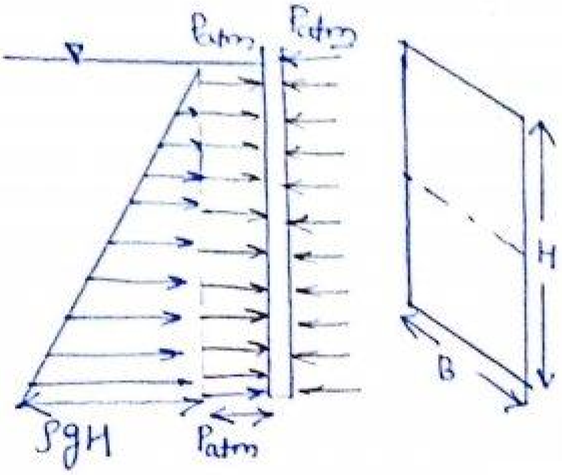
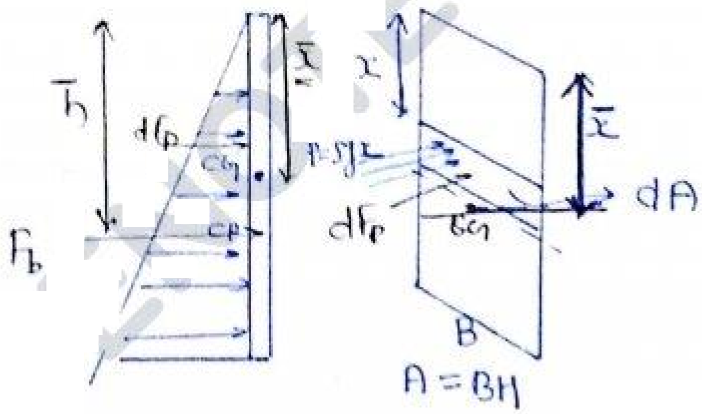
Hydrostatic force :- (Total pressure force)

This concept is used for the design of hydraulic structure.
like dams, hydraullic gate, ships, ~~summin etc~~
summariens etc.





$$F_p = P \times A$$

$$dF_p = P\,dA$$

$$\int dF_p = \rho g \int x\,dA$$

$$F_p = \rho g \int x\,dA \qquad A\bar{x} = \int x\,dA$$

$$\boxed{F_p = wA\bar{x}}$$

$\bar{x}$ = Vertical depth of flat surface from free surfact

Hydrostatic force = the net force exerted by the fluid on the body is know as total pressure force. for flat surface in any orientation

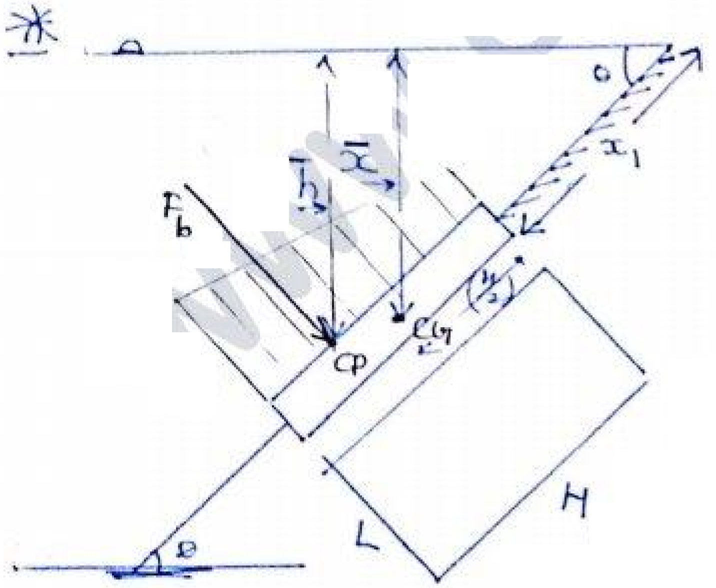
$$\boxed{F_p = wA\bar{x}}$$

Centre of pressure (CP):- The point of application of total pressure force is known as centre of pressure. It is measured in verticle depth from free surface.

$$\boxed{\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}}$$

I_{GG} = MOI of body about axis ||el to freesurface

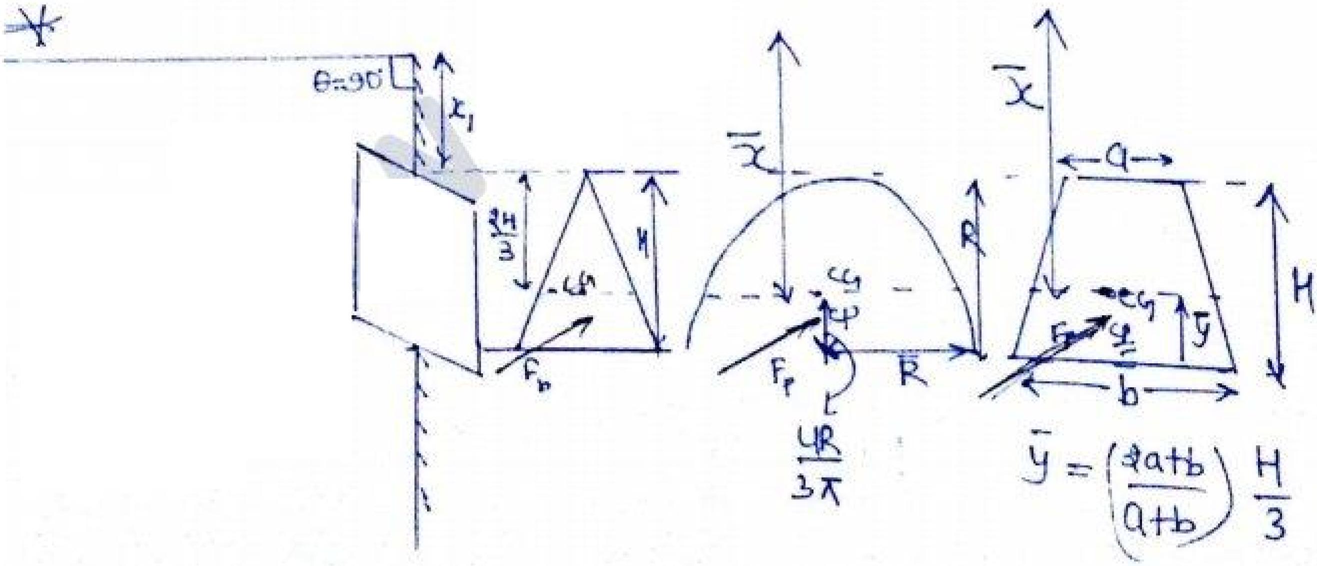
θ is the angle of flat surface from the free surface / Horizontal surface.



$F_p = wA\bar{x}$

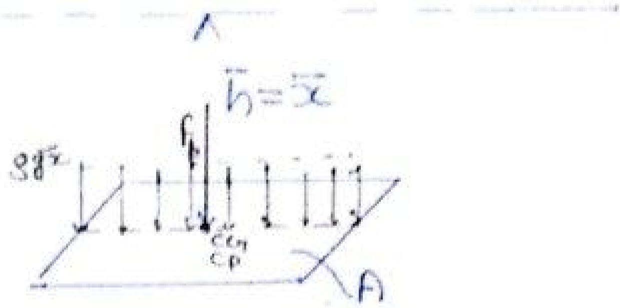
$$\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$I_{GG} = \frac{LH^3}{12}$$



$$\bar{y} = \left(\frac{2a+b}{a+b}\right)\frac{H}{3}$$

Horizontal surface



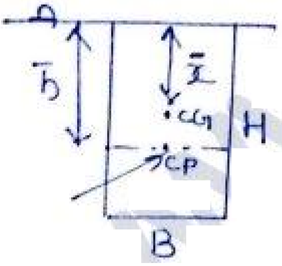
$\theta = 0, \sin\theta = 0$

$\bar{h} = \bar{x}$

$$F_p = p \times A = \rho g \bar{x} A$$

$$\boxed{F_p = w A \bar{x}}$$

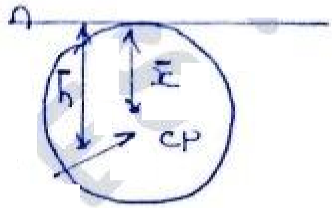
Note:-

* 

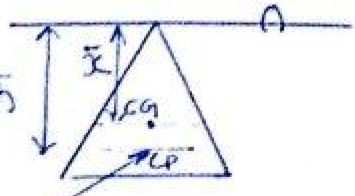
$$\bar{h} = \frac{I_{CG}}{A\bar{x}} + \bar{x}$$

$$\bar{h} = \frac{BH^3/12}{BH \times H/2} + \frac{H}{2}$$

$$\boxed{\bar{h} = \frac{2H}{3}}$$

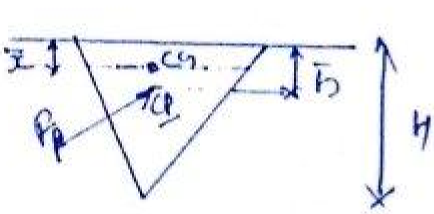
* 

$$\boxed{\bar{h} = \frac{5D}{8}}$$

* 

$$\bar{x} = \frac{2}{3}H$$

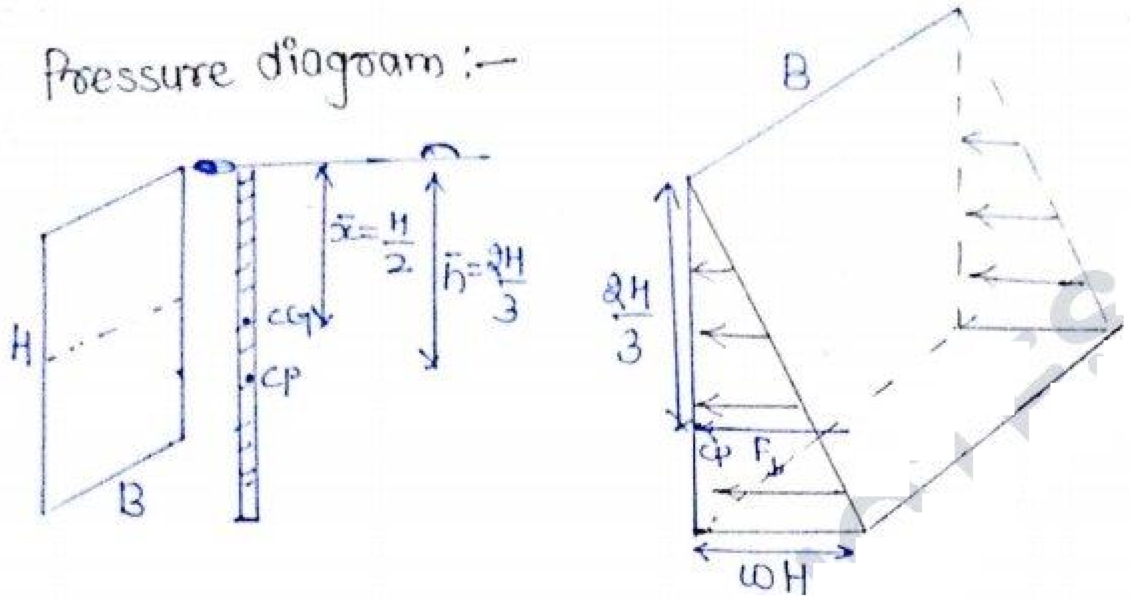
$$\boxed{\bar{h} = \frac{3}{4}H}$$

* 

$$\bar{x} = \frac{H}{3}$$

$$\boxed{\bar{h} = H/2}$$

Pressure diagram :-

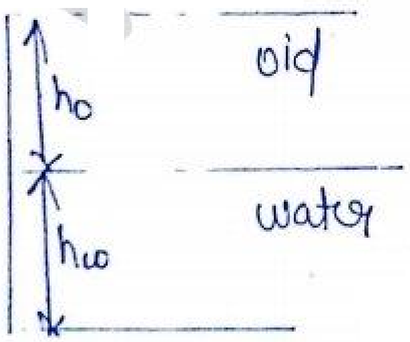


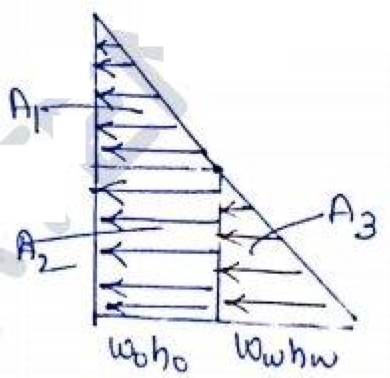
$$\text{Vol. of PD} = \frac{1}{2}\left[\omega H \times H\right] \times B$$

$$= \omega (H \times B) \times \frac{H}{2}$$

$$= \omega . A . \bar{x}$$

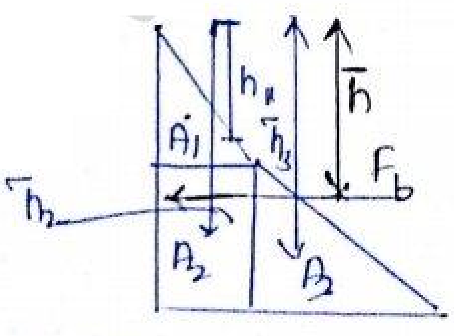
$$\text{Vol. of PD} = F_p = P \times A$$





$$w_w h'_w = h_o w_o$$

$$\text{Vol.} = (A_1 + A_2 + A_3)\, B$$

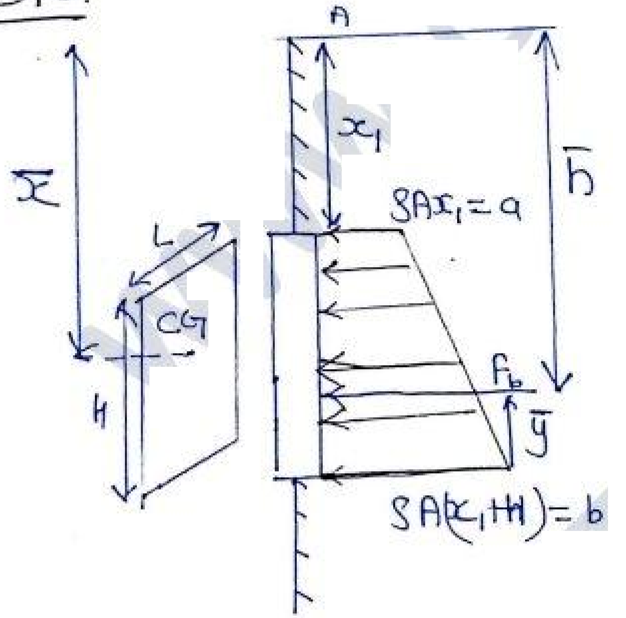


$$\boxed{A\bar{h} = A_1\bar{h}_1 + A_2\bar{h}_2 + A_3\bar{h}_3}$$

Note: → The Volume of pressure diagram is the net hydrostatic force and C.G. of pressure diagram is the ~~pa~~ centre of pressure i.e. point of application of FP.

→ The expression $wA\bar{x}$ is valid for single fluid so for two or more immersed fluids the hydrostatic force is should be obtained by pressure diagram.

Note:

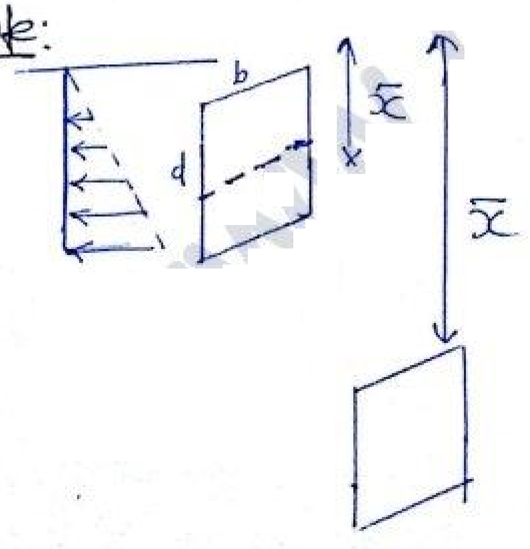


$$F_p = wA\bar{x}$$

$$\bar{y} = \left(\frac{2a+b}{a+b}\right)\frac{H}{3}$$

$$\bar{y} = \left(\frac{2x_1 + (x_1+H)}{x_1 + (x_1+H)}\right)\frac{H}{3}$$

Note:

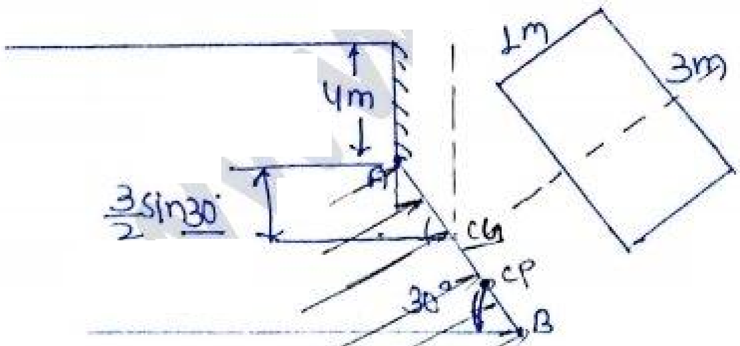


* On vertical plane surface the pressure diagram become uniform as depth of immerse increase i.e. C.P. shifted towards C.G of body but it will never coinside.

$$\bar{h} = \frac{I_{G}}{A\bar{x}} + \bar{x}$$

$$x \uparrow \Rightarrow \bar{h} \simeq \bar{x} \text{ (Never Coinsides)}$$

Que Find hydrostatic force on Gate AB and its point of application.



$$\bar{h} = \frac{I_{G}\sin^2\theta}{A\bar{x}} + \bar{x}$$

$$\theta = 30°$$

$$F_p = wA\bar{x}$$

$$= \rho g\left(4 + \frac{3}{2}\sin 30°\right)(3\times 1)$$

$$F_p = 9810\left(\frac{4}{1} + \frac{3}{4}\right)(3\times 1)$$

$$F_p = 139.7925 \text{ KN}$$

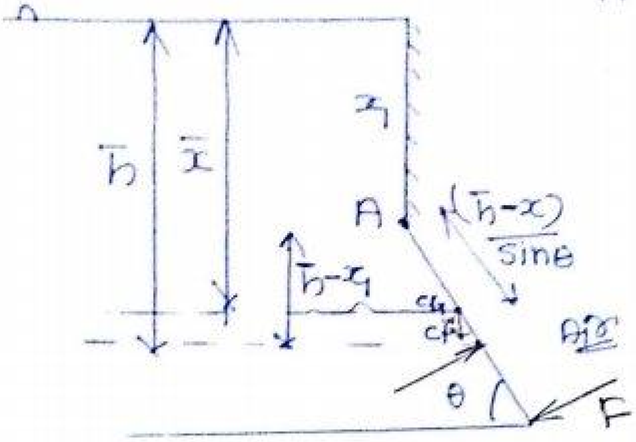
$$\bar{h} = \frac{\frac{1(3)^3}{12}\left(\frac{1}{2}\right)^2}{3\times\frac{19}{4}} + \frac{19}{4}$$

$$\bar{h} = \frac{27\times 4}{4\times 12\times 3\times 19} + \frac{19}{4}$$

$$\cancel{\bar{h} = 4.86} \text{ m}$$

$$\bar{h} = 4.789 \text{ m}$$

Note:

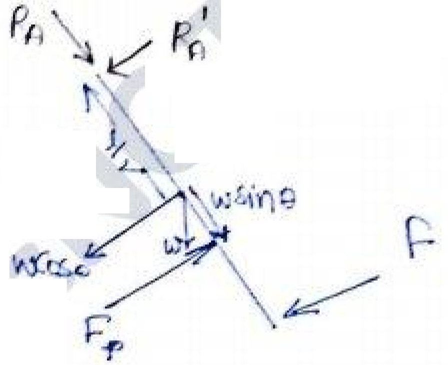


Force require to stop Gate by Air

if W weight of Gate

$F_p = WA\bar{x}$

$\Sigma M_A = 0$



$$-w\cos\theta \times \frac{l}{2} + F_p \cdot \frac{\bar{h}-x_1}{\sin\theta} - F \times l = 0$$

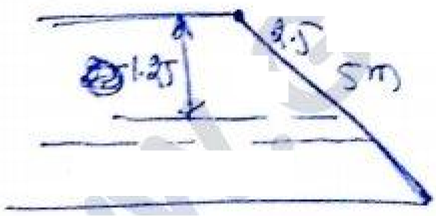
$$F = ?$$

if $w = 0$ (not Given)

$$F_p \frac{\bar{h}-x_1}{\sin\theta} - F \times l = 0$$

$$F = ?$$

Q.26



$$w \times \frac{l}{2}\cos\theta = F_p \frac{\bar{h}-x_1}{\sin\theta}$$

$$w \times \frac{5}{2}\cos 30° = 9810 \times \frac{10}{3 \times \sin 30°} \times 5$$

$$W = \frac{2 \times 2 \times 9810}{5 \times \sqrt{3}} \times \frac{10 \times 2 \times 5 \times 1.25}{3 \times \cancel{\phi}}$$

$\bar{h} = \frac{2 \times 5}{3} \times \frac{1}{2} = \frac{5}{3}$

$\cancel{F_p = \rho g A \bar{x}}$

$\cancel{= 24525 A}$

$F_p = wA\bar{x}$

$F_p = 9810 \times (5 \times 1) \times 2.5 \sin 30°$

$F_p = 61312.5$

$$\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$$

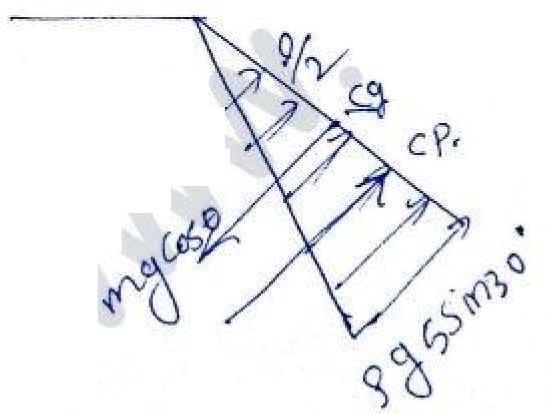
$$\bar{h} = \frac{1 \times (5)^3 \times \frac{1}{4}}{12 \times 5 \times 1 \times 1.25} + 1.25$$

$\bar{h} = 1.66$

$$w \times \frac{l}{2} \cos\theta = F_p \frac{\bar{h}}{\sin 30°}$$

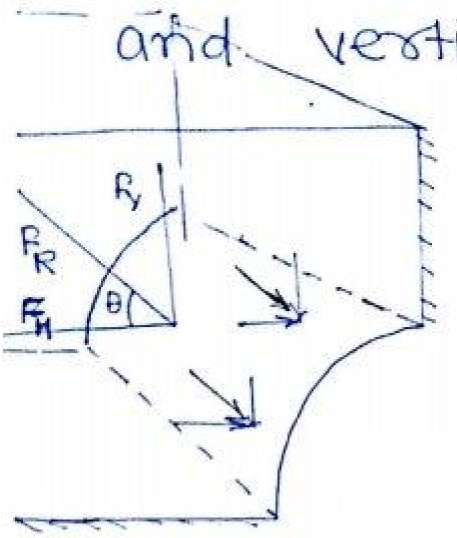
$w \simeq \underline{\underline{9623}}$

or $F_p \times \frac{10}{3} - mg\cos\theta \times 2.5 = 0$



Hydrostatic forces on Curved surface:—

Hydrostatic forces of curved surface are in different orientation so the resultant force can be obtain by horizontal and vertide component.

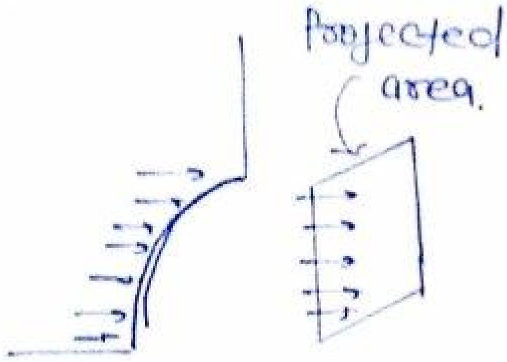


$$F_R = \sqrt{F_H^2 + F_V^2}$$

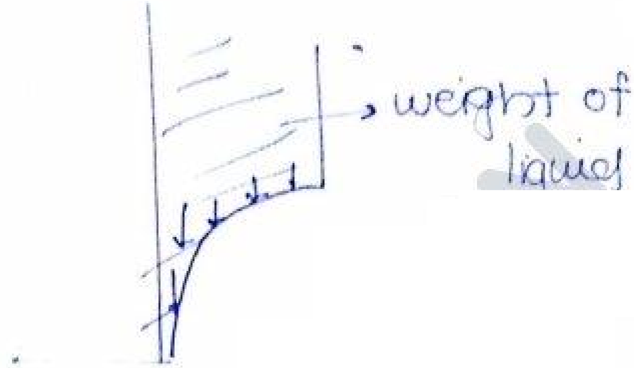
$$\tan\theta = \frac{F_V}{F_H}$$

<u>Horizontal Component</u>:- The hz Component is the net hydrostatic force on projected area of curved surface in vertical plane. It's point of application is centre of pressure of corresponding area.

<u>Vertical Component</u>: It is the weight of the liquid above the curved surface upto the free surface. It point of application is C.G. of Corresponding Volume

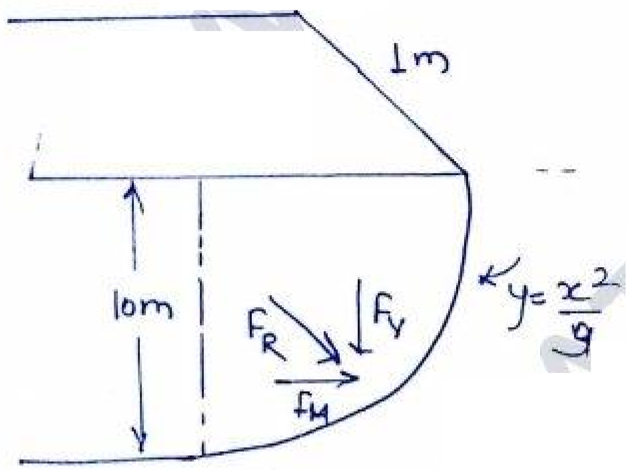


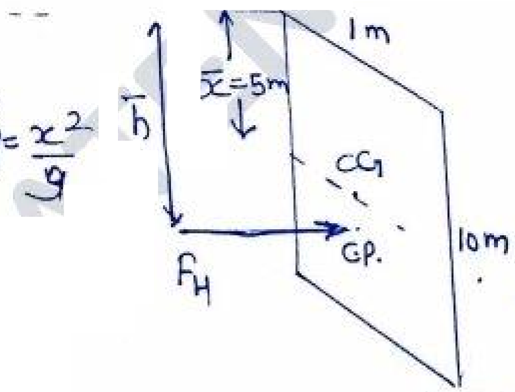
$F_{Hz} = F_p = \cdot w A \bar{x}$



$F_V =$ weight of liquid above curve surface.

Ques Find the total hydrostatic force on the dam structure which is shaped according to the relation $y = \frac{x^2}{9}$ having water upto height of 10 and the width of dam is 1 meter.



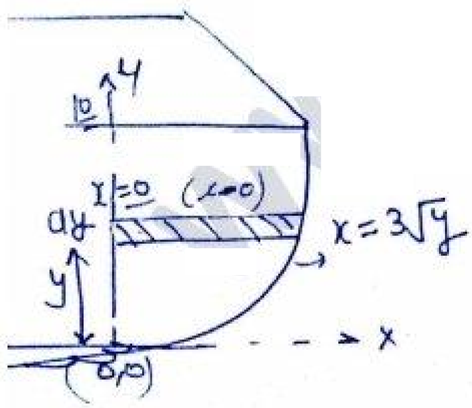


$F_H = w A_P \bar{x}$

$F_H = 9810 \times (10 \times 1) \times 5$

$F_H = 490.5\,kN$

$\bar{h} = \frac{I_{CG}}{A\bar{x}} + \bar{x}$



$F_V = wt.$

$= w \times Vol$

$= w \times A \times 1$

$= 9810 \times 2(10)^{3/2}$

$F_V = 620.43\,KN$

$A = \int dA$

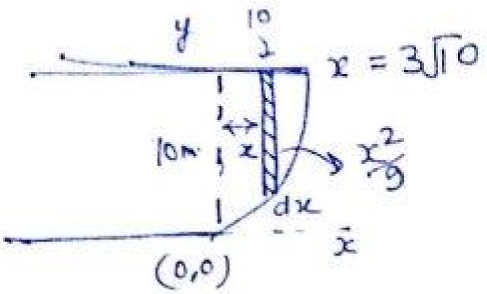
$= \int_0^{10} (x-0)\, dy$

$A = \int_0^{10} 3\sqrt{y}\, dy$

$A = 2(10)^{3/2}$

$$F_R = \sqrt{F_V^2 + F_H^2}$$

$$F_R = 790.9 \text{ kN}$$



$$A\bar{x} = \int dA \cdot x$$

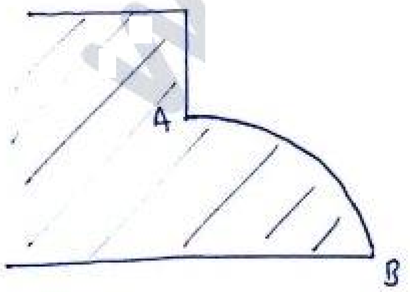
$$A\bar{x} = \int_0^{3\sqrt{10}} \frac{x^2}{9} \cdot x \, dx$$

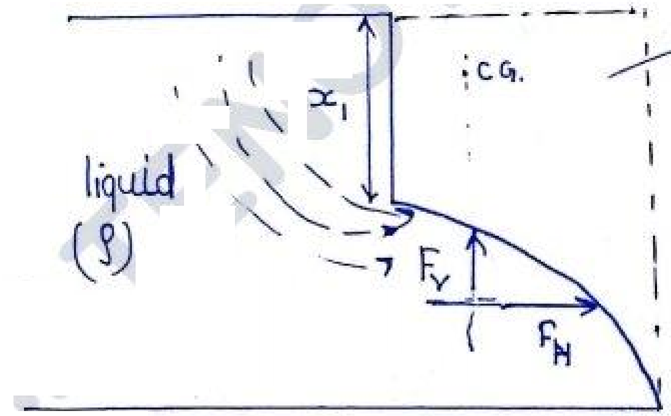
$$2 \times (10)^{3/2} \bar{x} = \frac{1}{9}\left[\frac{x^4}{4}\right]_0^{3\sqrt{10}}$$

point of application of F_V $\bar{x} = 3.557$ mm

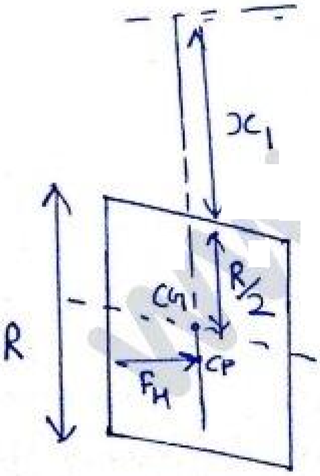
Case 1

* If there is No liquid above the curved surface:-





imaginary liquid upto to free surface



$$F_H = \underline{WA_p\bar{x}}$$

$$\bar{x} = x_1 + R/2$$

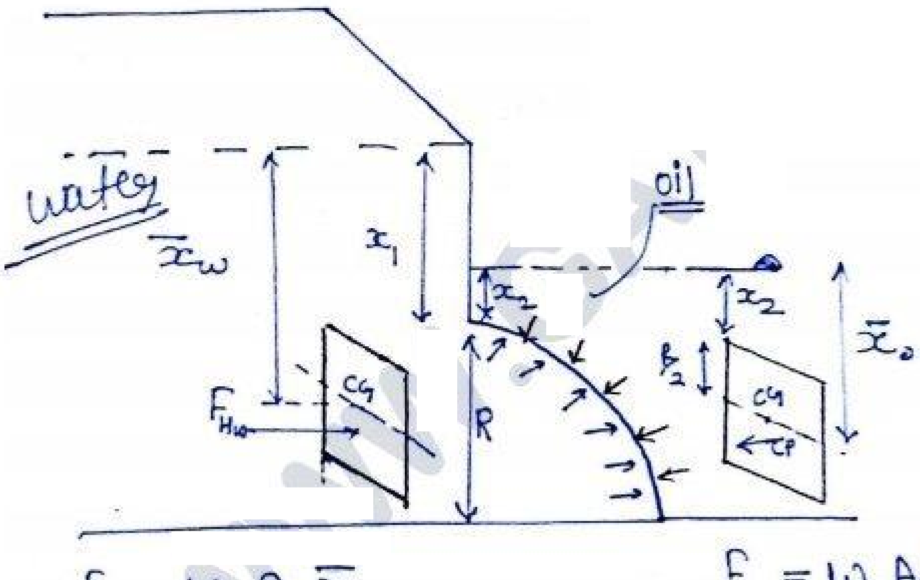
A_p – projected Area

$$F_R = \sqrt{F_H^2 + F_V^2}$$

If there is no liquid above curved surface the verticle component will be the weight an imagianary column of same liquid at same free surface above the curved surface and it's line of Action is about the the C.G. of and imagionary column in upward direction.

eg

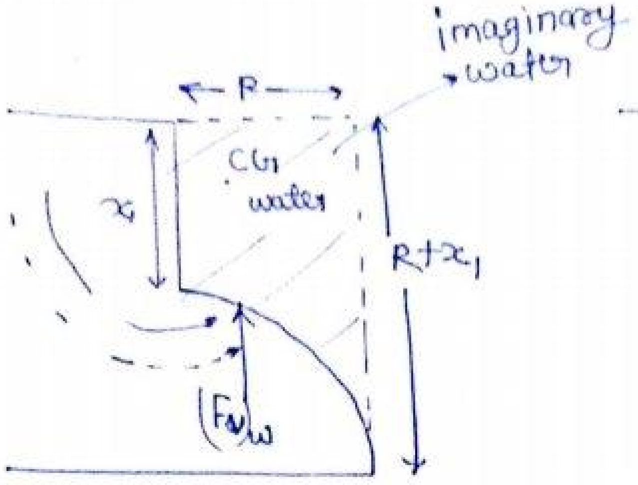
Case-2 fluid on both side of curved surface.

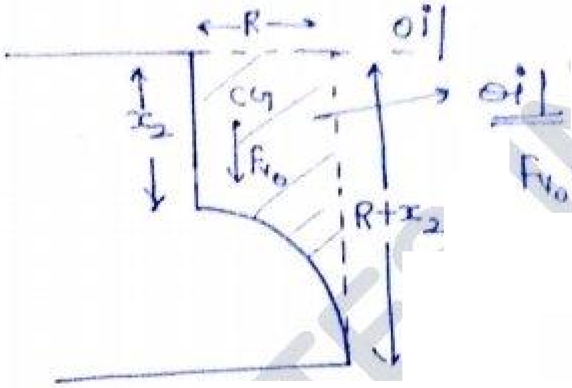


$F_{H_W} = W_w A_p \bar{x}_w$

$F_{H_o} = W_o A_p \bar{x}_o$

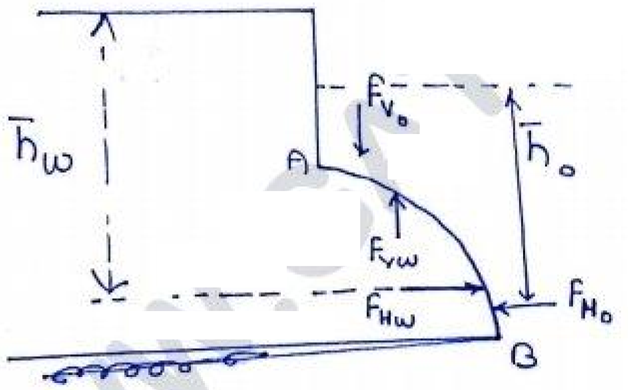
$$\bar{h} = \frac{I_{nG}}{A\bar{x}} + \bar{x}$$





F_{V_0} = wt of oil

*

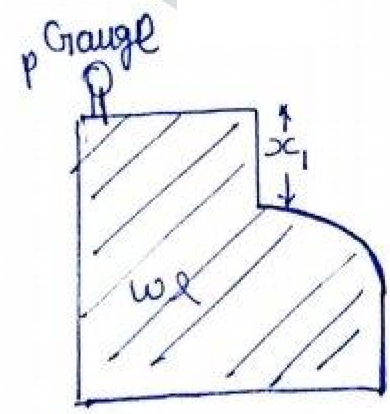


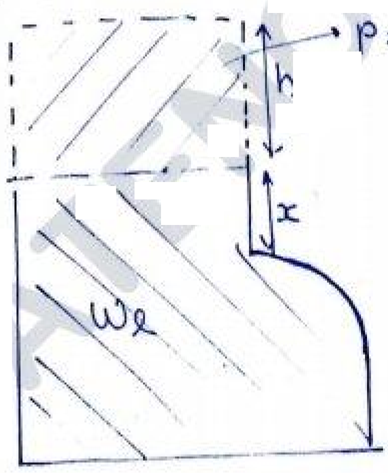
$$F_{V_{net}} = F_{V_W} - F_{V_0}$$

$$F_{H_{net}} = F_{H_W} - F_{H_0}$$

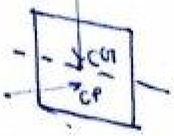
$$F_R = \sqrt{F_{H_{net}}^2 + F_{V_{net}}^2}$$

Case (3) Pressurized fluid in container.





$\bar{x} = h + x_1 + \frac{R}{2}$



$F_H = w A_p \bar{x}$

$$\bar{x} = h + x_1 + R/2$$

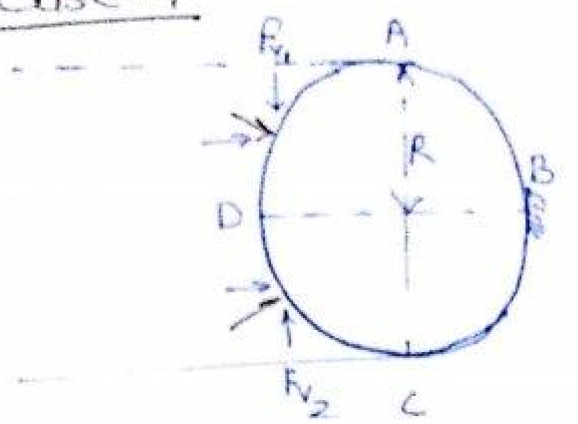
$$h = \frac{P}{w_l} = \frac{P}{\rho g}$$

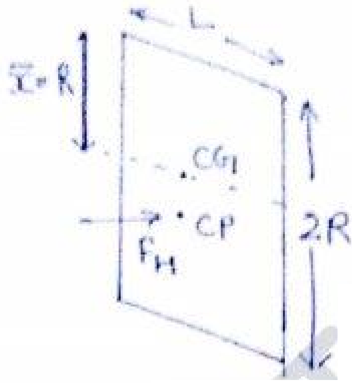
$$F_H = w A_p \bar{x}$$

if gauge pressure is -ve reading (-p)

$$\bar{x} = x_1 - \frac{p}{\rho g} + R/2$$

Case-4

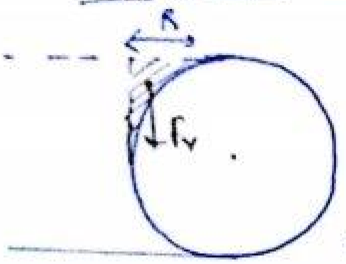




$F_H = w A_p \bar{x}$

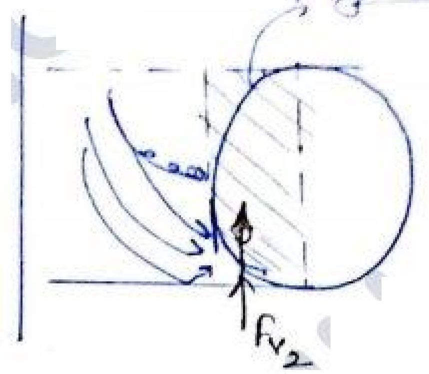
$A_p = (L \times 2R)$

Vertical Component



$F_{V_1} = w \times Vol$

$F_{V_1} = w\left[R^2 - \frac{\pi R^2}{4}\right] \times L\,(\downarrow)$



$\Rightarrow F_{V_2} = w \times Vol.$

$= w\left[R^2 + \frac{\pi R^2}{4}\right] \times L\,(\uparrow)$

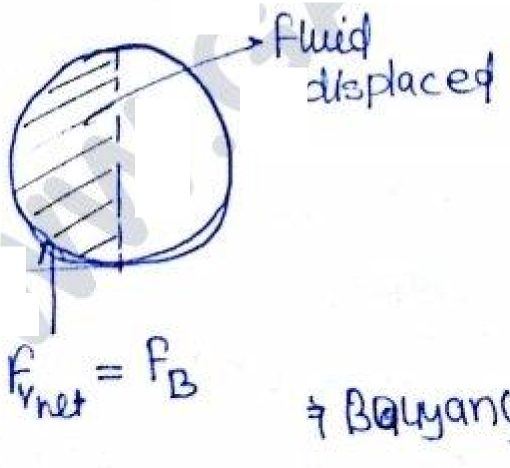
net verticle force.

$F_{net} = F_{V_2}(\uparrow) - F_{V_1}(\downarrow)$

$= w\left[R^2 + \frac{\pi R^2}{4}\right]L - w\left[R^2 - \frac{\pi R^2}{4}\right]L$

$F_{net_V} = w\left[\frac{\pi}{2} R^2 L\right](\uparrow)$

*



$F_{V_{net}} = F_B$

Wt. of fluid displaced

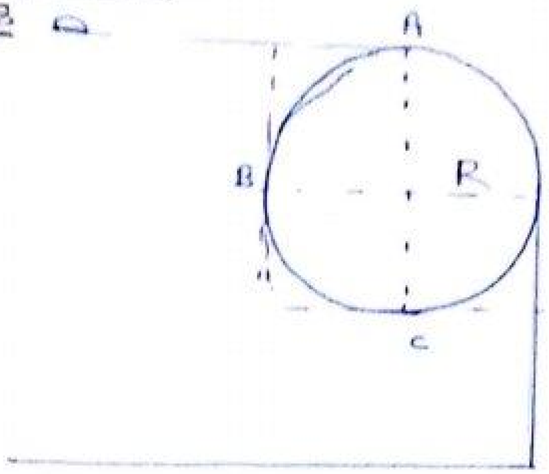
$= \rho g \times V_{fd}$

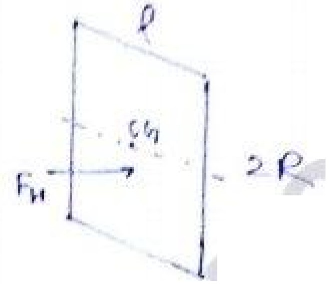
$F_B = \rho g\left(\frac{\pi R^2}{2} \times L\right)$

⇒ Bouyancy force = Net vertical upward hydrostatic force

H.W. Case 5

Ques:



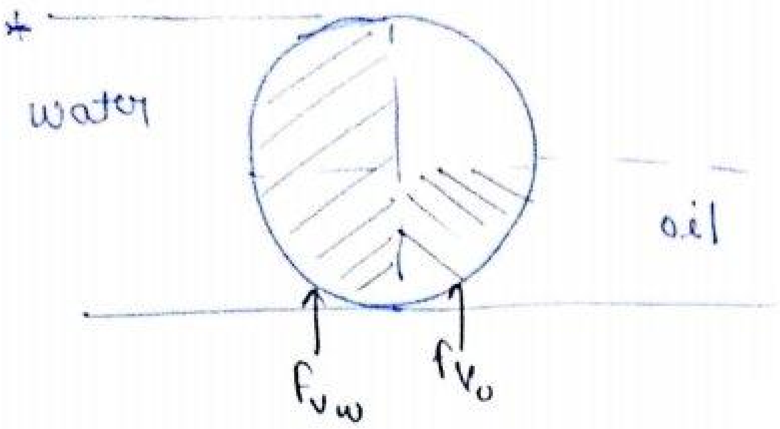


$F_H = w A_p . \bar{x}$

$F_H = \rho g (l \times 2R) \times R = \rho g l (2R^2)$

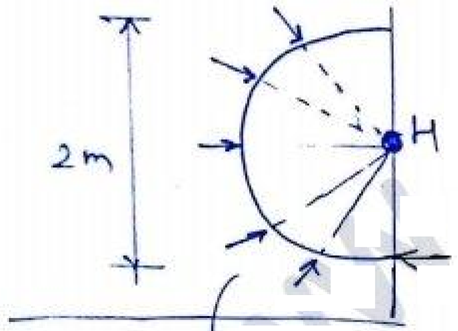
$F_V =$ (wt.) of fluid displaced

$= (\rho g) \frac{3\pi R^2}{4} \times l$



$$f_v = \left(w_o \frac{\pi R^2}{8} + w_o \frac{\pi R^2}{4}\right)$$

Ques



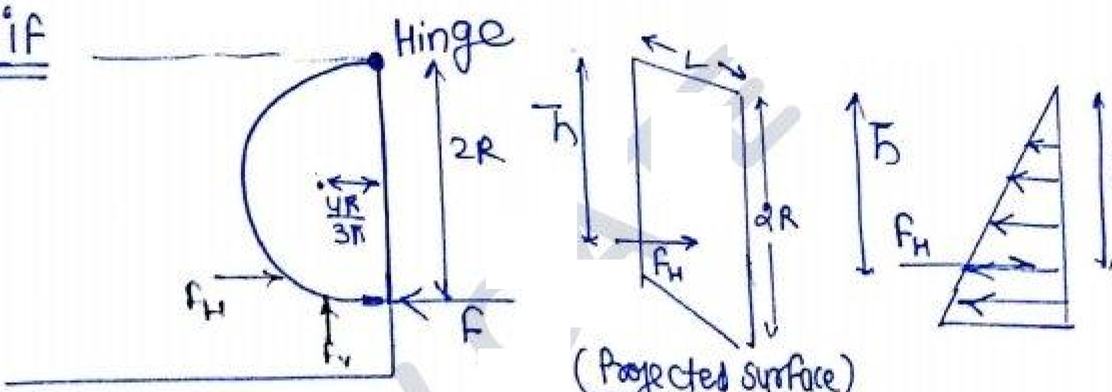
$M_H = 0$

$F \times R = 0$

$F = 0$

Resultant force will pass through centre

if



$$\frac{2}{3}(2R) = \frac{4R}{3} = \bar{h}$$

$$\Sigma_H = 0$$

$$-F \times 2R - F_v \times \frac{4R}{3\pi} + F_H \times \bar{h} = 0$$

T5

H.W