III d. I, IV **(IIT-JEE 2003)**

14. A block P of mass m is placed on a smooth horizontal surface. A block Q of the same mass is placed over the block P and the coefficient of static friction between them is μ_s. A spring of spring constant K is attached to block Q. The blocks are displaced together to a distance A and released. The upper block oscillates without slipping over the lower block. The maximum frictional force between the block is:



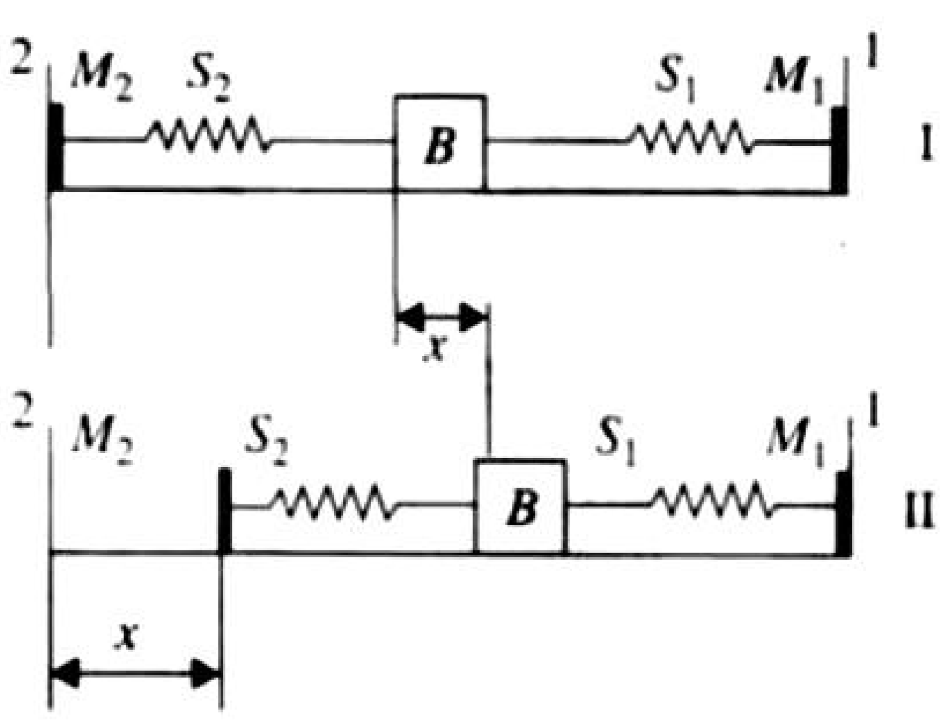
a. zero b. kA c. $\frac{1}{2}kA$ d. μg

(IIT-JEE 2004)

15. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$ ($K = 1$ m/s^2), where y is the vertical displacement. The time period now becomes T_2. The ratio of T_1^2/T_2^2 is ($g = 10$ m/s^2)

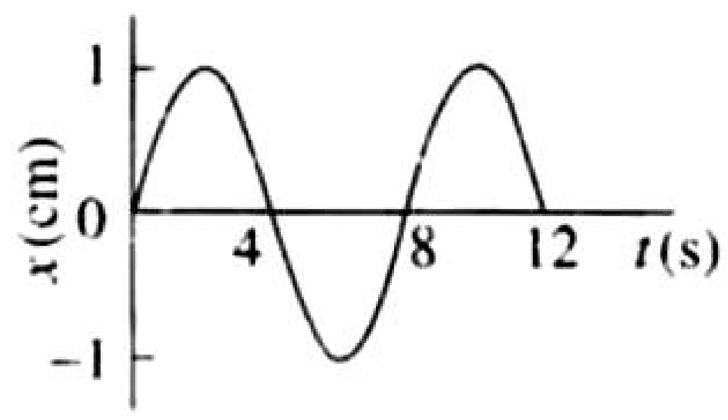
a. 5/6 b. 6/5
c. 1 d. 4/5 **(IIT-JEE 2005)**

16. A block B is attached to two unstretched springs S_1 and S_2 with spring constant k and $4k$, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. Block B is displaced towards wall 1 by a small distance x [figure (II)] and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of block B. The ratio y/x is



a. 4 b. 2
c. 1/2 d. 1/4 **(IIT-JEE 2008)**

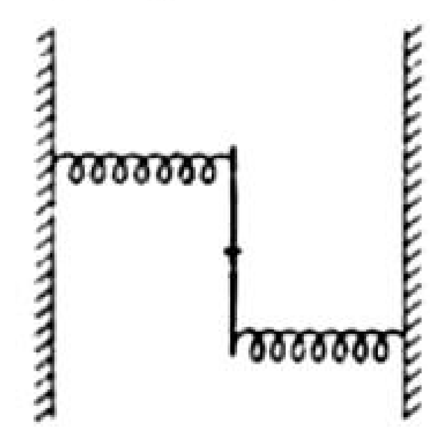
17. The x–t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is



a. $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 b. $\frac{-\pi^2}{32}$ cm/s^2
c. $\frac{\pi^2}{32}$ cm/s^2 d. $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

(IIT-JEE 2009)

18. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



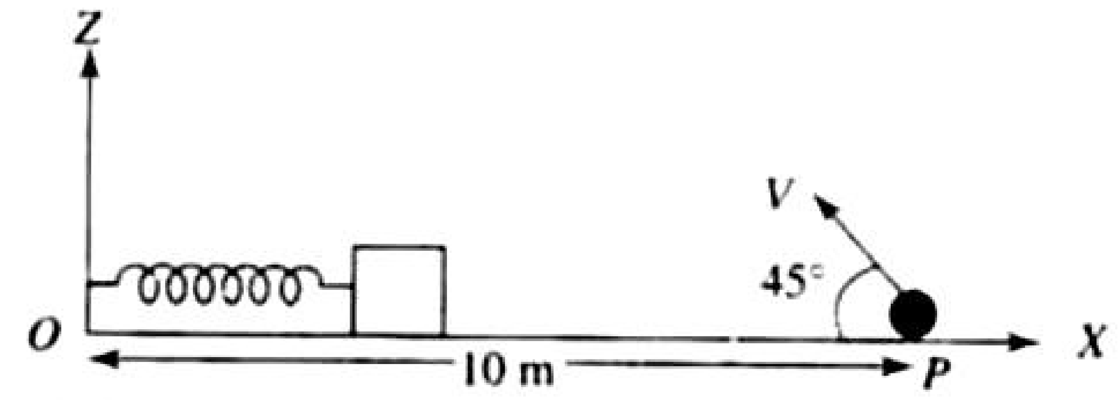
a. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ b. $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$
c. $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ d. $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

(IIT-JEE 2009)

19. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A\sin(\omega t + 2\pi/3)$. Adding a third sinusoidal displacement $x_3(t) = B \sin (\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

a. $\sqrt{2}A, \frac{3\pi}{4}$ b. $A, \frac{4\pi}{3}$
c. $\sqrt{3}A, \frac{5\pi}{6}$ d. $A, \frac{\pi}{3}$ **(IIT-JEE 2011)**

20. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2)



a. $\sqrt{50}$ m/s b. $\sqrt{51}$ m/s

c. $\sqrt{52}$ m/s d. $\sqrt{53}$ m/s **(IIT-JEE 2012)**

Multiple Correct Answer Type

1. The linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its

a. maximum potential energy is 100 J
b. maximum kinetic energy of 100 J
c. maximum potential energy is 160 J
d. minimum potential energy of zero

(IIT-JEE 1989)

2. As a wave propagates,

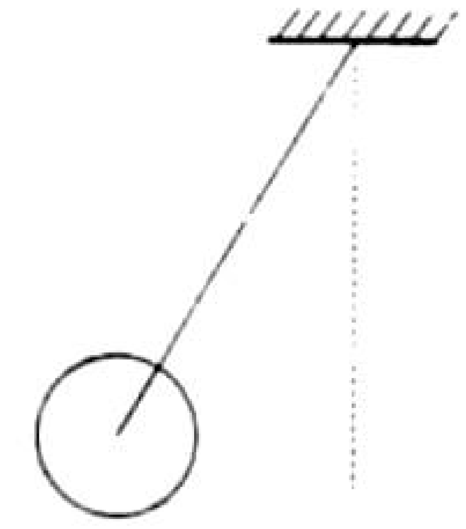
a. the wave intensity remains constant for a plane wave.
b. the wave intensity decreases as the inverse of the distance from the source for a spherical wave.
c. the wave intensity decreases as the inverse square of the distance from the source for a spherical wave.
d. total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times. **(IIT-JEE 1999)**

3. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then

a. the resultant amplitude is $(1+\sqrt{2})a$
b. the phase of the resultant motion relative to the first is 90°
c. the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated any single motion
d. the resulting motion is not simple harmonic

(IIT-JEE 1999)

4. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' ($<L$) is attached at its centre to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre. The rod–disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



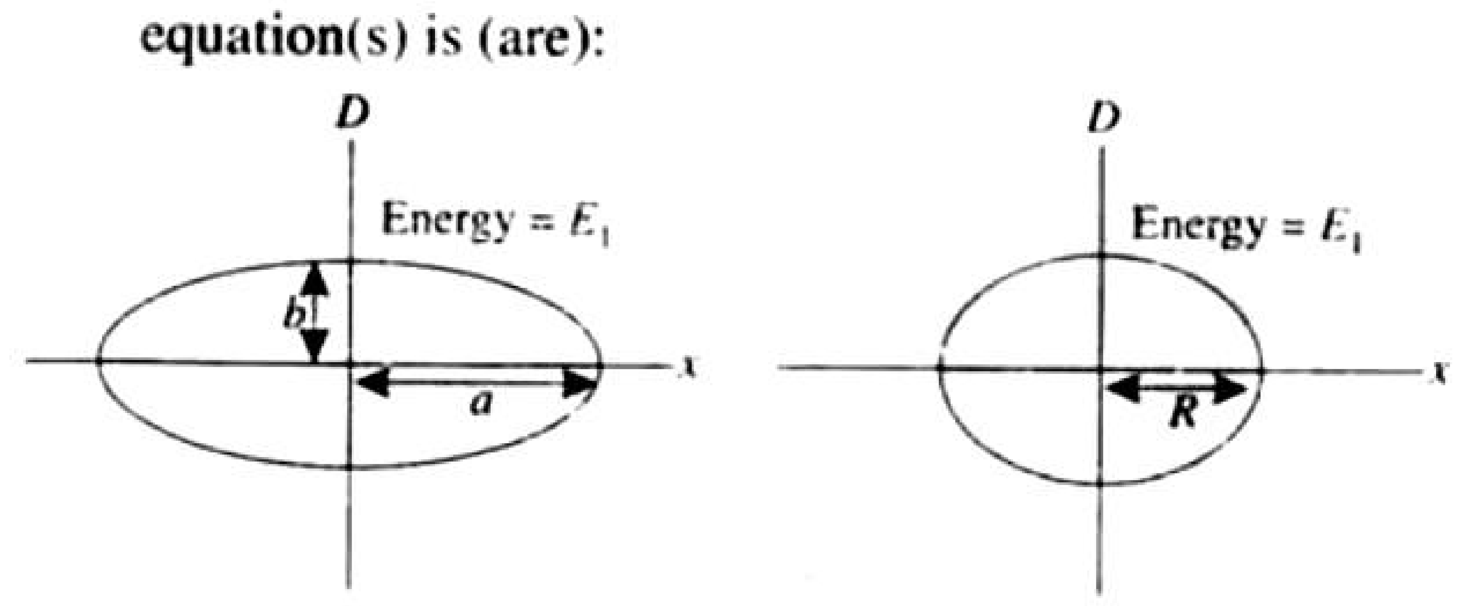
a. Restoring torque in case A = Restoring torque in case B
b. Restoring torque in case A < Restoring torque in case B
c. Angular frequency for case A > Angular frequency for case B
d. Angular frequency for case A < Angular frequency for case B **(IIT-JEE 2011)**

5. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\,u_0$. It collides elastically with a rigid wall. After this collision,

a. the speed in of the particle when it returns to its equilibrium position is u_0.
b. the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\dfrac{m}{k}}$
c. the time at which the maximum compression of the spring occurs is $t = \dfrac{4\pi}{3}\sqrt{\dfrac{m}{k}}$
d. the time at which the particle passes through the equilibrium position for the second time is $t = \dfrac{5\pi}{3}\sqrt{\dfrac{m}{k}}$

(JEE Advanced 2013)

6. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in figures. If $\dfrac{a}{b} = n^2$ and $\dfrac{a}{R} = n$, then the correct equation(s) is (are):



a. $E_1\omega_1 = E_2\omega_2$ **b.** $\dfrac{\omega_2}{\omega_1} = n^2$

c. $\omega_1\omega_2 = n^2$ **d.** $\dfrac{E_1}{\omega_1} = \dfrac{E_2}{\omega_2}$

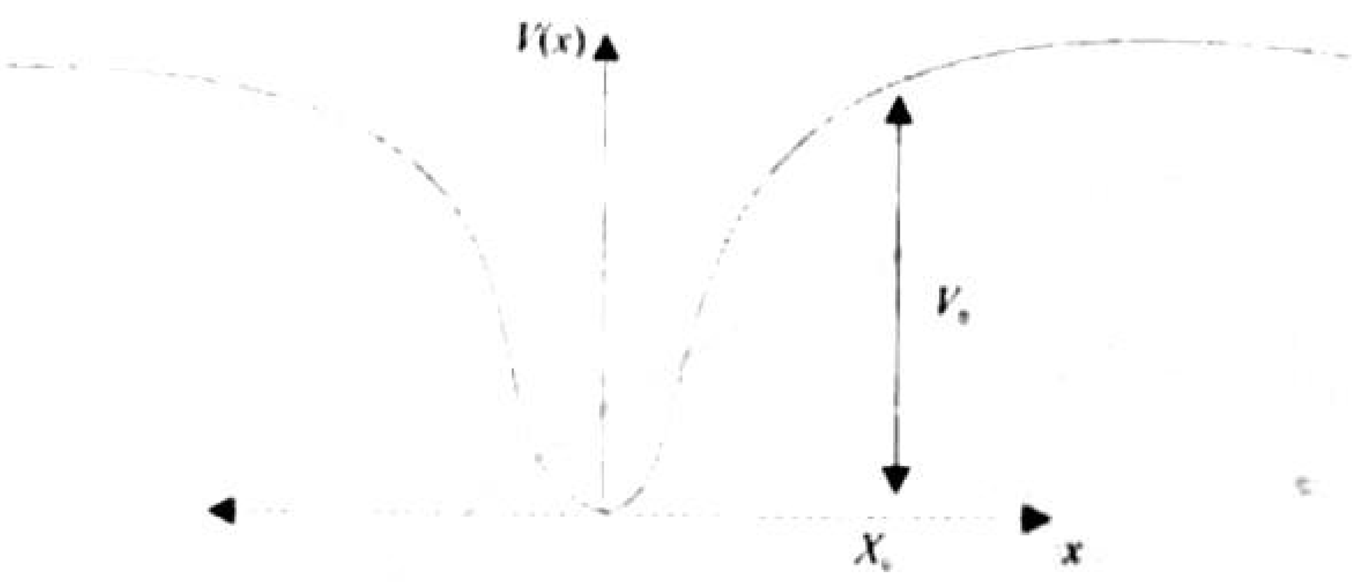
(JEE Advanced 2015)

Linked Comprehension Type

For Problems 1–3

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to m, k as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).

(IIT-JEE 2010)



1. If the total energy of the particle is E, it will perform periodic motion only if

a. $E < 0$ b. $E > 0$

c. $V_0 > E > 0$ d. $E > V_0$

2. For periodic motion of small amplitude A, the time period T of this particle is proportional to

a. $A\sqrt{\dfrac{m}{\alpha}}$ b. $\dfrac{1}{A}\sqrt{\dfrac{m}{\alpha}}$

c. $A\sqrt{\dfrac{\alpha}{m}}$ d. $A\sqrt{\dfrac{2\alpha}{m}}$

3. The acceleration of this particle for $|x| > X_0$ is

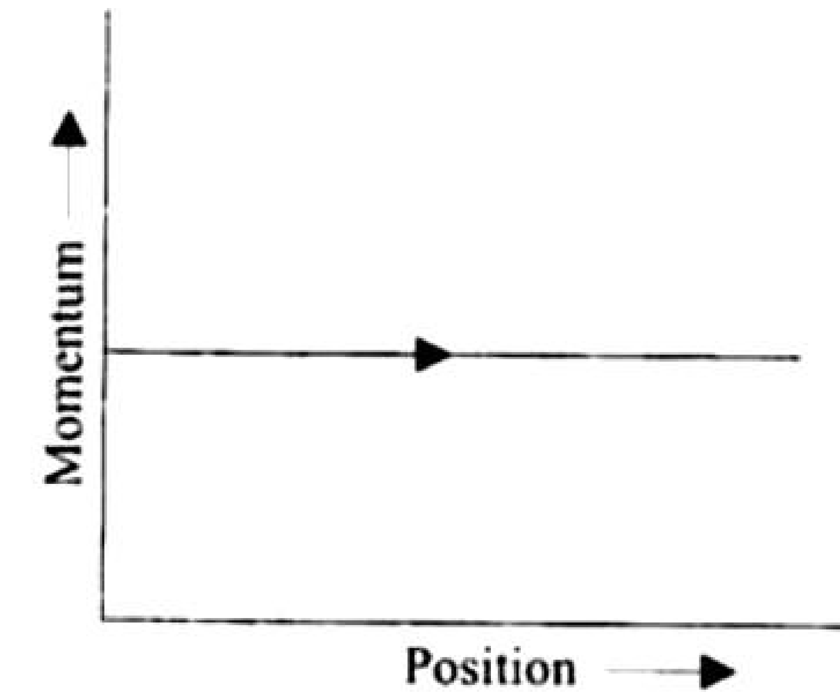
a. proportional to V_0 b. proportional to $\dfrac{V_0}{mX_0}$

c. proportional to $\sqrt{\dfrac{V_0}{mX_0}}$ d. zero

For Problems 4–6

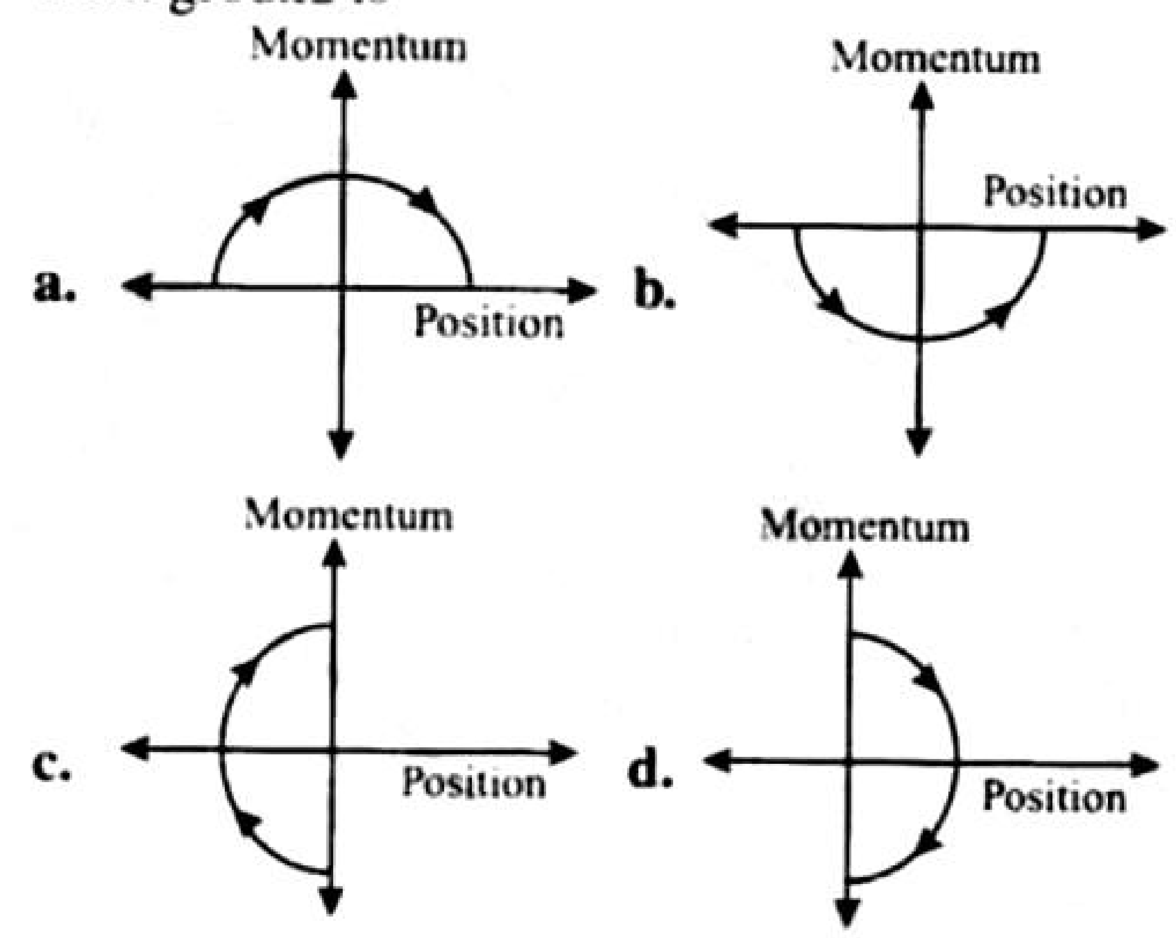
Phase space diagrams are useful tools in analysing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical the axis.

The phase space diagram is $x(t)$ vsersus $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

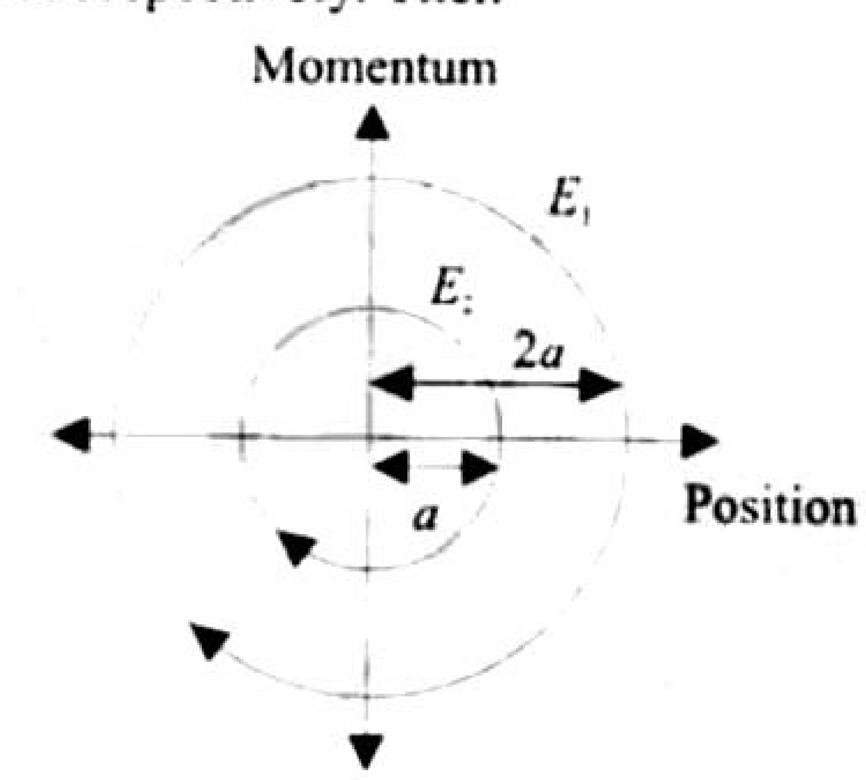


(IIT-JEE 2011)

4 The phase space diagram for a ball thrown vertically up from ground is



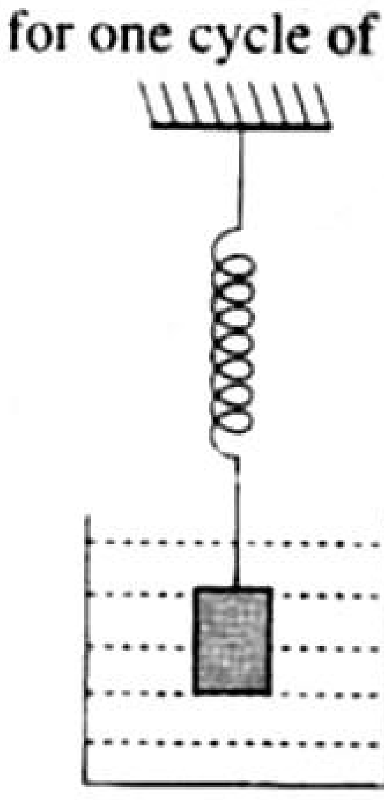
5. The phase space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

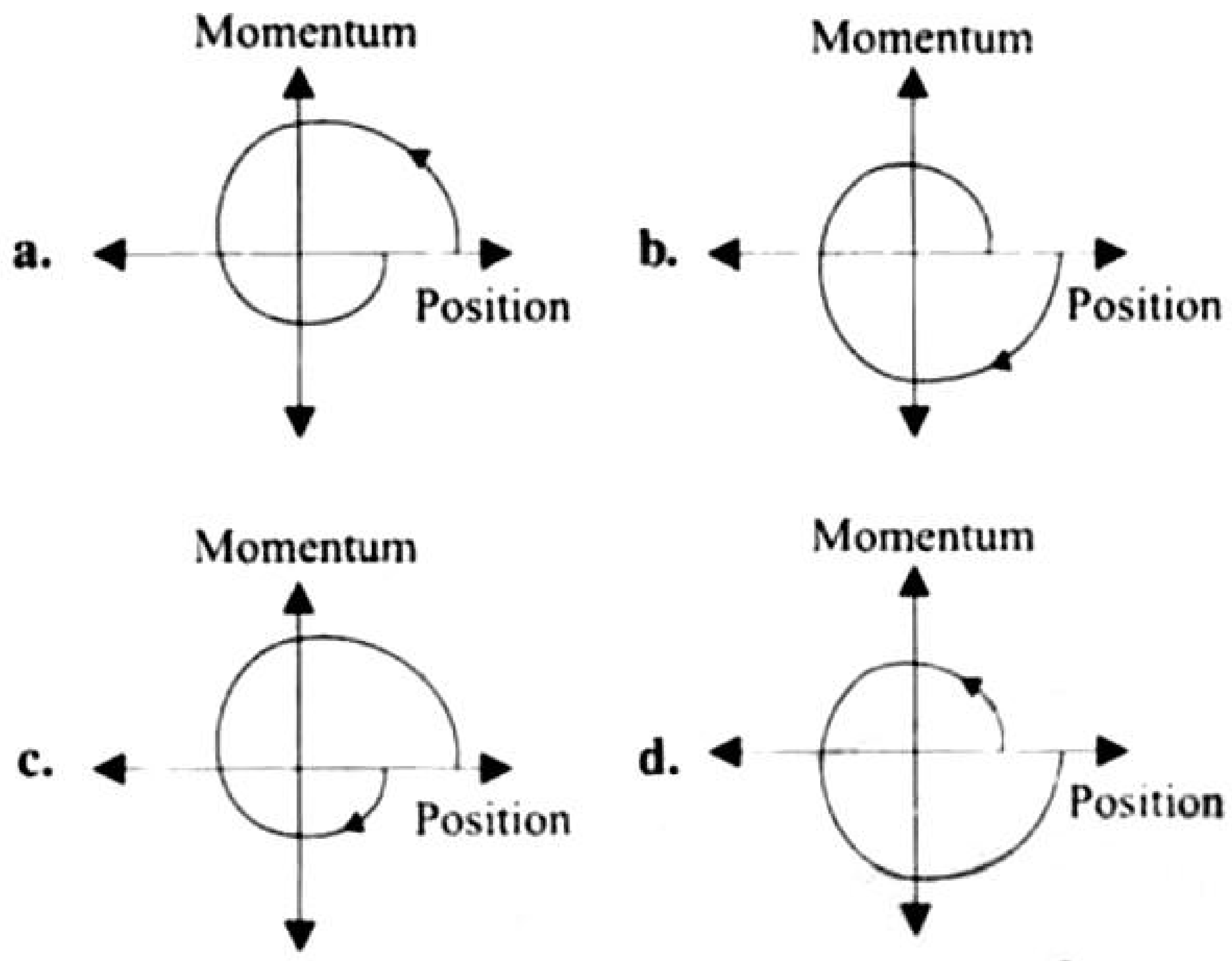


a. $E_1 = \sqrt{2}E_2$ b. $E_1 = 2E_2$

c. $E_1 = 4E_2$ d. $E_1 = 16E_2$

6. Consider the spring–mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is:





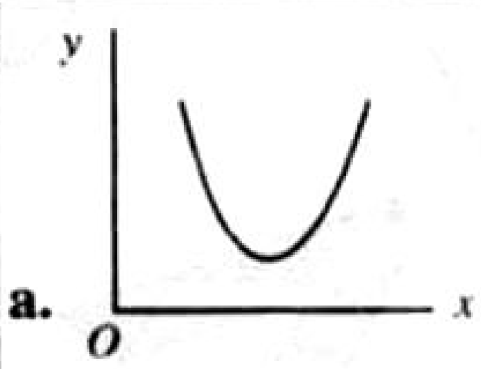
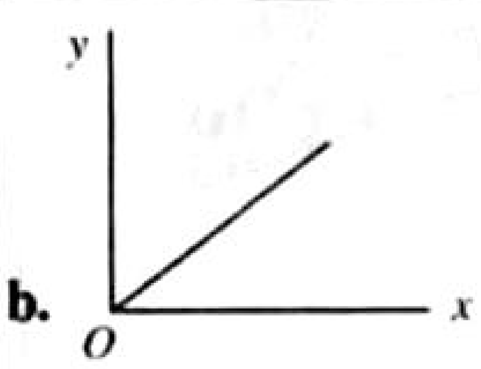
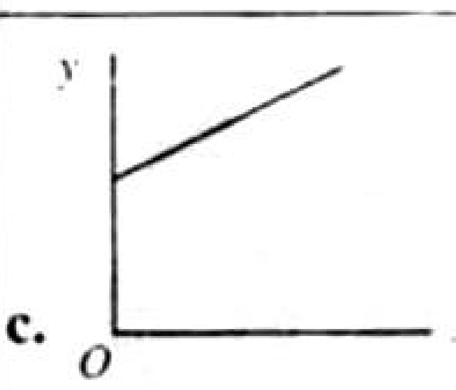
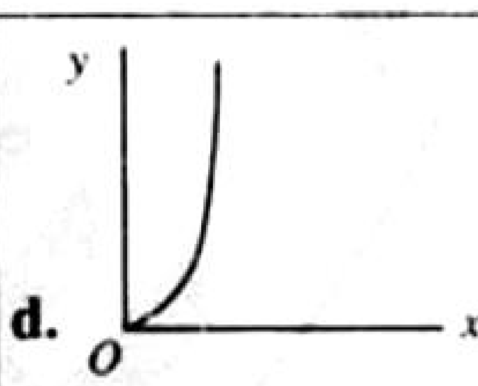
Matching Column Type

1. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match situations in Column I with the characteristics in Column II.

Column I	Column II
i. The object moves on the x-axis under a conservative force in such a way that its 'speed' and 'position' satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	**a.** The object executes a simple harmonic motion.
ii. The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	**b.** The object does not change its direction.
iii. The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	**c.** The kinetic energy of the object keeps on decreasing.
iv. The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	**d.** The object can change its direction only once.

(IIT-JEE 2007)

2. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graph given in Column II.

Column I	Column II
i. Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis).	**a.** 
ii. Displacement (y-axis) as a function of time (x-axis) for a one-dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	**b.** 
iii. Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle.	**c.** 
iv. The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis).	**d.** 

(IIT-JEE 2008)

Integer Answer Type

1. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs

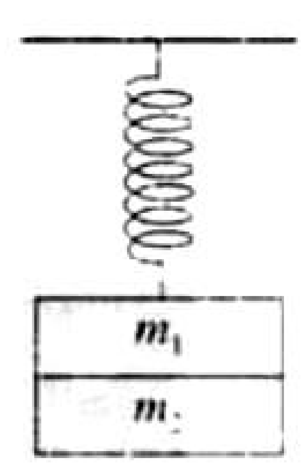
simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is **(IIT-JEE 2010)**

Fill in the Blanks Type

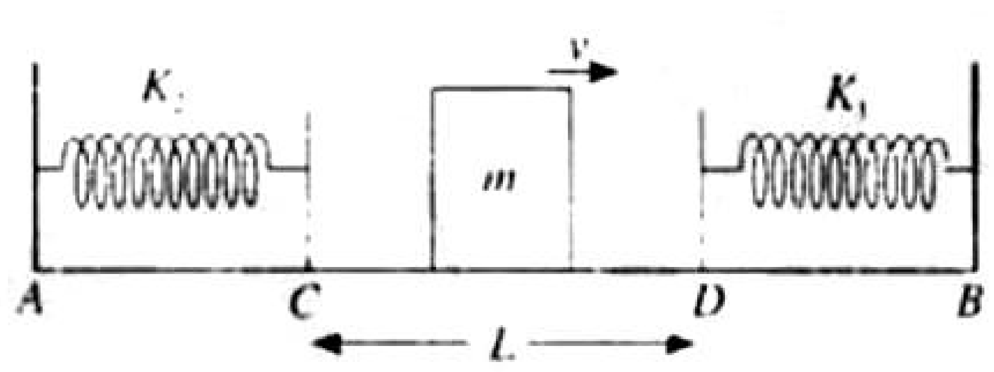
1. Two simple harmonic motions are represented by the equations $y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = (5\sin 3\pi t + \sqrt{3}\cos 3\pi t)$. Their amplitudes are in the ratio of ____. **(IIT-JEE 1986)**
2. An object of mass 0.2 kg executes simple harmonic oscillation along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$ m, the object has kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ____ m. **(IIT-JEE 1994)**

Subjective Type

1. A mass M attached to a spring oscillates with a period of 1 s. If the mass is increased by 3 kg, the period increases by 1 s. Find the value of M (in kg) assuming that Hooke's law is obeyed. **(IIT-JEE 1979)**
2. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2. **(IIT-JEE 1981)**

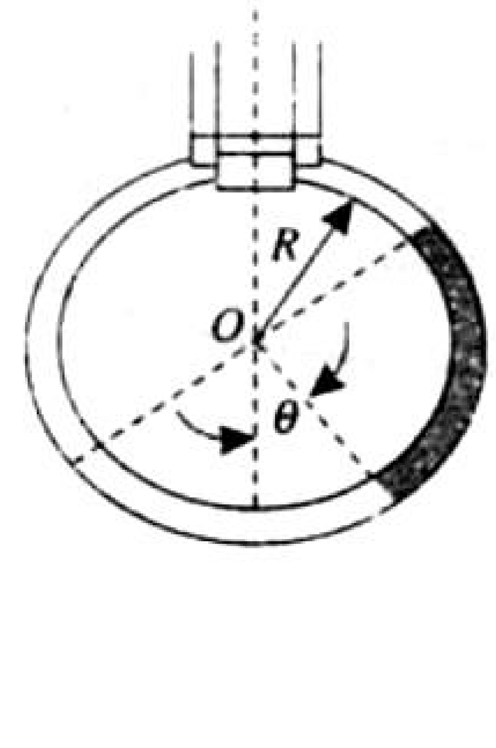


3. Two light springs of force constant k_1 and k_2 and a block of mass m are in the line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in figure.

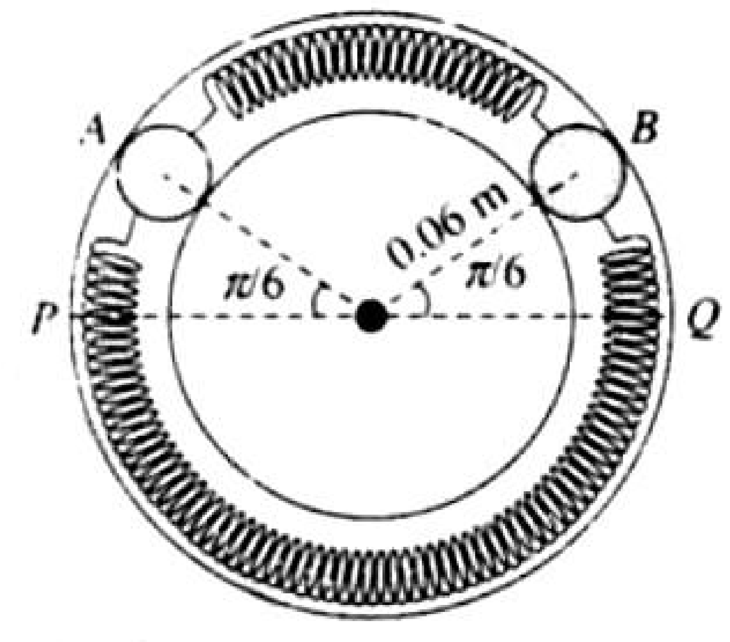


The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm/sec. in between the springs, calculate the period of oscillation of the block. (k_1 = 1.8 N/m, k_2 = 3.2 N/m and m = 200 g) Is the motion simple harmonic? **(IIT-JEE 1985)**

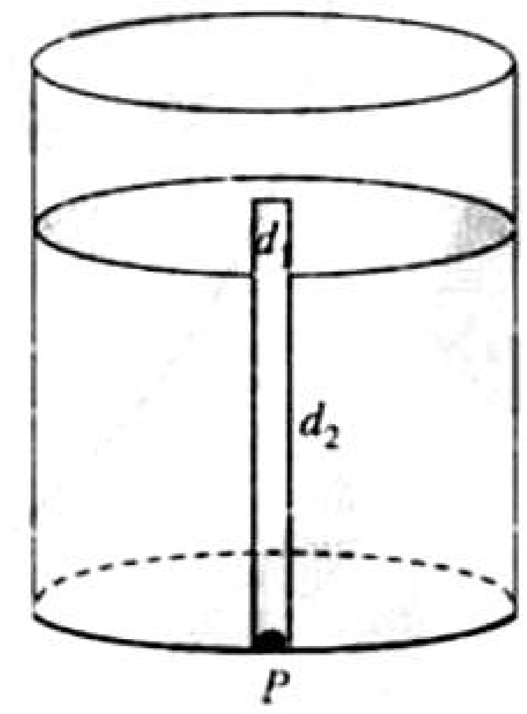
4. Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius R and small cross section kept fixed in a vertical plane as shown in the figure. Each liquid occupies one fourth the circumstances of the tube.



   a. Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.
   b. If the whole is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations **(IIT-JEE 1991)**
5. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle 0.06 π metre. Each spring has a natural length of 0.60 π metre and spring constant 0.1 N/m. Initially, both the balls are displaced an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in figure) and released from rest.



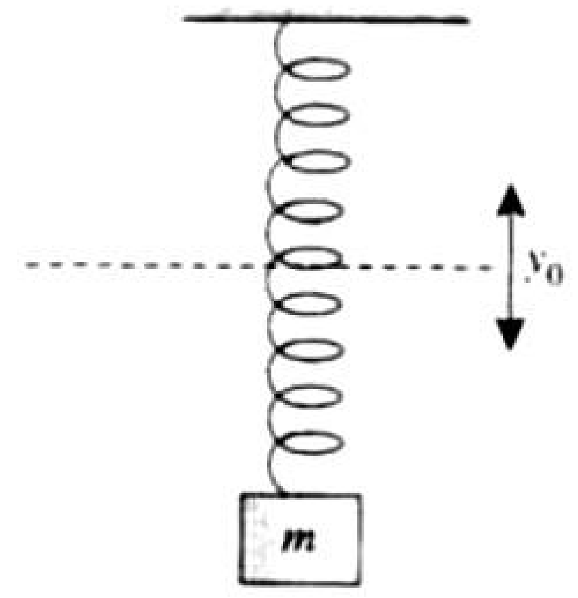
   i. Calculate the frequency of oscillation of ball B.
   ii. Find the speed of ball A when A and B are at the two ends of the diameter PQ.
   iii. What is the total energy of the system? **(IIT-JEE 1993)**
6. A thin rod of length L and area of cross section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



7. A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table is 0.72. Find the maximum amplitude in cm of the table for which the block does not slip on the surface of the table. **(IIT-JEE 1996)**
8. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. It makes time t_1 to go from 0 to $A/2$ and t_2 to go from $A/2$ to A. Find the ratio T_2/T_1. **(IIT-JEE 2001)**

9. A sphere of radius R is half submerged in liquid of density ρ. If the sphere is slightly pushed down and released, find the frequency of oscillation. **(IIT-JEE 2004)**

10. A mass m is undergoing SHM in the vertical direction about the mean position y_0 with amplitude A and angular frequency ω. At a distance y from the mean position, the mass detaches from the spring. Assume that the spring contracts and does not obstruct the motion of m. Find the distance y^*(measured from the mean position) such that the height h attained by the block is maximum. $(A\omega^2 > g)$



(IIT-JEE 2005)

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. c.	2. d.	3. b.	4. d.	5. b.
6. b.	7. a.	8. d.	9. a.	10. a.
11. d.	12. c.	13. a.	14. c.	15. b.
16. c.	17. d.	18. c.	19. b.	20. a.

Multiple Correct Answers Type

1. b., c.	2. a., c., d.
3. a., c.	4. a., d.
5. a., d.	6. b, d.

Linked Comprehension Type

1. c.	2. b.	3. d.	4. d.	5. c.
6. b.				

Matching Column Type

1. i.→a.; ii.→b., c.; iii.→a.; iv.→b., c.
2. i.→a., d.; ii.→b., c. d.; iii.→d.; iv.→b.

Integer Answer Type

1. (4)

Fill in the Blanks Type

1. 1:1 2. 0.06

Subjective Type

1. $M = 1$ kg 2. $\omega = \sqrt{\dfrac{k}{m_2}}$, $A = \dfrac{m_1 g}{k}$

3. 2.82 s 4. (a) $\theta = \tan^{-1}\left(\dfrac{1}{5}\right)$, (b) $2.5\sqrt{R}$ sec

5. (i) $\dfrac{1}{\pi}$ Hz, (ii) 0.0628 m/s, (iii) 3.9×10^{-4} m/s

6. $\omega = \sqrt{\dfrac{3g}{2L}\left(\dfrac{d_2 - d_1}{d_1}\right)}$ 7. 2 cm

8. $\dfrac{t_2}{t_1} = 2$ 9. $\dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$

10. $\dfrac{g}{\omega^2}$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. c. During one complete oscillation, the kinetic energy will become maximum twice. Therefore, the frequency of kinetic energy will be $2f$.

2. d. Both the bodies oscillate in simple harmonic motion for which the maximum velocities will be

$$v_1 = a_1\omega_1 = a_1 \times \frac{2\pi}{T_1}$$

$$v_2 = a_2\omega_2 = a_2 \times \frac{2\pi}{T_2}$$

Given that $v_1 = v_2$

$$a_1 \times \frac{2\pi}{T_1} = a_2 \times \frac{2\pi}{T_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}} = \sqrt{\frac{k_2}{k_1}}$$

3. b. When the cylinder is given a small push downwards, say x, then two forces start acting on the cylinder trying to bring it to its mean position. Restoring force = – (upthrust + spring force)

$$= -[\rho Axg + kx]$$

$$= -[\rho Ag + k]x$$

$$M\omega^2 = \rho Ag + k \quad \Rightarrow \quad \omega = \left[\frac{\rho Ag + k}{M}\right]^{1/2}$$

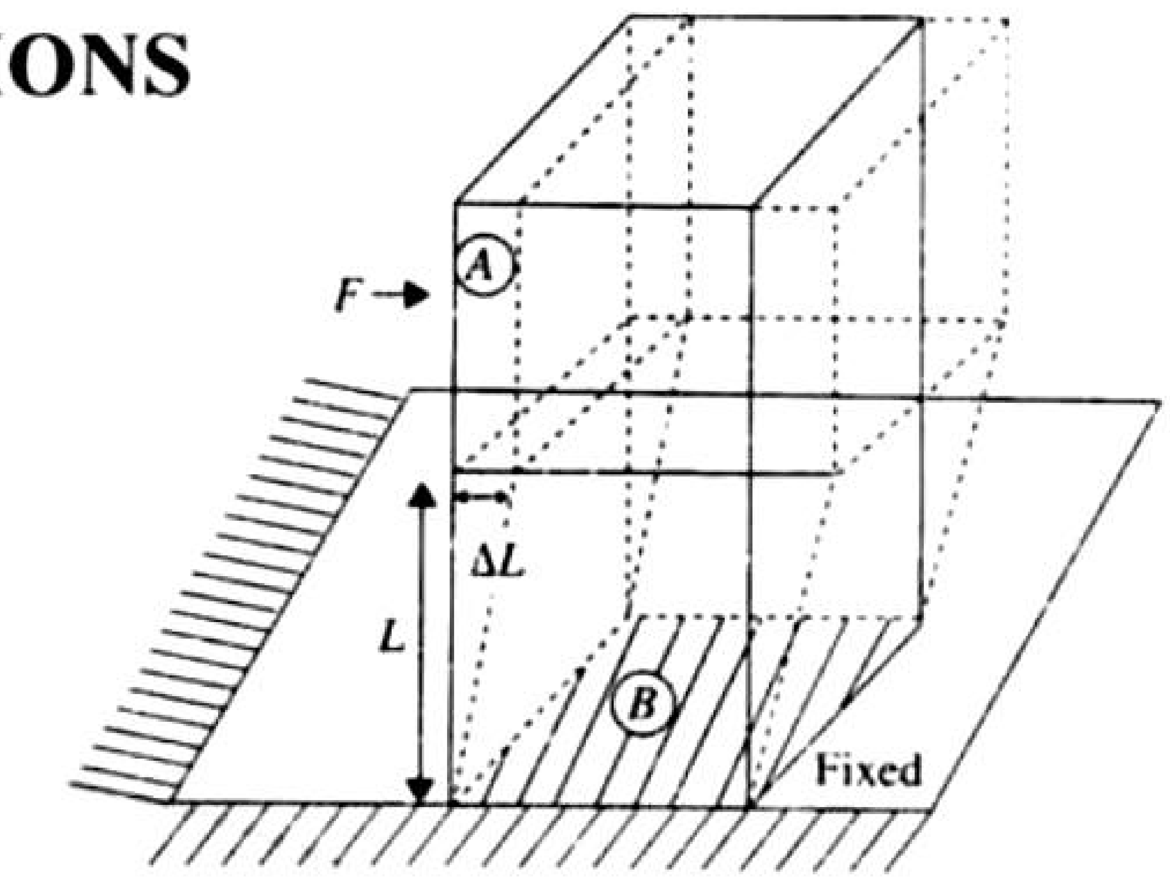
$$\Rightarrow v = \frac{1}{2\pi}\left[\frac{\rho Ag + k}{M}\right]^{1/2}$$

4. d. When a force is applied on cubical block A in the horizontal direction, then the lower block B will get distorted as shown by the dotted lines and A will attain a new position (without distortion as A is a rigid body) as shown by the dotted lines.

For cubical block B,

$$\eta = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L} = \frac{F}{L^2} \times \frac{L}{\Delta L} = \frac{F}{L \times \Delta L}$$

$$\Rightarrow F = \eta L \Delta L$$



ηL is a constant

$\Rightarrow$ $F \propto \Delta L$ and directed towards the mean position

$\Rightarrow$ oscillation will be simple harmonic in nature. Here, $M\omega^2 = \eta L$

$$\Rightarrow \omega = \sqrt{\frac{\eta L}{M}} = \frac{2\pi}{T}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{M}{\eta L}}$$

5. b. $$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$$

$$= 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$$

$$= 2[\cos t + 1]\sin 1000t$$
$$= 2\cos t \sin 1000t + 2\sin 1000t$$
$$= \sin 1001t + \sin 999t + 2\sin 1000t$$

6. b. Let us consider the wire also as a spring. Then the case becomes two springs attached in series. The equivalent spring constant is

$$\frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{K'}$$

where K' is the spring constant of the wire

$\therefore$ $$\therefore K_{eq} = \frac{KK'}{K + K'}$$

Now, $$Y = \frac{F/A}{\Delta L/L} = \frac{F}{A} \times \frac{L}{\Delta L}$$

$$\frac{F}{\Delta L} = \frac{YA}{L} = K'$$

We know that time period of the system

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m(K + K')}{KK'}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{K}\left[\frac{K + YA/L}{YA/L}\right]}$$

$$= 2\pi\sqrt{\frac{m(KL + YA)}{KYA}}$$

7. a. $V(x) = k|x|^3$

$\therefore$ $$\therefore [k] = \frac{[V]}{[x]^3} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Now time period on T and $(\text{mass})^x$ $(\text{amplitude})^y$ $(k)^z$

$$\therefore \quad \therefore [M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z$$

$$= [M^{x+z}L^{y-z}T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$
$$y - z = 0 \quad y = z = -1/2$$

Hence $T \propto (\text{amplitude})^{-1/2}$

or $$T \propto \frac{1}{\sqrt{a}}$$

8. d. Let us plot the graph of the mathematical equation

$$U(x) = K\left[1 - e^{-x^2}\right]$$

$$\therefore \quad F = -\frac{dU}{dx} = -2kxe^{-x^2}$$

It is clear that the potential energy is minimum at $x = 0$. Therefore, $x = 0$ is the state of stable equilibrium. Now if we displace the particle from $x = 0$, then for small displacements the particle tends to regain the position $x = 0$ with a force $F = 2kx/e^{x^2}$ for x to be small $F \propto x$.

9. a. Effective gravity $= g\cos\alpha$

$$\therefore \quad \therefore T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

10. a. Method 1: **Qualitative**. The velocity of a body executing SHM is maximum at its centre and decreases as the body proceeds to the extremes. Therefore, if the time taken for the body to go from O to $A/2$ is T_1 and to go A is T_2, then obviously $T_1 < T_2$.
Method 2: Quantitative. Any SHM is given by the equation $x = \sin \omega t$, where x is the displacement of the body at any instant t. a is the amplitude and ω is the angular frequency.

When $x = 0$, $\omega t_1 = 0$

$$\therefore \quad t_1 = 0$$

When $x = a/2$, $\omega t_2 = \pi/6$, $t_2 = \pi/6\omega$

When $x = a$, $\omega t_3 = \pi/2$, $t_3 = \pi/2\omega$

Time taken from O to $A/2$ will be

$$t_2 - t_1 = \frac{\pi}{6\omega} = T_1$$

Time taken from $A/2$ to A will be

$$t_3 - t_2 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega} = T_2$$

Hence $T_2 > T_1$

11. d. We know that

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and $$T_2 = 2\pi\sqrt{\frac{l}{g'}}$$

$$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g'}} \qquad \text{(i)}$$

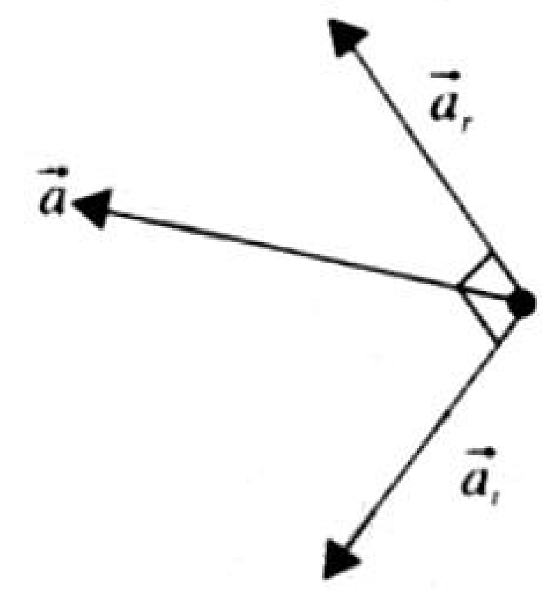
Also $$g = \frac{GM}{R^2}$$

$$\therefore \quad g' = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$$

$$\therefore \quad \frac{g}{g'} = 4 \Rightarrow \frac{T_2}{T_1} = 2$$

12. c. The components of acceleration are as shown.

$\vec{a} = \vec{a}_r + \vec{a}_t$



13. a. We know that in SHM, at extreme position, PE is maximum when $t = 0$, $x = A$, i.e., at time $t = 0$, the particle executing SHM is at its extreme position. Therefore PE is maximum. Graphs I and III represent the above characteristics.

14. c. Angular frequency of system

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration $a_{max} = \omega^2 A$

Frictional force between P and Q

= force exerted on lower block

$$= m\omega^2 A = m\left(\frac{k}{2m}\right)A = \frac{kA}{2}$$

15. b. $y = kt^2$

$$\therefore \quad \frac{dy}{dt} = 2kt$$

$$\Rightarrow \frac{d^2y}{dt^2} = 2k = 2 \text{ m/s}^2 \qquad \text{(i)}$$

$(\because k = 1 \text{ m/s}^2, \text{given})$

We know that

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g_2}{g_1} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$$

$$\left[\because g_1 = 10 \text{ m/s}^2, g_2 = g + 2 = 12 \text{ m/s}^2\right]$$

16. c. $\frac{1}{2}kx^2 = \frac{1}{2}4ky^2$

$$\Rightarrow \frac{y}{x} = \frac{1}{2}$$

17. d. The given motion is represented by

$$\left(\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\right)$$

$$x = 1\sin\left(\frac{\pi}{4}t\right), \quad \frac{d^2x}{dt^2} = \frac{-\pi^2}{16}\sin(\pi/4)t$$

At $\quad t = 4/3$ s, $\quad \dfrac{d^2x}{dt^2} = -\dfrac{\sqrt{3}}{32}\pi^2 \quad \text{cm/s}^2$

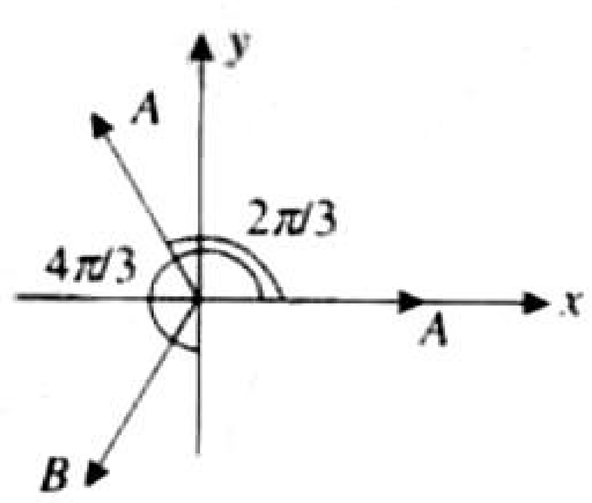
18. c. Restoring torque $= -2\times k\left(\dfrac{l}{2}\theta\right)\dfrac{l}{2} = \dfrac{Id^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\frac{kl^2}{2}(-\theta)}{\frac{Ml^2}{12}}$$

$$\Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

19. b. So, $B = A$, $\phi = 240^\circ = \dfrac{4\pi}{3}$



20. a. Time of flight for projectile

$$T = \frac{2u\sin\theta}{g} = 1 \text{ sec}$$

$$\frac{2u\sin 45}{g} = 1\text{ sec}$$

$$u = \frac{g}{\sqrt{2}}$$

$$u = \sqrt{50} \text{ m/s}$$

Multiple Correct Answer Type

1. **b., c.** $KE_{max} = \dfrac{1}{2}kA^2 = \dfrac{1}{2}\times 2\times 10^6 \times (0.01)^2 = 100$ J

   $U_{max} = TE = 160$ J

2. **a., c., d.** For a plane wave, intensity (energy crossing per unit area per unit time) is constant at all points. But for a spherical wave, intensity at a distance r from a point source of power (P), is given by

$$I = \frac{P}{4\pi r^2} \quad \Rightarrow I \propto \frac{1}{r^2}$$

   But the total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

3. **a., c.** From superposition principle.

$$y = y_1 + y_2 + y_3$$

$$= a\sin\omega t + a\sin(\omega t + 45^\circ) + a\sin(\omega t + 90^\circ)$$

$$= a\{\sin\omega t + \sin(\omega t + 90^\circ)\} + a\sin(\omega t + 45^\circ)$$

$$= 2a\sin(\omega t + 45^\circ)\cos 45^\circ + a\sin(\omega t + 45^\circ)$$

$$= (\sqrt{2}+1)a\sin(\omega t + 45^\circ) = A\sin(\omega t + 45^\circ)$$

   Therefore, resultant motion is simple harmonic of amplitude $A = (\sqrt{2}+1)a$ and which differs in phase by 45° relative to the first.

   Energy in SHM $\propto$ (amplitude)^2

$$\left[E = \frac{1}{2}mA^2\omega^2\right]$$

$$\therefore \quad \frac{E_{resultant}}{E_{single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\therefore \quad E_{resultant} = (3+2\sqrt{2})E_{single}$$

4. **a., d.** Torque is same for both the cases.

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}}$$

$$I_A = \frac{mL^2}{3} + \left(\frac{MR^2}{2} + ML^2\right)$$

   When the disc is free to rotate about its centre, it will not rotate w.r.t. ground. It will act as a point mass, so moment of inertia is given by

$$I_B = \frac{mL^2}{3} + ML^2$$

$$I_A > I_B \Rightarrow T_A > T_B \Rightarrow \omega_A < \omega_B$$

5. **a., d.** $v = u_0 \sin\omega t$

   (suppose t_{01} is the time of collision)

$$\frac{u_0}{2} = u_0\cos\omega t_1 \Rightarrow t_1 = \frac{\pi}{3\omega}$$

   Now the particle returns to equilibrium position at time $t_2 = 2t_1$, i.e., $\dfrac{2\pi}{3\omega}$ with the same mechanical energy, i.e., its speed will be u_0. Let t be the time at which the particle passes through the equilibrium position for the second time.

   Energy of particle and spring remains conserved.

$$t_3 = \frac{T}{2} + 2t_1 = \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

6. **b, d.** $x = A\sin(\omega t)$

   $V = A\omega\cos(\omega t)$

   $p_1 = mA_1\omega_1 = b$, $A_1 = a$ (From ellipse)

   $p_2 = mA_2\omega_2 = R$, $A_2 = R$ (From circle)

   Also, $\dfrac{a}{b} = n^2$ and $\dfrac{a}{R} = n \Rightarrow \dfrac{R}{b} = n$

$$E_1 = \frac{p_1^2}{2m} \Rightarrow E_2 = \frac{p_2^2}{2m}$$

$$\frac{E_1}{E_2} = \left(\frac{b}{R}\right)^2 = \frac{1}{n^2} \Rightarrow \frac{m\omega_1^2A_1^2}{m\omega_2^2A_2^2} = \frac{1}{n^2}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{n^2} \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Linked Comprehension Type

1. c. Energy must be less than V_0, so that KE becomes zero before PE becomes maximum and particle returns back.

2. b. Dimension of α can be found as

$[\alpha] = ML^{-2}T^{-2}$

Only option (b) has dimension of time

Alternatively:

From conservation of energy:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \alpha x^4 = \alpha A^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2\alpha}{m}(A^4 - x^4)$$

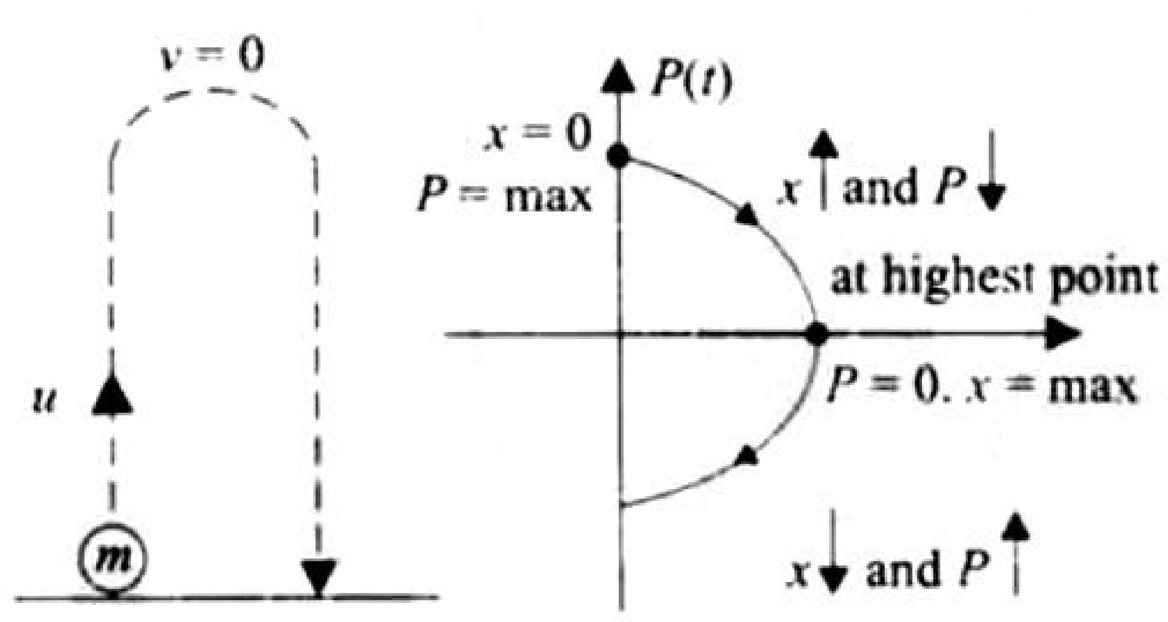
$$\int dt = \sqrt{\frac{m}{2\alpha}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}}$$

$$\Rightarrow t = \frac{1}{A}\sqrt{\frac{m}{2\alpha}}\int_0^1 \frac{du}{\sqrt{1-u^4}} \qquad \text{[Substitute } x = Au\text{]}$$

$$\Rightarrow t \propto \frac{1}{A}\sqrt{\frac{m}{2\alpha}}$$

3. d. As potential energy is constant for $|x| > X_0$, the force on the particle is zero hence acceleration is zero.

4. d.



5. c. In the 1st case amplitude of SHM is a.

In the 2nd case amplitude of SHM is $2a$

Total energy = 1/2 k (amplitude)^2

$$E_1 = \frac{1}{2}k(2a)^2 \quad \Rightarrow \quad E_2 = \frac{1}{2}k(a)^2$$

$$E_1 = 4E_2$$

6. b. Amplitude of oscillation inside liquid will decrease due to viscous force. So radius of circular arcs will decrease as position change. Let initially mass is taken to maximum upward position and released, then its initial momentum is zero and then afterwards momentum will become negative first which is correctly shown in option (b).

Matching Column Type

1. i.→a.; ii.→b., c.; iii.→a.; iv.→b., c.

i. Given $v = c_1\sqrt{c_2 - x^2}$.

Comparing with $v = \omega\sqrt{A^2 - x^2}$, we find that this is a case of simple harmonic motion. Hence only option (a) is correct.

ii. $v = -kx$

$$\Rightarrow \frac{dx}{dt} = -kx \Rightarrow \int_{x_0}^{x}\frac{dx}{x} = -\int_0^t k\,dt$$

$$\Rightarrow \ln\left(\frac{x}{x_0}\right) = -kt \quad \Rightarrow \quad x = x_0e^{-kt}$$

At $\quad t = 0, x = x_0$

At $\quad t = \infty, x = 0$

So x decreases with time and hence v decreases in magnitude always. So KE keeps on decreasing. v always remains negative, so the object does not change its direction of motion.

iii. Here spring–block system will execute simple harmonic motion

iv. Given $v = 2\sqrt{\frac{GM_e}{R_e}} > v_{escape}$. So the object keeps on moving away from the earth. Its speed goes on decreasing and it never changes its direction of motion.

2. i.→a., d.; ii.→b., c. d.; iii.→d.; iv.→b.

i. Potential energy is minimum at mean position.

ii. For $a = 0$, $s = vt$ → option (b).

So, $s = s_0 + vt$ → option (c).

For a = constant, $s = ut + \frac{1}{2}at^2$ → option (d).

iii. $R = \frac{v^2 \sin 2\theta}{g}$

$\therefore R \propto v^2$ → option (d).

iv. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore T^2 \propto l$ → option (b).

Integer Answer Type

1. (4) When a wire of length L, area of cross-section A, Young's modulus Y is stretched by suspending a mass m, then the mass performs simple harmonic motion with angular frequency

$$\omega = \sqrt{\frac{YA}{mL}}$$

Substituting the given values, we get

$$140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}}$$

or $\quad 140 \times 140 = \frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}$

$$\Rightarrow \quad n = \frac{14 \times 14 \times 10^2}{49 \times 10^2} = 4$$

Fill in the Blanks Type

1. $y_1 = 10\sin(3\pi t + \pi/4)$ (i)

$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$ (ii)

In Eq. (ii), let

$5 = a\cos\theta$ (iii)

and $5\sqrt{3} = a\sin\theta$ (iv)

$\therefore \quad y_2 = a\sin 3\pi t\cos\theta + a\sin\theta\cos 3\pi t$

$y_2 = a\sin(3\pi t + \theta)$ (v)

Squaring and adding Eqs. (iii) and (iv), we get

$$5^2 + (5\sqrt{3})^2 = a^2\cos^2\theta + a^2\sin^2\theta$$

$\Rightarrow 100 = a^2$

$\Rightarrow \quad a = 10$

Therefore, Eq. (v) can be written as

$y_2 = 10\sin(3\pi t + \theta)$ (vi)

From Eqs. (i) and (vi), the ratio of amplitudes is 10:10, i.e., 1:1.

2. $x = 0.04$ m, KE = 0.5 J and PE = 0.4 J

Also $\nu = \dfrac{25}{\pi}$ Hz

Now, $\text{TE} = \dfrac{1}{2}m\omega^2a^2 = \dfrac{1}{2}m \times 4\pi^2\nu^2a^2$

$$\Rightarrow 0.9 = \frac{1}{2}\times 0.2 \times 4\pi^2 \times \frac{25}{\pi}\times\frac{25}{\pi}\times a^2$$

$$\Rightarrow \quad a = \frac{3}{50} = 0.06 \text{ m}$$

Alternate Method:

$$\frac{1}{2}mv^2 = 0.5 \Rightarrow \frac{1}{2}(0.2)v^2 = 0.5 \Rightarrow v^2 = 5$$

Now $v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow \quad 5 = (2\pi\nu)^2\,[A^2 - (0.04)^2]$

$$\Rightarrow \quad 5 = 4\pi^2 \times \frac{25}{\pi}\times\frac{25}{\pi}[A^2 - (0.04)^2]$$

$\Rightarrow \quad A = 0.06$

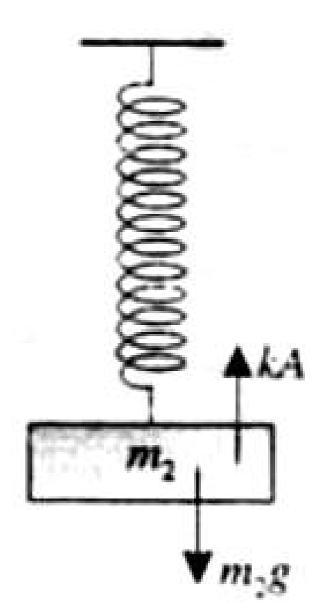
Subjective Type

1. $1 = 2\pi\sqrt{\dfrac{M}{k}}$ and $2 = 2\pi\sqrt{\dfrac{M+3}{k}}$

Dividing these equations we get

$$4 = \frac{M+3}{M} \Rightarrow M = 1 \text{ kg}$$

2. When mass m_1 is removed, then the equilibrium will get disturbed. There will be a restoring force in the upward direction and will be equal to m_1g. The body will undergo S.H.M. now. The position where m_1 is removed will be one of the extreme positions (or amplitude position) of S.H.M. Therefore we can write



$F = m_1g = kA$

where A is the amplitude of S.H.M

$\Rightarrow \quad \dfrac{m_1g}{k}$

Let at any instant the mass m_2 be having a displacement x. From mean position, the restoring force at this situation will be, $F = -kx$

$$m_2a = -kx \Rightarrow a = -\frac{k}{m_2}x$$

Comparing this with $a = -\omega^2x$ we get $\omega^2 = \dfrac{k}{m_2}$

$$\therefore \quad \omega = \sqrt{\frac{k}{m_2}}$$

or $\quad 2\pi f = \sqrt{\dfrac{k}{m_2}} \Rightarrow f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_2}}$

3. When the block touches D, it will compress the spring and its KE will be converted into elastic energy of the spring. The compressed spring will push the block to D with same speed; so time taken by block to move from D towards B and block to D will be

$$t_1 = \frac{T_1}{2} = \pi\sqrt{\frac{m}{k_1}} = \pi\sqrt{\frac{0.2}{1.8}} = \frac{\pi}{3}\text{sec}$$

Similarly time t_2 taken by block in contact with spring between AC.

$$t_2 = \frac{T_2}{2} = \pi\sqrt{\frac{m}{k_2}} = \pi\sqrt{\frac{0.2}{3.2}} = \frac{\pi}{4}\text{sec}$$

Moreover during complete oscillation between A and B, the block moves the distance CD twice with uniform velocity v, once from C to D and again from D to C.

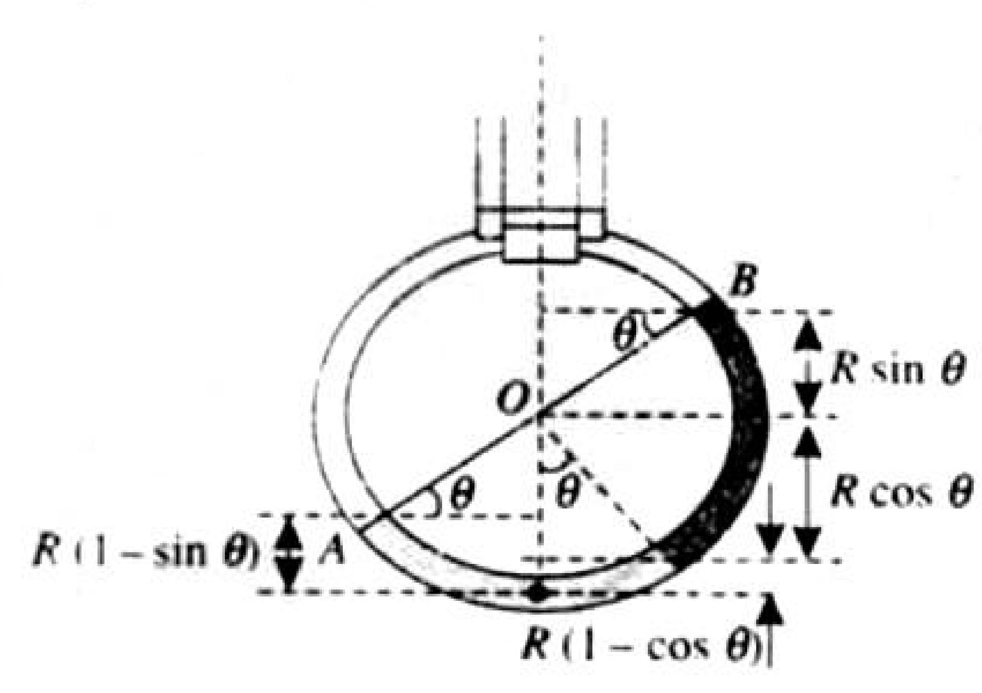
So $\quad t_3 = \dfrac{2L}{v} = \dfrac{2\times 0.6}{1.2} = 1$ sec

$$T = t_1 + t_2 + t_3 = \pi\left(\frac{1}{3}+\frac{1}{4}\right) + 1 = 2.82 \text{ sec}$$

Now a motion is simple harmonic only and only if throughout the motion $F = -kx$. Here between C and D, $F = 0$ (as v = constant); the motion is not simple harmonic but oscillatory.

4. The pressure due to liquid on left limb (at bottom)

$P_1 = R(1 - \sin\theta)\,1.5\,\rho g$ (i)



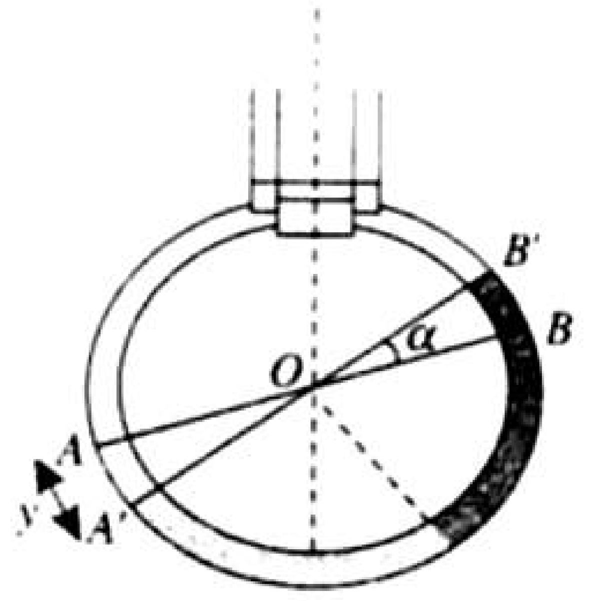
The pressure due to liquids on right limb

$P_2 = (R\sin\theta + R\cos\theta)\,\rho g + R(1 - \cos\theta)\,1.5\rho g$ (ii)

In equilibrium, $P_1 = P_2$

Which gives, $\tan\theta = \left(\dfrac{1}{5}\right) \Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{5}\right)$

If the liquid is given a small angular displacement α.



the pressure difference, $dP = P_1 - P_2$

$dP = [R\sin(\theta+\alpha) + R\cos(\theta+\alpha)]\rho g + R[1-\cos(\theta+\alpha)]\,1.5\rho g - R[1-\sin(\theta+\alpha)]\,1.5\rho g$

as α is small, $\sin\alpha = \alpha$, $\cos\alpha = 1$

$dP = R\rho g\,[2.5\sin\theta + 2.5\cos\theta\cdot\alpha - 0.5\cos\theta + 0.5\sin\theta\cdot\alpha]$

$$\tan\theta = 0.2,\ \sin\theta = \frac{0.2}{\sqrt{1.04}} \text{ and } \cos\theta = \frac{1}{\sqrt{1.04}}$$

$$dP = 2.55\,R\rho g\,\alpha = 2.55\rho g\,y \ (\text{as } R\alpha = y)$$

Restoring force $F = dP \times \text{area} = -2.55\,\rho g y\,A$

Mass of the liquid in tube

$$m = \frac{2\pi R}{4}A\rho + \frac{2\pi r}{4}A\times 1.5\rho = 1.25\pi RA\rho$$

Hence acceleration $a = \frac{F}{m} = -\frac{2.55\,\rho g y\,A}{1.25\,\pi RA\rho}$

$$a = -2.04\left(\frac{g}{\pi R}\right)y; \qquad a = -\omega^2 y$$

Hence, $\omega = \sqrt{2.04\left(\frac{g}{\pi R}\right)}$

Time period, $T = \frac{2\pi}{\omega} = 2.5\sqrt{R}$ sec

5. i. At any moment when angular displacement of the balls from equilibrium is θ, total energy of the system is given by

$$E = 2\left(\frac{1}{2}mv^2\right) + 2\cdot\frac{1}{2}k(2R\theta)^2 = mv^2 + 4KR^2\theta^2$$

$\because$ E = constant, hence $\frac{dE}{dt} = 0$

$$\Rightarrow \quad 0 = 2mv\frac{dv}{dt} + 8KR^2\cdot\theta\frac{d\theta}{dt}$$

$$= mv\frac{dv}{dt} + 4KR\theta\left(\frac{d\theta}{dt}R\right)$$

putting $R\theta = x$, and $\frac{d\theta}{dt}R = v$

$$0 = mv\frac{dv}{dt} + 4kxv \Rightarrow \frac{d^2x}{dt^2} = -\frac{4K}{m}x$$

Hence $T = 2\sqrt{\frac{m}{4k}} \Rightarrow f = \frac{1}{T} = \frac{1}{\pi}$ Hz

ii. Since one spring is compressed while the other is stretched by the same amount, therefore,

$$E = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 = kx^2 \qquad [k_1 = k_2 = k]$$

$$x = x_1 + x_2 = R\theta_1 + R\theta_2 = 2R\theta \quad [\theta_1 = \theta_2 = \theta]$$

$\therefore$ $x = 2(0.6)(\pi/6) = 0.02\pi$ m

Thus, $E = (0.1)(0.02\pi)^2 = 4\pi^2\times 10^{-5}$ J

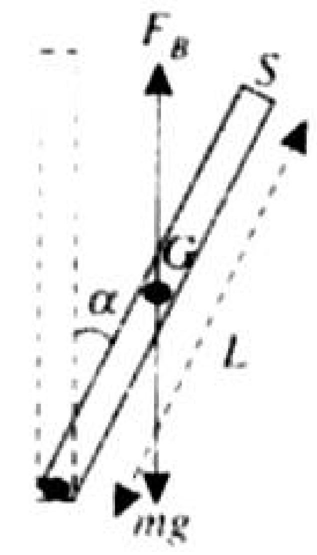
iii. Since at P and Q springs are relaxed so whole energy becomes kinetic.

$$\therefore \quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = mv^2 = E = 4\pi^2\times 10^{-5}$$

or $(0.1)v^2 = 4\pi^2\times 10^{-5}$

$v = 2\pi\times 10^{-2}$ m/s.

6. If the rod is displaced through an angle α from its equilibrium position, i.e., vertical position then it will be under the thrust F_B and its own weight mg. There will be a net torque, which will try to restore the rod to its equilibrium position.



Weight of the rod acting downward $mg = SLd_1g$

Buoyant force acting upwards $F_B = SLd_2g$

Net force acting of the rod upward $= SL(d_2 - d_1)g$

If the area of cross-section of the rod is S

Net restoring torque $\tau = SL(d_2 - d_1)gx\frac{L}{2}\sin\alpha$

$$\tau = SL(d_2 - d_1)gx\frac{L}{2}\alpha \qquad (\because \sin\alpha = \alpha)$$

If α is in clockwise direction, then τ will be in anticlockwise direction. Thus

$$\tau = -\frac{1}{2}SL^2(d_2 - d_1)gx$$

$$\tau = I\alpha = \left(\frac{ML^2}{3}\right)\frac{d^2\alpha}{dt^2} = \left(\frac{SLd_1\times L^2}{3}\right)\frac{d^2\alpha}{dt^2}$$

$$\therefore \quad \frac{d^2\alpha}{dt^2} = \frac{3}{SL^3d_1}\times\frac{1}{2}SL^2(d_2 - d_1)g\alpha$$

$$\text{or} \quad \frac{d^2\alpha}{dt^2} = -\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)\alpha$$

This is the equation of S.H.M. Hence the rod performs simple harmonic motion.

Where $\omega^2 = \frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)$

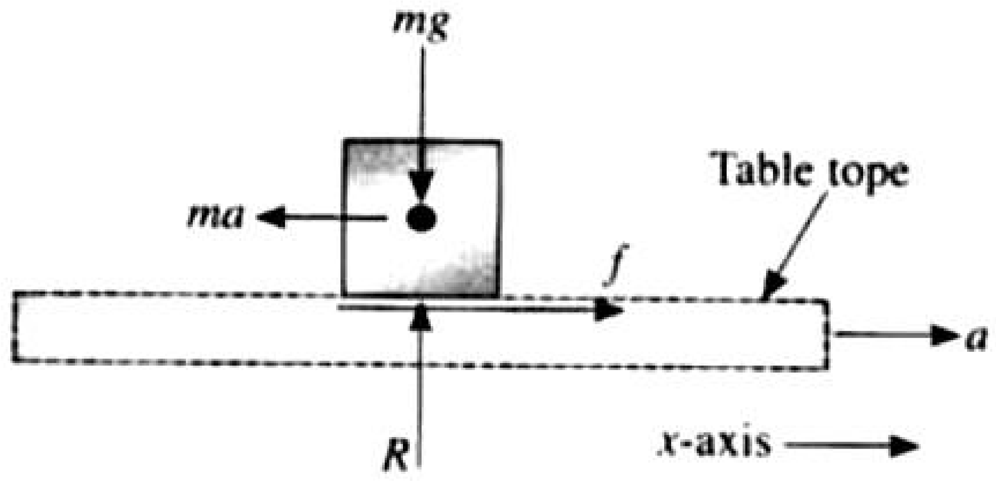
The angular frequency $\omega = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$

Therefore time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2L}{3g}\left(\frac{d_1}{d_2 - d_1}\right)}$

7. Let the mass of the block be m and let, at a certain instant of time, the direction of acceleration a of the table (executing simple harmonic motion) be along the positive x-direction. Considering the block w.r.t to table, the block will experience a pseudo force ma directed along the negative x-axis. Consequently, the force of friction f will act along the positive x-axis. The weight mg of the block will be balanced by the normal reaction R.

For block is not sliding w.r.t table $f = ma$ (i)

The block will not slip on the surface of the table, if friction is static for this $f \le \mu mg$ or $ma \le \mu mg$



Therefore, for no slipping the table can have a maximum acceleration $a_{max} = \mu g$. Where $a_{max} = \omega^2 A$. Therefore, the maximum amplitude is given by

$$\omega^2 A_{max} = \mu g$$

$$A_{max} = \frac{\mu g}{\omega^2} = \frac{\mu g}{4\pi^2 f^2} \qquad (\because \omega = 2\pi f)$$

$$= \frac{0.72 \times 10}{4\pi^2 \times (3)^2} = 0.02\,\text{m} = 2\,\text{cm}$$

8. From $x = A \sin \omega t$, we have

$$\frac{A}{2} = A \sin \omega t_1 \Rightarrow \omega t_1 = \frac{\pi}{6} \qquad \text{(i)}$$

Also $A = A \sin \omega (t_1 + t_2)$

$$\omega(t_1 + t_2) = \frac{\pi}{2} \qquad \text{(ii)}$$

From (i) and (ii), we get $\frac{t_1}{t_1 + t_2} = \frac{1}{3}$ which gives $\frac{t_2}{t_1} = 2$.

9. Half of the volume of sphere is submerged.

At equilibrium net force is zero

$$\therefore \quad F_{mg} = F_{buoyancy}$$

$$\left(\frac{4}{3}\pi r^3 \rho_s\right) \times g = \left(\frac{2}{3}\pi r^3 \rho\right) g \Rightarrow \rho_s = \frac{\rho}{2}$$

Let the sphere is slightly displaced downward by x, weight will remain as it is while upthrust will increase. The volume of submerged portion of sphere increases by $\pi R^2 x$, hence buoyancy increases by $\pi R^2 x \rho g$. This increased upthrust will becomes the net restoring force (upwards).

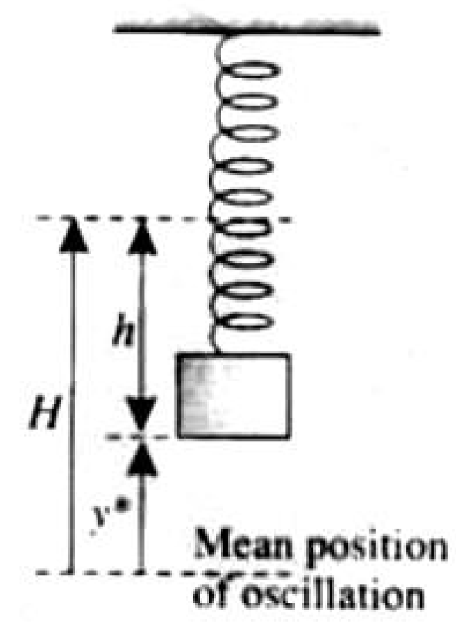
$$\therefore \quad F_{restoring} = -\pi R^2 x \rho g = ma$$

$$\therefore \quad a = \frac{-\pi R^2 \rho g}{\frac{4}{3}\pi R^2 \rho_s} = -\frac{3g}{2R} x$$

$$\therefore \quad \omega = \sqrt{\frac{3g}{2R}} \quad [\text{Comparing with } a = -\omega^2 x]$$

or $\quad v = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

10. Speed of block at $y^* = \omega\sqrt{A^2 - y^{*2}}$



Height h attained by the block after detachment

$$= \frac{\omega^2 (A^2 - y^{*2})}{2g}$$

Total height attained by the block

$$H = \frac{\omega^2 (A^2 - y^{*2})}{2g} + y^*$$

For H to be maximum, $\frac{dH}{dy^*} = 0 \Rightarrow y^* = \frac{g}{\omega^2}$