10. Polynomials

Questions Pg-239

1 A. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = x^2 – 7x + 12$

Answer

Given, $p(X) = x^2 – 7x + 12$

Now, we need to write the given polynomial as a product of first degree polynomial and p(x) = 0

⇒ $x^2 – 7x + 12 = 0$

⇒ The given equation can be written as follows

⇒ (x – 4)(x – 3) = 0

⇒ Since, we know that $x^2 – 7x + 12$ can be written as (x – a)(x – b)

⇒ $x^2 – 7x + 12 = x^2 – (a + b)x + ab$

∴ coefficient on either side of the polynomial is same and a + b = 7 and ab = 12

⇒ we must find two numbers that satisfy a + b and ab we get, 4 and 3 as the numbers

∴ (x – 4)(x – 3) are the product of first degree polynomial

⇒ Substitute x = 4 and x = 3, We get p(x) as 0

⇒ $x^2 – 7x + 12 = 4^2 – 7(4) + 1\ 2 = 16 – 28 + 12 = 0$

⇒ $x^2 – 7x + 12 = 3^2 – 7(3) + 12 = 9 – 21 + 12 = 0$

Hence, (x – 4)(x – 3) are the first degree factors of the polynomial and 4, 3 are the solutions of the given polynomial $x^2 – 7x + 12$

1 B. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = x^2 + 7x + 12$

Answer

Given, $p(X) = x^2 + 7x + 12$

Now, we need to write the given polynomial as a product of first degree polynomial and p(x) = 0

⇒ $x^2 + 7x + 12 = 0$

⇒ the given equation can be written as follows

⇒ (x + 4)(x + 3) = 0

⇒ Since, we know that $x^2 + 7x + 12$ can be written as (x + a)(x + b)

⇒ $x^2 + 7x + 12 = x^2 + (a + b)x + ab$

∴ coefficient on either side of the polynomial is same and a + b = 7 and ab = 12

⇒ we must find two numbers that satisfy a + b and ab we get, 4 and 3 as the numbers

∴ (x + 4)(x + 3) are the product of first degree polynomial

⇒ P(x) = 0 if (x + 3) is 0 and (x + 4) is 0

∴ x + 4 = 0

⇒ x = – 4

⇒ $x^2 + 7x + 12 = (-4)^2 + 7(-4) + 12 = 16 - 28 + 12 = 0$

And x + 3 = 0

⇒ x = – 3

⇒ $x^2 + 7x + 12 = (-3)^2 + 7(-3) + 12 = 9 - 21 + 12 = 0$

Hence, (x + 4)(x + 3) are the first degree factors of the polynomial and – 4, – 3 are the solutions of the given polynomial $x^2 + 7x + 12$

1 C. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = x^2 - 8x + 12$

Answer

Given, $p(X) = x^2 - 8x + 12$

Now, we need to write the given polynomial as a product of first degree polynomial and p(x) = 0

⇒ $x^2 - 8x + 12 = 0$

⇒ The given equation can be written as follows

⇒ (x – 6)(x – 2) = 0

⇒ Since, we know that $x^2 - 8x + 12$ can be written as (x – a)(x – b)

⇒ $x^2 - 8x + 12 = x^2 - (a + b)x + ab$

∴ coefficient on either side of the polynomial is same and a + b = 8 and ab = 12

⇒ we must find two numbers that satisfy a + b and ab we get, 6 and 2 as the numbers

∴ (x – 6)(x – 2) are the product of first degree polynomial

⇒ P(x) = 0 if (x – 6) is 0 and (x – 2) is 0

∴ x – 6 = 0

⇒ x = 6

⇒ $x^2 - 8x + 12 = 6^2 - 8(6) + 12 = 36 - 48 + 12 = 0$

And x – 2 = 0

⇒ x = 2

⇒ $x^2 - 8x + 12 = 2^2 - 8(2) + 12 = 4 - 16 + 12 = 0$

Hence, (x – 6)(x – 2) are the first degree factors of the polynomial and 6, 2 are the solutions of the given polynomial $x^2 - 8x + 12$

1 D. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = x^2 + 13x + 12$

Answer

Given, $p(X) = x^2 + 13x + 12$

Now, we need to write the given polynomial as a product of first degree polynomial and $p(x) = 0$

⇒ $x^2 + 13x + 12 = 0$

⇒ the given equation can be written as follows

⇒ $(x + 12)(x + 1) = 0$

⇒ Since, we know that $x^2 + 12x + 13$ can be written as $(x + a)(x + b)$

⇒ $x^2 + 13x + 12 = x^2 + (a + b)x + ab$

∴ coefficient on either side of the polynomial is same and $a + b = 7$ and $ab = 12$

⇒ we must find two numbers that satisfy a + b and ab we get, 12 and 1 as the numbers

∴ $(x + 12)(x + 1)$ are the product of first degree polynomial

⇒ $P(x) = 0$ if $(x + 12)$ is 0 and $(x + 1)$ is 0

∴ $x + 1 = 0$

⇒ $x = – 1$

⇒ $x^2 + 13x + 12 = (– 1)^2 + 13(– 1) + 12 = 1 – 13 + 12 = 0$

And $x + 12 = 0$

⇒ $x = – 12$

⇒ $x^2 + 13x + 12 = (– 12)^2 + 13(– 12) + 12 = 144 – 156 + 12 = 0$

Hence, $(x + 12)(x + 1)$ are the first degree factors of the polynomial and – 12, – 1 are the solutions of the given polynomial $x^2 + 13x + 12$

1 E. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation $p(x) = 0$ in each.

$p(x) = x^2 – 2x + 1$

Answer

Given, $p(X) = x^2 – 2x + 1$

Now, we need to write the given polynomial as a product of first degree polynomial and $p(x) = 0$

⇒ $x^2 – 2x + 1 = 0$

⇒ The given equation can be written as follows

⇒ $(x – 1)(x – 1) = 0$

⇒ Since, we know that $x^2 – 2x + 1$ can be written as $(x – a)(x – b)$

⇒ $x^2 – 2x + 1 = x^2 – (a + b)x + ab$

∴ coefficient on either side of the polynomial is same and $a + b = 2$ and $ab = 1$

⇒ we must find two numbers that satisfy a + b and ab we get, 1 and 1 as the numbers

∴ $(x – 1)(x – 1)$ are the product of first degree polynomial

⇒ $p(x)$ is 0 if $(x – 1)$ is 0

∴ $x – 1 = 0$

⇒ $x^2 – 2x + 1 = 1^2 – 2(1) + 1 = 1 – 2 + 1 = 0$

Hence, (x – 1)(x – 1) are the first degree factors of the polynomial and 1 is the solution of the given polynomial $x^2 - 2x + 1$

1 F. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = x^2 + x - 1$

Answer

Given, $p(x) = x^2 + x - 1$

Now, we need to write the given polynomial as a product of first degree polynomial and p(x) = 0

⇒ The given equation can be written as = $x^2 + x + (-1)$ which is of the form $x^2 + (a + b)x + ab$

∴ a + b = 1 and ab = – 1

⇒ we know that $(a + b)^2 - (a - b)^2 = 4ab$

∴ $(a + b)^2 - 4ab = (a - b)^2$

⇒ we know a + b = 1 and ab = – 1

⇒ $1 - 4(-1) = (a - b)^2$

⇒ $(a - b)^2 = 5$

⇒ $a - b = \pm\sqrt{5}$

⇒ Solving the equation both equation a + b and a – b we get as follows

⇒ $a + b + a - b = 1 + \sqrt{5}$

⇒ $a = \frac{1}{2}(1 + \sqrt{5})$

⇒ $(a + b) - (a - b) = 1 + \sqrt{5}$

⇒ $b = \frac{1}{2}(1 - \sqrt{5})$

∴ $x^2 + x + (-1)$ has factors $(x + \frac{1}{2}(1 + \sqrt{5}))(x + \frac{1}{2}(1 - \sqrt{5}))$

⇒ p(x) = 0 if $(x + \frac{1}{2}(1 + \sqrt{5}))$is 0 and $(x + \frac{1}{2}(1 - \sqrt{5}))$is 0

⇒ $(x + \frac{1}{2}(1 + \sqrt{5})) = 0$

⇒ $x = -\frac{1}{2}(1 + \sqrt{5})$

⇒ $(-\frac{1}{2}(1 + \sqrt{5}))^2 + (-\frac{1}{2}(1 + \sqrt{5})) + (-1) = 0$

And

⇒ $(x + \frac{1}{2}(1 - \sqrt{5})) = 0$

⇒ $x = -\frac{1}{2}(1 - \sqrt{5})$

⇒ $(-\frac{1}{2}(1 - \sqrt{5}))^2 + (-\frac{1}{2}(1 - \sqrt{5})) + (-1) = 0$

Hence, $(x + \frac{1}{2}(1 + \sqrt{5}))(x + \frac{1}{2}(1 - \sqrt{5}))$ are the first degree factors of the polynomial and $-\frac{1}{2}(1 + \sqrt{5})$ and $-\frac{1}{2}(1 - \sqrt{5})$ are the solution of the given polynomial $x^2 + x - 1$

1 G. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation p(x) = 0 in each.

$p(x) = 2x^2 – 5x + 2$

Answer

Given, $p(x) = 2x^2 – 5x + 2$

Now, we need to write the given polynomial as a product of first degree polynomial and p(x) = 0

⇒ $2x^2 – 5x + 2$ can be written as $2(x^2 – \frac{5}{2}x + 1)$

⇒ Now, we will find out the factors for $x^2 – \frac{5}{2}x + 1$

⇒ $x^2 – \frac{5}{2}x + 1$ is in the form $x^2 – (a + b)x + ab$ and (x – a)(x – b) are the factors

∴ coefficient on either side of the polynomial is same and $a + b = \frac{5}{2}$

and ab = 1

⇒ we know that $(a + b)^2 –(a – b)^2 = 4ab$

∴ $(a + b)^2 – 4ab = (a – b)^2$

⇒ $\left(\frac{5}{2}\right)^2 – 4(1) = (a – b)^2$

⇒ $\frac{25}{4} – 4 = (a – b)^2$

⇒ $\frac{25-16}{4} = (a – b)^2$

⇒ $\frac{9}{4} = (a – b)^2$

⇒ $a – b = \pm\sqrt{\frac{9}{4}}$

⇒ $a – b = \pm\frac{3}{2}$

⇒ We need to find out the values of a and b by solving (a + b) + (a – b) and (a + b) – (a – b)

⇒ we take $a – b = \frac{3}{2}$

⇒ $(a + b) + (a – b) = \frac{5}{2} + \frac{3}{2}$

⇒ $2a = \frac{8}{2}$

⇒ a = 2

⇒ $(a + b) – (a – b) = \frac{5}{2} – \frac{3}{2}$

⇒ $2b = \frac{2}{2}$

⇒ $b = \frac{1}{2}$

∴ we get a = 2 and $b = \frac{1}{2}$

And if we take $a - b = -\frac{3}{2}$ we get as follows

$\Rightarrow (a + b) + (a - b) = \frac{5}{2} + (-\frac{3}{2})$

$\Rightarrow 2a = \frac{2}{2}$

$\Rightarrow a = \frac{1}{2}$

$\Rightarrow (a + b) - (a - b) = \frac{5}{2} - (-\frac{3}{2})$

$\Rightarrow 2b = \frac{8}{2}$

$\Rightarrow b = 2$

$\therefore a = \frac{1}{2}$ and $b = 2$

$\Rightarrow$ So, we have $2(x - \frac{1}{2})(x - 2)$

$\Rightarrow (2x - 1)(x - 2)$

$\therefore (2x - 1)(x - 2)$ are the factors

$\Rightarrow p(x) = 0$ if $(2x - 1)$ is 0 and $(x - 2)$ is 0

$\Rightarrow (2x - 1) = 0$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

$\Rightarrow$ Substitute, $x = \frac{1}{2}$ in $2x^2 - 5x + 2 = 2(\frac{1}{2})^2 - 5(\frac{1}{2}) + 2 = \frac{1}{2} - \frac{5}{2} + 2 = 0$

$\Rightarrow (x - 2) = 0$

$\Rightarrow x = 2$ substitute, $2x^2 - 5x + 2 = 2(2)^2 - 5(2) + 2 = 8 - 10 + 2 = 0$

Hence, $(2x - 1)(x - 2)$ are the first degree factors of the polynomial and $\frac{1}{2}$ and 2 are the solution of the given polynomial $2x^2 - 5x + 2$

1 H. Question

Write each polynomial below as a product of first degree polynomials. Write also the solutions of the equation $p(x) = 0$ in each.

$p(x) = 6x^2 - 7x + 2$

Answer

Given, $p(x) = 6x^2 - 7x + 2$

Now, we need to write the given polynomial as a product of first degree polynomial and $p(x) = 0$

$\Rightarrow 6x^2 - 7x + 2$ can be written as $6(x^2 - \frac{7}{6}x + \frac{2}{6}) = 6(x^2 - \frac{7}{6}x + \frac{1}{3})$

$\Rightarrow$ Now, we will find out the factors for $x^2 - \frac{7}{6}x + \frac{1}{3}$

$\Rightarrow x^2 - \frac{7}{6}x + \frac{1}{3}$ is in the form $x^2 - (a + b)x + ab$ and $(x - a)(x - b)$ are the factors

$\therefore$ coefficient on either side of the polynomial is same and $a + b = \frac{7}{6}$

and ab = $\frac{1}{3}$

⇒ we know that $(a + b)^2 - (a - b)^2 = 4ab$

∴ $(a + b)^2 - 4ab = (a - b)^2$

⇒ $\left(\frac{7}{6}\right)^2 - 4(\frac{1}{3}) = (a - b)^2$

⇒ $\frac{49}{36} - \frac{4}{3} = (a - b)^2$

⇒ $\frac{(49-48)}{36} = (a - b)^2$

⇒ $\frac{1}{36} = (a - b)^2$

⇒ $a - b = \pm\sqrt{\frac{1}{36}}$

⇒ $a - b = \pm\frac{1}{6}$

⇒ We need to find out the values of a and b by solving (a + b) + (a – b) and (a + b) – (a – b)

⇒ we take $a - b = \frac{1}{6}$

⇒ $(a + b) + (a - b) = \frac{7}{6} + \frac{1}{6}$

⇒ $2a = \frac{8}{6}$

⇒ $a = \frac{4}{6} = \frac{2}{3}$

⇒ $(a + b) - (a - b) = \frac{7}{6} - \frac{1}{6}$

⇒ $2b = \frac{6}{6}$

⇒ $b = \frac{1}{2}$

∴ we get $a = \frac{2}{3}$ and $b = \frac{1}{2}$

And if we take $a - b = -\frac{1}{6}$ we get as follows

⇒ $(a + b) + (a - b) = \frac{7}{6} + (-\frac{1}{6})$

⇒ $2a = \frac{6}{6}$

⇒ $a = \frac{1}{2}$

⇒ $(a + b) - (a - b) = \frac{7}{6} - (-\frac{1}{6})$

⇒ $2b = \frac{8}{6}$

⇒ $b = \frac{2}{3}$

∴ $a = \frac{1}{2}$ and $b = \frac{2}{3}$

⇒ So, we have $6(x - \frac{1}{2})(x - \frac{2}{3})$

⇒ $(6x - \frac{6}{2})(x - \frac{2}{3})$

⇒ $(6x - 3)(x - \frac{2}{3})$

∴ $(6x - 3)(x - \frac{2}{3})$ are the factors

⇒ p(x) = 0 if (6x – 3) is 0 and $(x - \frac{2}{3})$ is 0

⇒ (6x – 3) = 0

⇒ 6x = 3

⇒ $x = \frac{3}{6} = \frac{1}{2}$

⇒ Substitute, $x = \frac{1}{2}$ in $6x^2 - 7x + 2 = 6(\frac{1}{2})^2 - 7(\frac{1}{2}) + 2 = \frac{3}{2} - \frac{7}{2} + 2 = 0$

⇒ $(x - \frac{2}{3}) = 0$

⇒ $x = \frac{2}{3}$ substitute, $6x^2 - 7x + 2 = 6(\frac{2}{3})^2 - 7(\frac{2}{3}) + 2 = \frac{8}{3} - \frac{14}{3} + 2 = 0$

Hence, $(6x - 3)(x - \frac{2}{3})$ are the first degree factors of the polynomial and $\frac{1}{2}$ and $\frac{2}{3}$ are the solution of the given polynomial $6x^2 - 7x + 2$

2. Question

Find a second degree polynomial p(x) such that p(1) = 0 and p(–2) = 0.

Answer

Given, p(1) = 0, p(– 2) = 0

Need to find a polynomial p(x) of second degree

⇒ since we know that p(1) = 0

∴ if x = 1 is substituted in p(x) then it satisfies the equation

⇒ x – 1 = 0 , and x – 1 is one factor of p(x)

And p(– 2) = 0 is given

⇒ if x = – 2 is substituted in p(x) then it satisfies the equation

⇒ x + 2 = 0, and x + 2 is one factor of p(x)

⇒ since, x – 1 and x + 2 are the factors of p(x), it can be written as follows

⇒ p(x) = (x – 1)(x + 2)

⇒ $p(x) = x^2 - x + 2x - 2$

⇒ $p(x) = x^2 + x - 2$

∴ $x^2 + x - 2$ is the second degree polynomial which satisfies p(1) = 0 and p(– 2) = 0.

3. Question

Find a second degree polynomial p(x) such that $p(1+\sqrt{3}) = 0$ and $p(1-\sqrt{3}) = 0$.

Answer

Given, p(1 + √3) = 0, p(1 – √3) = 0

Need to find a polynomial p(x) of second degree

⇒ since we know that $p(1 + \sqrt{3}) = 0$

∴ if $x = 1 + \sqrt{3}$ is substituted in p(x) then it satisfies the equation

⇒ $x - (1 + \sqrt{3}) = 0$, and $((x - 1) - \sqrt{3})$ is one factor of p(x)

And $p(1 - \sqrt{3}) = 0$ is given

⇒ if $x = 1 - \sqrt{3}$ is substituted in p(x) then it satisfies the equation

⇒ $x - (1 - \sqrt{3}) = 0$, and $((x - 1) + \sqrt{3})$ is one factor of p(x)

⇒ since, $((x - 1) - \sqrt{3})$ and $((x - 1) + \sqrt{3})$ are the factors of p(x), it can be written as follows

⇒ $p(x) = ((x - 1) - \sqrt{3})((x - 1) + \sqrt{3})$

⇒ $p(x) = (x - 1 - \sqrt{3})(x - 1 + \sqrt{3})$

⇒ $p(x) = x^2 - x + \sqrt{3}x - x + 1 - \sqrt{3} - \sqrt{3}x + \sqrt{3} - 3$

⇒ $p(x) = x^2 - 2x - 2$

∴ $x^2 - 2x - 2$ is the second degree polynomial which satisfies $p(1 + \sqrt{3}) = 0$ and $p(1 - \sqrt{3}) = 0$.

4. Question

Find a third degree polynomial p(x) such that $p(1) = 0, p\left(\sqrt{2}\right) = 0 \text{ and } p\left(-\sqrt{2}\right) = 0.$

Answer

Given, $p(1) = 0$, $p(\sqrt{2}) = 0$ and $p(-\sqrt{2}) = 0$

Need to find the third degree polynomial p(x)

⇒ $p(1) = 0$ is given which satisfy p(x)

∴ if $x = 1$ is substituted in p(x) then it satisfies the equation

⇒ $x - 1$ is one factor of p(x)

⇒ $p(\sqrt{2}) = 0$ is given

∴ if $x = \sqrt{2}$ is substituted in p(x) then it satisfies the equation

⇒ $x - \sqrt{2}$ is another factor

⇒ $p(-\sqrt{2}) = 0$ is given

∴ if $x = -\sqrt{2}$ is substituted in p(x) then it satisfies the equation

⇒ $x + \sqrt{2}$ is third factor of the p(x)

⇒ Since, $(x - 1)(x - \sqrt{2})(x + \sqrt{2})$ are the factors of the third degree polynomials

∴ $p(x) = (x - 1)(x - \sqrt{2})(x + \sqrt{2})$

⇒ $p(x) = (x^2 - x - \sqrt{2}x + \sqrt{2})(x + \sqrt{2})$

⇒ $p(x) = (x^3 + \sqrt{2}x^2 - x^2 - \sqrt{2}x - \sqrt{2}x^2 - 2x + \sqrt{2}x + 2)$

⇒ $p(x) = (x^3 - x^2 - 2x + 2)$

Hence, $x^3 - x^2 - 2x + 2$ is the third degree polynomial which satisfies

$p(1) = 0$, $p(\sqrt{2}) = 0$ and $p(-\sqrt{2}) = 0$

5. Question

Prove that the polynomial $x^2 + x + 1$ cannot be written as a product of first degree polynomials.

Answer

Given, a second degree polynomial $x^2 + x + 1$

Need to prove the given equation cannot be written as product of first degree polynomial

⇒ we know that a polynomial equation of degree 2, $x^2 + (a + b)x + ab$ can be written as $(x + a)(x + b)$

⇒ Here, $x^2 + x + 1$ is written as $x^2 + (a + b)x + ab$

⇒ coefficient on either side are equal, we get

⇒ $a + b = 1$ and $ab = 1$

⇒ We need to find the values of a, b such that it satisfies the given equation to get the factors of first degree polynomial

⇒ Since, $a + b = 1$ and $ab = 1$ it is not possible to find out the values of a, b which satisfy the equation $x^2 + x + 1$

Hence, $x^2 + x + 1$ cannot be splited into factors of first degree polynomial

Questions Pg-245

1 A. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

$x – 1, x^3 + 4x^2 – 3x – 6$

Answer

Given, a pair of polynomial as $x – 1, x^3 + 4x^2 – 3x – 6$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check $x – 1$ is a factor of $x^3 + 4x^2 – 3x – 6$ we must substitute $x = 1$ in the second polynomial, we get as follows

⇒ $1 + 4 – 3 – 6 = – 4 \neq$ not equal to 0

∴ $x – 1$ is not a factor of $x^3 + 4x^2 – 3x – 6$

⇒ To find the remainder by using divide second polynomial by first polynomial

⇒ so, we can subtract a number from the second polynomial to get the remainder

∴ $x^3 + 4x^2 – 3x – 6 = (x – 1)q(x) + c$

⇒ $x^3 + 4x^2 – 3x – 6 –c = (x – 1)q(x)$

⇒ $c = ((x^3 + 4x^2 – 3x – 6) – (x – 1)) \times q(x)$

⇒ Now, substitute $x = 1$ in the above equation we get

⇒ $c = (1 + 4 – 3 – 6 – 1 + 1) \times q(1)$

∴ $c = – 4$

– 4 is the remainder

1 B. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

$x + 1, x^3 + 4x^2 – 3x – 6$

Answer

Given, a pair of polynomial as $x + 1, x^3 + 4x^2 – 3x – 6$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check x + 1 is a factor of $x^3 + 4x^2 – 3x – 6$ we must substitute x = – 1 in the second polynomial, we get as follows

⇒ – 1 + 4 + 3 – 6 = 0

∴ x + 1 is a factor

1 C. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

x – 2, $x^3 + 3x^2 – 4x – 12$

Answer

Given, a pair of polynomial as x – 2, $x^3 + 3x^2 – 4x – 12$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check x – 2 is a factor of $x^3 + 3x^2 – 4x – 12$ we must substitute x = 2 in the second polynomial, we get as follows

⇒ $2^3 + 3(2)^2 – 4(2) – 12 = 8 + 12 – 8 – 12 = 0$

∴ x – 2 is a factor

1 D. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

x + 2, $x^3 + 3x^2 – 4x – 12$

Answer

Given, a pair of polynomial as x + 2, $x^3 + 3x^2 – 4x – 12$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check x + 2 is a factor of $x^3 + 3x^2 – 4x – 12$ we must substitute x = – 2 in the second polynomial, we get as follows

⇒ $(– 2)^3 + 3(– 2)^2 – 4(– 2) – 12 = – 8 + 12 + 8 – 12 = 0$

∴ x + 2 is a factor

1 E. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

2x – 1, $3x^3 – x^2 – 8x + 6$

Answer

Given, a pair of polynomial as 2x – 1, $3x^3 – x^2 – 8x + 6$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check 2x – 1 is a factor of $3x^3 – x^2 – 8x + 6$ we must substitute $x = \frac{1}{2}$ in the second polynomial, we get as follows

⇒ $3(\frac{1}{2})^3 – (\frac{1}{2})^2 – 8(\frac{1}{2}) + 6 = \frac{3}{8} - \frac{1}{4} - \frac{8}{2} + 6 = \frac{3}{8} - \frac{1}{4} – 4 + 6 = \frac{1}{8} + 2 = \frac{17}{8} \neq 0$

∴ 2x – 1 is not a factor

⇒ To find the remainder divide second polynomial by first polynomial

⇒ so, we can subtract a number from the second polynomial to get the remainder

∴ $3x^3 - x^2 - 8x + 6 = (2x - 1)q(x) + c$

⇒ $3x^3 - x^2 - 8x + 6 - c = (2x - 1)q(x)$

⇒ $c = ((3x^3 - x^2 - 8x + 6) - (2x - 1)) \times q(x)$

⇒ Now, substitute $x = \frac{1}{2}$ in the above equation we get

⇒ $c = (3(\frac{1}{2})^3 - (\frac{1}{2})^2 - 8(\frac{1}{2}) + 6 - 2(\frac{1}{2}) + 1) \times q(1)$

$= \frac{3}{8} - \frac{1}{4} - 4 + 6$

∴ $c = \frac{17}{8}$

$\frac{17}{8}$ is the remainder

1 F. Question

For each pair of polynomial is below, check whether the first is a factor of the second. If not a factor, find the remainder on dividing the second by the first.

$3x - 1, 3x^3 - 10x^2 + 9x - 2$

Answer

Given, a pair of polynomial as $3x - 1, 3x^3 - 10x^2 + 9x - 2$

Need to find out the first polynomial is factor of second and if not a factor need to find the remainder

⇒ To check $3x - 1$ is a factor of $3x^3 - 10x^2 + 9x - 2$ we must substitute $x = \frac{1}{3}$ in the second polynomial, we get as follows

⇒ $3(\frac{1}{3})^3 - 10(\frac{1}{3})^2 + 9(\frac{1}{3}) - 2 = \frac{3}{27} - \frac{10}{9} - \frac{9}{3} - 2$

$= \frac{1}{9} - \frac{10}{9} - 3 - 2$

$= -\frac{1}{9} - 5$

$= -\frac{46}{9} \neq 0$

∴ $3x - 1$ is not a factor

⇒ To find the remainder divide second polynomial by first polynomial

⇒ so, we can subtract a number from the second polynomial to get the remainder

∴ $3x^3 - 10x^2 + 9x - 2 = (3x - 1)q(x) + c$

⇒ $3x^3 - 10x^2 + 9x - 2 - c = (3x - 1)q(x)$

⇒ $c = ((3x^3 - 10x^2 + 9x - 2) - (3x - 1)) \times q(x)$

⇒ Now, substitute $x = \frac{1}{3}$ in the above equation we get

⇒ $c = (3\left(\frac{1}{3}\right)^3 - 10\left(\frac{1}{3}\right)^2 + 9(\frac{1}{3}) - 2 - 3(\frac{1}{3}) + 1) \times q(1)$

$= \frac{3}{27} - \frac{10}{9} - \frac{9}{3} - 2 - 0$

∴ $c = -\frac{46}{9}$

$-\frac{46}{9}$ is the remainder

2 A. Question

For each pair of polynomials below, find the quotient and remainder on dividing the first by the second.

$x^3 - 1$, $x - 1$

Answer

Given, $x^3 - 1$, $x - 1$ as pair of polynomials

Need to find the quotient and remainder

⇒ To find the quotient and remainder the given equation can be written as $p(x) = (x - a)q(x) + b$

⇒ since, the polynomial is of third degree we can write the q(x) as $x^2 + ax + b$

$\therefore p(x) = (x - a)(x^2 + ax + b) + c$

$\Rightarrow x^3 - 1 = (x - 1)(x^2 + ax + b) + c$

$\Rightarrow x^3 - 1 = (x^3 - x^2 + ax^2 - ax + bx - b) + c$

$\Rightarrow x^3 - 1 = x^3 + (a-1)x^2 + (b - a)x + (c - b)$

$\therefore a - 1 = 0, b - a = 0, c - b = -1$

$\Rightarrow a = 1, b = 1$

$\Rightarrow c = b - 1 = 0$

Quotient = $x^2 + ax + b = x^2 + x + 1$

Remainder = 0

2 B. Question

For each pair of polynomials below, find the quotient and remainder on dividing the first by the second.

$x^3 - 1$, $x + 1$

Answer

Given, $x^3 - 1$, $x - 1$ as pair of polynomials

Need to find the quotient and remainder

⇒ To find the quotient and remainder the given equation can be written as $p(x) = (x - a)q(x) + b$

⇒ Since, the polynomial is of third degree we can write the q(x) as $x^2 + ax + b$

$\therefore p(x) = (x - a)(x^2 + ax + b) + c$

$\Rightarrow x^3 - 1 = (x + 1)(x^2 + ax + b) + c$

$\Rightarrow x^3 - 1 = (x^3 + x^2 + ax^2 + ax + bx + b) + c$

$\Rightarrow x^3 - 1 = x^3 + (a + 1)x^2 + (a + b)x + (c + b)$

$\therefore a + 1 = 0, a + b = 0, c + b = -1$

$\Rightarrow a = -1$

$\Rightarrow a + b = 0$

$\Rightarrow -1 + b = 0$

$\Rightarrow b = 1$

$\Rightarrow c + b = -1$

⇒ $c + 1 = – 1$

⇒ $c = – 2$

Quotient = $x^2 + ax + b = x^2 – x + 1$

Remainder = – 2

2 C. Question

For each pair of polynomials below, find the quotient and remainder on dividing the first by the second.

$x^3 + 1, x – 1$

Answer

Given, $x^3 – 1, x – 1$ as pair of polynomials

Need to find the quotient and remainder

⇒ To find the quotient and remainder the given equation can be written as $p(x) = (x – a)q(x) + b$

⇒ Since, the polynomial is of third degree we can write the q(x) as $x^2 + ax + b$

∴ $p(x) = (x – a)(x^2 + ax + b) + c$

⇒ $x^3 + 1 = (x – 1)(x^2 + ax + b) + c$

⇒ $x^3 + 1 = (x^3 –x^2 + ax^2 – ax + bx – b) + c$

⇒ $x^3 + 1 = x^3 + (a–1)x^2 + (b – a)x + (c – b)$

∴ $a – 1 = 0, b – a = 0, c – b = 1$

⇒ $a = 1, b = 1$

⇒ $c – b = 1$

⇒ $c = 2$

Quotient = $x^2 + ax + b = x^2 + x + 1$

Remainder = 2

2 D. Question

For each pair of polynomials below, find the quotient and remainder on dividing the first by the second.

$x^3 + 1, x + 1$

Answer

Given, $x^3 – 1, x – 1$ as pair of polynomials

Need to find the quotient and remainder

⇒ To find the quotient and remainder the given equation can be written as $p(x) = (x – a)q(x) + b$

⇒ Since, the polynomial is of third degree we can write the q(x) as $x^2 + ax + b$

∴ $p(x) = (x – a)(x^2 + ax + b) + c$

⇒ $x^3 + 1 = (x + 1)(x^2 + ax + b) + c$

⇒ $x^3 + 1 = (x^3 + x^2 + ax^2 + ax + bx + b) + c$

⇒ $x^3 + 1 = x^3 + (a + 1)x^2 + (b + a)x + (c + b)$

∴ $a + 1 = 0, b + a = 0, c + b = 1$

⇒ $a = – 1, b = 1$

⇒ c + b = 1

⇒ c = 0

Quotient = $x^2 + ax + b = x^2 - x + 1$

Remainder = 0

3 A. Question

By adding a number to p(x) = $x^3 + x^2 + x$, a new polynomial q(x) is to be formed.

What number should be added, so that x – 1 is a factor of q(x)?

Answer

Given p(x) = $x^3 + x^2 + x$

Let the number to be added be “k”.

Then, the new polynomial q(x) = $x^3 + x^2 + x + k$

Now, (x – 1) is a factor of $x^3 + x^2 + x + k$.

i.e. x = 1 is the root of the polynomial.

Then, put the polynomial to zero we get,

$x^3 + x^2 + x + k = 0$

⇒ $(1)^3 + (1)^2 + 1 + k = 0$

⇒ 1 + 1+1 + k = 0

⇒ k = – 3

Hence, “ – 3” should be added to the polynomial such that (x – 1) is a factor of q(x).

3 B. Question

By adding a number to p(x) = $x^3 + x^2 + x$, a new polynomial q(x) is to be formed.

What number should be added, so that x + 1 is a factor of q(x)?

Answer

Given p(x) = $x^3 + x^2 + x$

Let the number to be added be “k”.

Then, the new polynomial q(x) = $x^3 + x^2 + x + k$

Now, (x+1) is a factor of $x^3 + x^2 + x + k$.

i.e. x = – 1 is the root of the polynomial.

Then, put the polynomial to zero we get,

$x^3 + x^2 + x + k = 0$

⇒ $(-1)^3 + (-1)^2 + (-1) + k = 0$

⇒ – 1 + 1 – 1 + k = 0

⇒ k = – 1

Hence, “ – 1” should be added to the polynomial such that (x – 1) is a factor of q(x).

4 A. Question

In each pair of polynomials below find what kind of natural number n must be, so that the first is a factor of the second.

$x - 1, x^n - 1$

Answer

Given, $x - 1, x^n - 1$ pair of polynomials

Need to find out n such as first polynomial is factor of second

⇒ To check $x - 1$ is factor of $x^n - 1$ we must get $x^n - 1 = 0$ when substituted x with 1 from the first polynomial

⇒ since, when x is substituted with 1 it will satisfy irrespective of n in the given polynomial

Consider n as 1 then the polynomial equation itself wiil be $x - 1$ and $x - 1$ will be the factor

Hence, n is 1

4 B. Question

In each pair of polynomials below find what kind of natural number n must be, so that the first is a factor of the second.

$x - 1, x^n + 1$

Answer

Given, $x - 1, x^n + 1$ pair of polynomials

Need to find out n such as first polynomial is factor of second

⇒ To check $x - 1$ is factor of $x^n + 1$ we must get $x^n + 1 = 0$ when substituted x with 1 from the first polynomial

⇒ Consider n as 1 then the polynomial equation wiil be $x + 1$ which is not equal to zero and $x - 1$ will not be the factor.

⇒ For any n value $x - 1$ cannot be a factor

4 C. Question

In each pair of polynomials below find what kind of natural number n must be, so that the first is a factor of the second.

$x + 1, x^n - 1$

Answer

Given, $x + 1, x^n - 1$ pair of polynomials

Need to find out n such as first polynomial is factor of second

⇒ To check $x + 1$ is factor of $x^n - 1$ we must get $x^n - 1 = 0$ when substituted x with -1 from the first polynomial

⇒ Consider n as 1 then the polynomial equation will be $x - 1$ which is not equal to zero and $x + 1$ will not be the factor.

⇒ Then consider n as 2 then $x^2 - 1$ be the polynomial equation substitute $x = -1$ in the equation

We get, $(-1)^2 - 1 = 0$

∴ $x + 1$ is the factor of $x^n - 1$

Hence, $n = 2$

4 D. Question

In each pair of polynomials below find what kind of natural number n must be, so that the first is a factor of the second.

$x + 1, x^n + 1$

Answer

Given, $x + 1$, $x^n + 1$ pair of polynomials

Need to find out n such as first polynomial is factor of second

⇒ To check $x + 1$ is factor of $x^n + 1$ we must get $x^n + 1 = 0$ when substituted x with – 1 from the first polynomial

⇒ Consider n as 1 then the polynomial equation will be $x + 1$ which is equal to zero and $x + 1$ will is the factor.

We get, $(– 1) + 1 = 0$

∴ $x + 1$ is the factor of $x^n + 1$

Hence, $n = 1$

4 E. Question

In each pair of polynomials below find what kind of natural number n must be, so that the first is a factor of the second.

$x^2 – 1$, $x^n – 1$

Answer

Given, $x^2 – 1$, $x^n – 1$ pair of polynomials

Need to find out n such as first polynomial is factor of second

⇒ To check $x^2 – 1$ is factor of $x^n – 1$ we must get $x^n – 1 = 0$ when substituted with x values from the first polynomial

⇒ Here, $x^2 – 1$ so, $x = \sqrt{1} = 1$

⇒ Consider n as 1 then the polynomial equation will be $x^1 – 1$ which is equal to zero and $x^2 – 1$ is the factor.

We get, $1 – 1 = 0$

∴ $x^2 – 1$ is the factor of $x^n – 1$

Hence, $n = 1$

5. Question

Prove that if $x^2 – 1$ is a factor of $ax^3 + bx^3 + cx + d$, then $a = –c$ and $b = –d$.

Answer

Given, $ax^3 + bx^2 + cx + d$

Need to show $a = – c$ and $b = – d$ if $x^2 – 1$ is a factor

⇒ consider, $x^2 – 1$ is a factor of $ax^3 + bx^2 + cx + d$

⇒ then $x = + 1$, $x = – 1$

⇒ substitute x value in the equation $ax^3 + bx^2 + cx + d$

We get as follows

⇒ for $x = 1$ we get $a(1)^3 + b(1)^2 + c(1) + d$

$= a + b + c + d$ ....eq(1)

⇒ for $x = – 1$ we get $a(– 1)^3 + b(– 1)^2 + c(– 1) + d$

$= – a + b – c + d$ ....eq(2)

⇒ Solving the two equations we get

⇒ (a + b + c + d) + (– a + b – c + d) = 0

⇒ b + d = 0

∴ b = – d

And if (a + b + c + d) – (– a + b – c + d) = 0

⇒ a + c = 0

∴ a = – c

6. Question

What first degree polynomial added to $2x^3 - 3x^2 + 5x + 1$ gives a multiple of $x^2 - 1$?

Answer

Given, $2x^3 - 3x^2 + 5x + 1$

Need to find the first degree polynomial if added to the equation gets the multiple of $x^2 - 1$

⇒ $2x^3 - 3x^2 + 5x + 1$ can be written as $2(x^3 - \frac{3}{2}x^2 + \frac{5}{2}x + \frac{1}{2})$

⇒ $x^3 - \frac{3}{2}x^2 + \frac{5}{2}x + \frac{1}{2} = (x^2 - 1)(x - a) + b$

$= x^3 - ax^2 - x + a + b$

∴ $a = \frac{3}{2}$ and $a + b = \frac{1}{2}$

⇒ $b = -1$