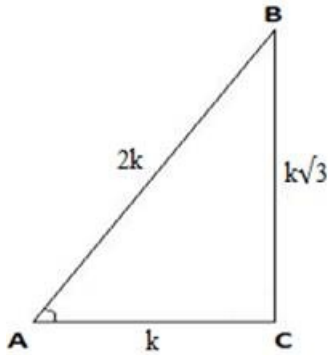


Chapter : 5. TRIGONOMETRIC RATIOS

Exercise : 5

Question: 1

If We have, $\sin\theta = \frac{\sqrt{3}}{2} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$ Let Perpendicular = $\sqrt{3}$ and hypotenuse = $2k$ where 'k' is some integer.



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$(2k)^2 = (k\sqrt{3})^2 + AC^2$$

$$4k^2 = 3k^2 + AC^2$$

$$AC^2 = (4 - 3)k^2$$

$$AC^2 = k^2$$

$\rightarrow AC = k$, for some number k

Hence, the trigonometric ratios for the given θ are:

$$\sin\theta = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = AC/AB = k/(2k) = 1/2$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = (k\sqrt{3})/k = \sqrt{3}$$

$$\cot\theta = 1/\tan\theta = AC/BC = k/(k\sqrt{3}) = 1/\sqrt{3}$$

$$\operatorname{cosec}\theta = 1/\sin\theta = AB/BC = (2k)/(k\sqrt{3}) = 2/\sqrt{3}$$

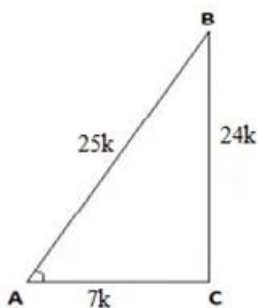
$$\sec\theta = 1/\cos\theta = AB/AC = (2k)/k = 2$$

Question: 2

If $\cos\theta = 7/25$, find

Solution:

We have, $\cos\theta = (7k)/(25k) = \text{base/hypotenuse}$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (25k)^2 = BC^2 + (7k)^2 \text{ (for some value of } k)$$

$$= 625k^2 = BC^2 + 49k^2$$

$$= BC^2 = 576k^2$$

$$= BC^2 = (24k)^2$$

$$\rightarrow BC = 24k$$

Hence, the trigonometric ratios of the given θ are:

$$\sin\theta = BC/AB = (24k)/(25k) = 24/25$$

$$\cos\theta = 7/25$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = 24/7$$

$$\cot\theta = AC/BC = 1/\tan\theta = 7/24$$

$$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = 25/24$$

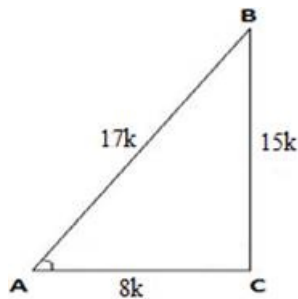
$$\sec\theta = AB/AC = 1/\cos\theta = 25/7$$

Question: 3

If $\tan\theta = 15/8$, find

Solution:

We have, $\tan\theta = 15k/8k = \text{perpendicular/base}$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$AB^2 = (15k)^2 + (8k)^2$$

$$AB^2 = 225k^2 + 64k^2$$

$$AB^2 = 289k^2 = (17k)^2$$

$$\rightarrow AB = 17k$$

Hence, the trigonometric ratios for the given θ are:

$$\sin\theta = BC/AB = (15k)/(17k) = 15/17$$

$$\cos\theta = AC/AB = (8k)/(17k) = 8/17$$

$$\tan\theta = 15/8$$

$$\cot\theta = AC/BC = 1/\tan\theta = 8/15$$

$$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = 17/15$$

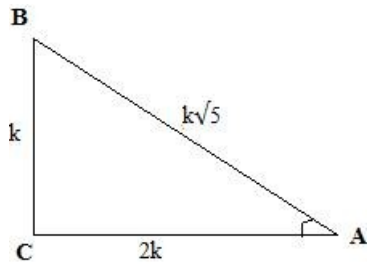
$$\sec\theta = AB/AC = 1/\cos\theta = 17/8$$

Question: 4

If $\cot\theta = 2$, find

Solution:

We have, $\cot\theta = 2k/k = \text{base/perpendicular}$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$AB^2 = (k)^2 + (2k)^2$$

$$AB^2 = k^2 + 4k^2$$

$$AB^2 = 5k^2 = (k\sqrt{5})^2$$

$$\rightarrow AB = k\sqrt{5}$$

Hence, the trigonometric ratios for the given θ are:

$$\sin\theta = BC/AB = k/(k\sqrt{5}) = 1/\sqrt{5}$$

$$\cos\theta = AC/AB = (2k)/(k\sqrt{5}) = 2/\sqrt{5}$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = k/(2k) = 1/2$$

$$\cot\theta = 2$$

$$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = \sqrt{5}$$

$$\sec\theta = AB/AC = 1/\cos\theta = \sqrt{5}/2$$

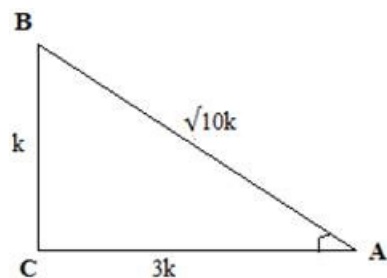
Question: 5

If $\operatorname{cosec}\theta = \sqrt{10}$,

Solution:

We have, $\operatorname{cosec}\theta = (k\sqrt{10})/k = 1/\sin\theta$ (For some value of k)

$$\rightarrow \sin\theta = k/(k\sqrt{10}) = \text{perpendicular/hypotenuse}$$



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$(\sqrt{10}k)^2 = (k)^2 + AC^2$$

$$AC^2 = 10k^2 - k^2$$

$$AC^2 = 9k^2 = (3k)^2$$

$$\rightarrow AC = 3k$$

Hence, the trigonometric ratios for the given θ are:

$$\sin\theta = 1/\sqrt{10}$$

$$\cos\theta = AC/AB = (3k)/(k\sqrt{10}) = 3/\sqrt{10}$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = 1/3$$

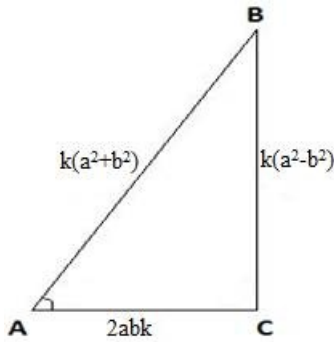
$$\cot\theta = AC/BC = 1/\tan\theta = 3$$

$$\operatorname{cosec}\theta = \sqrt{10}$$

$$\sec\theta = AB/AC = 1/\cos\theta = \sqrt{10}/3$$

Question: 6

If We have, $\sin\theta = \frac{(a^2 - b^2)k}{(a^2 + b^2)k} = \text{perpendicular/hypotenuse}$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$\{(a^2 + b^2)k\}^2 = \{(a^2 - b^2)k\}^2 + AC^2$$

$$a^4k^2 + b^4k^2 + 2a^2b^2k^2 = a^4k^2 + b^4k^2 - 2a^2b^2k^2 + AC^2$$

$$AC^2 = 4a^2b^2k^2 = (2abk)^2$$

$$\rightarrow AC = 2abk$$

Hence, the trigonometric ratios for the given θ are:

$$\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\cos\theta = AC/AB = \frac{2abk}{(a^2 + b^2)k} = \frac{2ab}{a^2 + b^2}$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = \frac{a^2 - b^2}{2ab}$$

$$\cot\theta = AC/BC = 1/\tan\theta = \frac{2ab}{a^2 - b^2}$$

$$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = \frac{a^2 + b^2}{a^2 - b^2}$$

$$\sec\theta = AB/AC = 1/\cos\theta = \frac{a^2 + b^2}{2ab}$$

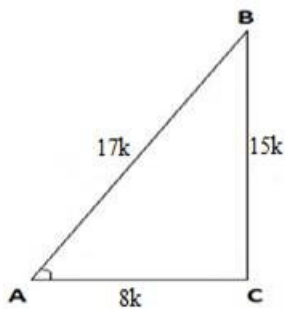
Question: 7

If $15 \cot A = 8$, f

Solution:

We have, $15 \cot A = 8$

$$\rightarrow \cot A = (8k)/(15k) = 1/\tan A = AC/BC \text{ (For some value of k)}$$



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$AB^2 = (15k)^2 + (8k)^2$$

$$AB^2 = 225k^2 + 64k^2$$

$$AB^2 = 289k^2$$

$$= (17k)^2$$

$$\rightarrow AB = 17k$$

$$\therefore \sin A = BC/AB = (15k)/(17k) = 15/17$$

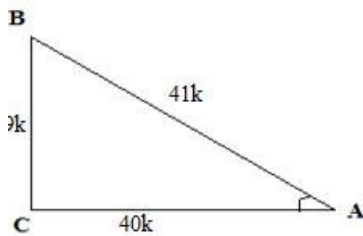
$$\sec A = 1/\cos A = AB/AC = (17k)/(8k) = 17/8$$

Question: 8

If $\sin A = 9/41$, find

Solution:

We have, $\sin A = (9k)/(41k) = BC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (41k)^2 = (9k)^2 + AC^2$$

$$= 1681k^2 = 81k^2 + AC^2$$

$$AC^2 = 1600k^2$$

$$= (40k)^2$$

$$\rightarrow AC = 40k$$

$$\therefore \cos A = AC/AB = (40k)/(41k) = 40/41$$

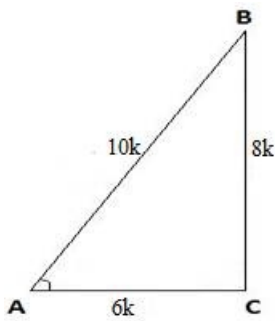
$$\tan A = BC/AC = \sin A / \cos A = 9/40$$

Question: 9

If $\cos \theta = 0.6$, find

Solution:

We have $\cos \theta = 0.6 = (6k)/(10k) = AC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (10k)^2 = BC^2 + (6k)^2$$

$$= 100k^2 = BC^2 + 36k^2$$

$$= BC^2 = 64k^2$$

$$= (8k)^2$$

$$\rightarrow BC = 8k$$

$$\therefore \sin\theta = BC/AB = (8k)/(10k) = 0.8$$

$$\tan\theta = \sin\theta / \cos\theta = 0.8/0.6$$

consider, the LHS,

$$5\sin\theta - 3\tan\theta = 5(0.8) - 3(0.8/0.6)$$

$$= 4 - 3(0.4/0.3)$$

$$= 4(0.3) - 3(0.4)$$

$$= 1.2 - 1.2$$

$$= 0$$

$$= \text{RHS}$$

HENCE PROVED

Question: 10

If $\operatorname{cosec}\theta =$

Solution:

We have, $\operatorname{cosec}\theta = 2 = 1/\sin\theta$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ \quad \Rightarrow \cos\theta = \cos 30 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cot\theta = \cot 30 = \sqrt{3}$$

$$\text{consider LHS} = \cot\theta + \frac{\sin\theta}{1 + \cos\theta} = \sqrt{3} + \frac{1/2}{1 + \frac{\sqrt{3}}{2}}$$

$$= \sqrt{3} + \frac{1}{2 + \sqrt{3}}$$

$$= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{2(2 + \sqrt{3})}{2 + \sqrt{3}}$$

$$= 2$$

$$= \text{RHS}$$

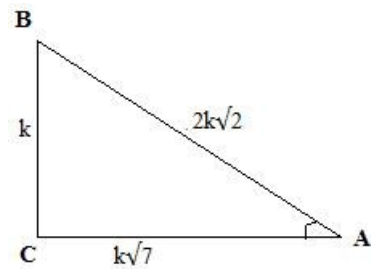
HENCE PROVED

Question: 11

If $\tan\theta = 1/\sqrt{7}$,

Solution:

We have, $\tan\theta = k/(k\sqrt{7}) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= AB^2 = (k)^2 + (k\sqrt{7})^2$$

$$= AB^2 = k^2 + 7k^2$$

$$= 8k^2 = (2k\sqrt{2})^2$$

$$\rightarrow AB = 2k\sqrt{2}$$

$$\therefore \operatorname{cosec}\theta = AB/BC = 2k\sqrt{2}$$

$$\sec\theta = AB/AC = \frac{2k\sqrt{2}}{k\sqrt{7}} = \frac{2\sqrt{2}}{\sqrt{7}}$$

consider the LHS,

$$\text{LHS} = \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{56 - 8}{56 + 8}$$

$$= 48/64$$

$$= 3/4$$

$$= \text{RHS}$$

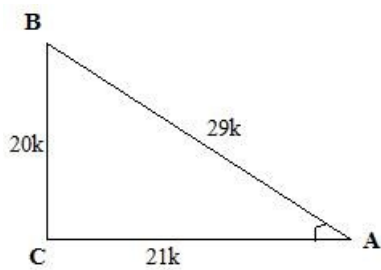
HENCE PROVED

Question: 12

If $\tan\theta = 20/21$,

Solution:

We have, $\tan\theta = (20k)/(21k) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= AB^2 = (20k)^2 + (21k)^2$$

$$= AB^2 = 400k^2 + 441k^2$$

$$= AB^2 = 841k^2$$

$$= (29k)^2$$

$$\rightarrow AB = 29k$$

$$\therefore \sin\theta = BC/AB = (20k)/(29k) = 20/29$$

$$\cos\theta = AC/AB = (21k)/(29k) = 21/29$$

consider, the LHS

$$\text{LHS} = \frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$$

$$= \frac{29 - 20 + 21}{29 + 20 + 21}$$

$$= 30/70$$

$$= 3/7$$

$$= \text{RHS}$$

HENCE PROVED

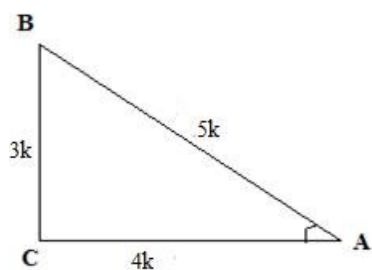
Question: 13

If $\sec\theta = 5/4$, sh

Solution:

We have, $\sec\theta = 5/4 = 1/\cos\theta$

$\rightarrow \cos\theta = (4k)/(5k) = AC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (5k)^2 = BC^2 + (4k)^2$$

$$= 25k^2 = BC^2 + 16k^2$$

$$= BC^2 = 9k^2$$

$$\rightarrow BC = 3k$$

$$\therefore \sin\theta = BC/AB = (3k)/(5k) = 3/5$$

consider the LHS

$$\text{LHS} = \frac{\sin\theta - 2\cos\theta}{\tan\theta + \cot\theta} = \frac{\sin\theta - 2\cos\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{(\sin\theta - 2\cos\theta)\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\left(\frac{3}{5} - 2\left(\frac{4}{5}\right)\right)\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)}{\frac{9}{25} - \frac{16}{25}}$$

$$= \frac{\frac{-5}{5} \times \frac{3}{5} \times \frac{4}{5}}{\frac{-7}{25}}$$

$$= \frac{-12}{-7}$$

$$= 12/7$$

$$= \text{RHS}$$

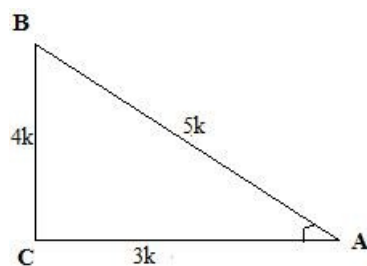
HENCE PROVED

Question: 14

If $\cot\theta = 3/4$, sh

Solution:

We have, $\cot\theta = (3k)/(4k) = AC/BC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= AB^2 = (4k)^2 + (3k)^2$$

$$= AB^2 = 16k^2 + 9k^2$$

$$= AB^2 = 25k^2$$

$$= (5k)^2$$

$$\rightarrow AB = 5k$$

$$\therefore \sin\theta = BC/AB = (4k)/(5k) = 4/5$$

$$\cos\theta = AC/AB = (3k)/(5k) = 3/5$$

consider the LHS

$$\text{LHS} = \sqrt{\frac{\sec\theta - \text{cosec}\theta}{\sec\theta + \text{cosec}\theta}} = \sqrt{\frac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}}$$

$$= \sqrt{\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}}$$

$$= \sqrt{\frac{\frac{4}{5} - \frac{3}{5}}{\frac{4}{5} + \frac{3}{5}}}$$

$$= \sqrt{\frac{1}{7}}$$

$$= 1/\sqrt{7}$$

$$= \text{RHS}$$

HENCE PROVED

Question: 15

If $\sin\theta = 3/4$, sh

Solution:

We have, $\sin\theta = (3k)/(4k) = BC/AB$ (For some value of k)

Consider the LHS

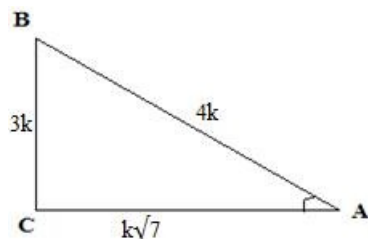
$$\text{LHS} = \sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - 1}}$$

$$= \sqrt{\frac{(1 - \cos^2\theta)\cos^2\theta}{(1 - \cos^2\theta)\sin^2\theta}}$$

$$= \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \cos\theta / \sin\theta$$

$$= \cot\theta$$



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (4k)^2 = (3k)^2 + AC^2$$

$$= 16k^2 = 9k^2 + AC^2$$

$$= AC^2 = 7k^2$$

$$\rightarrow AC = k\sqrt{7}$$

$$\therefore \cot\theta = AC/BC = k/(k\sqrt{7}) = 1/\sqrt{7}$$

$$\therefore \text{LHS} = \text{RHS}$$

HENCE PROVED

Question: 16

If $\sin\theta = a/b$, sh

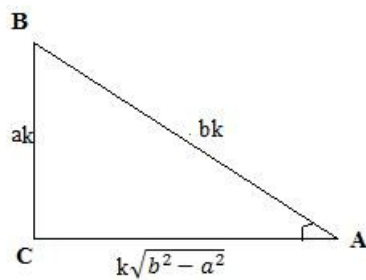
Solution:

Consider LHS,

$$\text{LHS} = \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta} \quad (1)$$

We have, $\sin\theta = (ak)/(bk) = BC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (bk)^2 = (ak)^2 + AC^2$$

$$= AC^2 = b^2k^2 - a^2k^2$$

$$\rightarrow AC = k\sqrt{b^2 - a^2}$$

$$\therefore \cos\theta = AC/AB = \frac{k\sqrt{b^2 - a^2}}{bk} = \frac{\sqrt{b^2 - a^2}}{b}$$

$\therefore$ from(1)

$$\text{LHS} = \frac{1 + \sin\theta}{\cos\theta} = \frac{1 + \frac{a}{b}}{\frac{\sqrt{b^2 - a^2}}{b}}$$

$$= \frac{b + a}{\sqrt{(b-a)(b+a)}}$$

$$= \sqrt{\frac{b+a}{b-a}}$$

$$= \text{RHS}$$

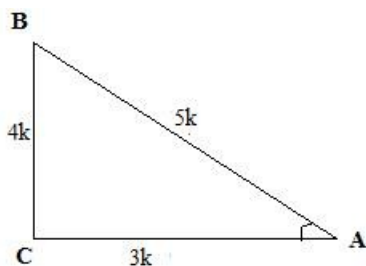
HENCE PROVED

Question: 17

If $\cos\theta = 3/5$, sh

Solution:

We have, $\cos\theta = (3k)/(5k) = AC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (5k)^2 = BC^2 + (3k)^2$$

$$= 25k^2 = BC^2 + 9k^2$$

$$= BC^2 = 16k^2$$

$$= (4k)^2$$

$$\rightarrow BC = 4k$$

$$\therefore \sin\theta = BC/AB = (4k)/(5k) = 4/5$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = (4k)/(3k) = 4/3$$

$$\cot\theta = 1/\tan\theta = 3/4$$

consider the LHS

$$\text{LHS} = \frac{\sin\theta - \cot\theta}{2\tan\theta} = \frac{\frac{4}{5} - \frac{3}{4}}{2\left(\frac{4}{3}\right)}$$

$$= \frac{(16-15)3}{20(8)}$$

$$= 3/160$$

$$= \text{RHS}$$

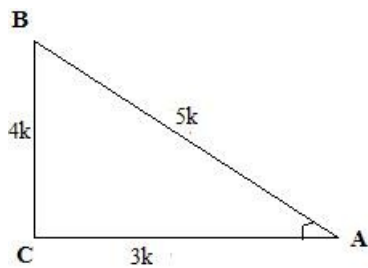
HENCE PROVED

Question: 18

If $\tan\theta = 4/3$, sh

Solution:

We have, $\tan\theta = (4k)/(3k) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= AB^2 = (4k)^2 + (3k)^2$$

$$= AB^2 = 16k^2 + 9k^2$$

$$= AB^2 = 25k^2$$

$$= (5k)^2$$

$$\rightarrow AB = 5k$$

$$\sin\theta = BC/AB = (4k)/(5k) = 4/5$$

$$\cos\theta = AC/AB = (3k)/(5k) = 3/5$$

$$\text{consider LHS} = \sin\theta + \cos\theta = \frac{4}{5} + \frac{3}{5}$$

$$= 7/5$$

$$= \text{RHS}$$

HENCE PROVED

Question: 19

If $\tan\theta = a/b$, sh

Solution:

Consider the LHS = $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$

(Dividing the numerator and denominator by $\cos \theta$)

$$= \frac{\frac{a \sin \theta}{\cos \theta} - \frac{b \cos \theta}{\cos \theta}}{\frac{a \sin \theta}{\cos \theta} + \frac{b \cos \theta}{\cos \theta}}$$

$$= \frac{a \tan \theta - b}{a \tan \theta + b}$$

$$= \frac{a \left(\frac{a}{b} \right) - b}{a \left(\frac{a}{b} \right) + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

$$= \text{RHS}$$

HENCE PROVED

Question: 20

If $3 \tan \theta = 4$

Solution:

We have, $3 \tan \theta = 4$

$$\rightarrow \tan \theta = \frac{4}{3} \quad (1)$$

Consider the LHS = $\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$

(Dividing the numerator and denominator by $\cos \theta$)

$$= \frac{4 - \tan \theta}{2 + \tan \theta}$$

$$= \frac{4 - \frac{4}{3}}{2 + \frac{4}{3}} \quad (\text{from (1)})$$

$$= \frac{12 - 4}{6 + 4}$$

$$= \frac{8}{10}$$

$$= \frac{4}{5}$$

$$= \text{RHS}$$

HENCE PROVED

Question: 21

If $3 \cot \theta = 2$, show

Solution:

We have $3 \cot \theta = 2$

$$\rightarrow \cot \theta = \frac{2}{3} \quad (1)$$

Consider the LHS = $\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$

(Dividing the numerator and denominator by $\sin \theta$)

$$= \frac{\frac{4 \sin \theta - 3 \cos \theta}{\sin \theta}}{\frac{2 \sin \theta + 6 \cos \theta}{\sin \theta}}$$

$$= \frac{4-3\cot\theta}{2+6\cot\theta}$$

$$= \frac{4-3\left(\frac{2}{1}\right)}{2+6\left(\frac{2}{1}\right)} \text{ (from (1))}$$

$$= \frac{4-2}{2+4}$$

$$= 2/6$$

$$= 1/3$$

$$= \text{RHS}$$

HENCE PROVED

Question: 22

If $3\cot\theta = 4$, sho

Solution:

$$\text{Consider the LHS} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1-\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{1+\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \cos^2\theta - \sin^2\theta \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

$$= \text{RHS}$$

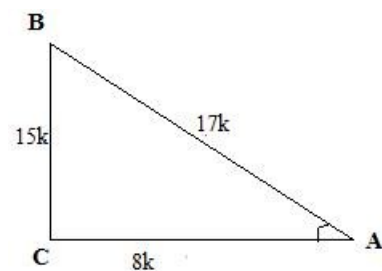
HENCE PROVED

Question: 23

If $\sec\theta = 17/8$, v

Solution:

We have, $\sec\theta = (17k)/(8k) = AB/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (17k)^2 = BC^2 + (8k)^2$$

$$= 289k^2 = BC^2 + 64k^2$$

$$= BC^2 = 225k^2$$

$$\rightarrow BC = 15k$$

$$\therefore \sin\theta = BC/AB = (15k)/(17k) = 15/17$$

$$\cos\theta = AC/AB = (8k)/(17k) = 8/17$$

$$\tan\theta = BC/AC = \sin\theta / \cos\theta = 15/8$$

consider the LHS = $\frac{3-4\sin^2\theta}{4\cos^2\theta-3}$

$$= \frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$$

$$= \frac{3(289)-4(225)}{4(64)-3(289)}$$

$$= \frac{867-900}{256-867}$$

$$= (-33)/(-611)$$

$$= 33/611$$

Now consider RHS = $\frac{3-\tan^2\theta}{1-3\tan^2\theta}$

$$= \frac{3-\left(\frac{15}{8}\right)^2}{1-3(15/8)^2}$$

$$= \frac{3(64)-225}{64-675}$$

$$= (-33)/(-611)$$

$$= 33/611$$

$\therefore$ RHS = LHS

HENCE PROVED

Question: 24

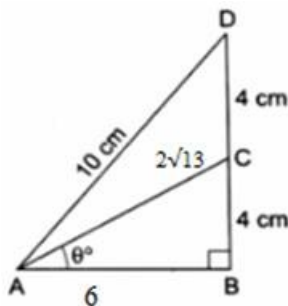
In the adjoining

Solution:

Clearly, ΔABC and ΔABD are right angled triangles

where $AD = 10\text{cm}$ $BC = CD = 4\text{cm}$

$BD = BC + CD = 8\text{cm}$



Applying Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AD^2 = BD^2 + AB^2$$

$$= (10)^2 = (8)^2 + AB^2$$

$$= 100 = 64 + AB^2$$

$$= AB^2 = 36$$

$$= (6)^2$$

$$\rightarrow AB = 6\text{cm}$$

Now applying Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AC^2 = BC^2 + AB^2$$

$$= AC^2 = (4)^2 + (6)^2$$

$$= AC^2 = 16 + 36$$

$$= 52$$

$$\rightarrow AC = \sqrt{52}$$

$$= 2\sqrt{13}\text{cm}$$

$$\text{i. } \sin\theta = BC/AC = \frac{4}{2\sqrt{13}}$$

$$= 2/\sqrt{13}$$

$$= (2\sqrt{13})/13$$

$$\text{ii. } \cos\theta = AB/AC = \frac{6}{2\sqrt{13}}$$

$$= 3/\sqrt{13}$$

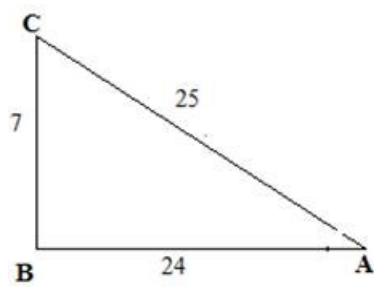
$$= (3\sqrt{13})/13$$

Question: 25

In a ΔABC ,

Solution:

Clearly ΔABC is a right angled triangle,



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AC^2 = BC^2 + AB^2$$

$$= AC^2 = (7)^2 + (24)^2$$

$$= AC^2 = 49 + 576$$

$$= AC^2 = 625$$

$$\rightarrow AC = 25$$

$$\text{a) } \sin A = BC/AC = 7/25$$

$$\text{b) } \cos A = AB/AC = 24/25$$

$$\text{c) } \sin C = AB/AC = 24/25$$

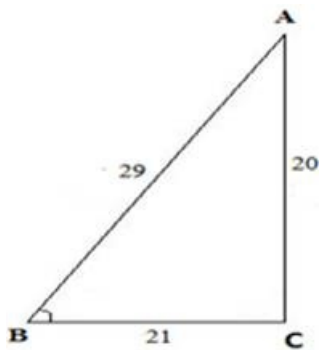
$$\text{d) } \cos C = BC/AC = 7/25$$

Question: 26

In a ΔABC ,

Solution:

ΔABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = AC^2 + BC^2$$

$$= (29)^2 = AC^2 + (21)^2$$

$$= 841 = AC^2 + 441$$

$$= AC^2 = 400$$

$$\rightarrow AC = 20$$

$$\therefore \sin\theta = AC/AB = 20/29$$

$$\cos\theta = BC/AB = 21/29$$

$$\cos^2\theta - \sin^2\theta = (21/29)^2 - (20/29)^2$$

$$= \frac{441-400}{841}$$

$$= 41/841$$

$$= \text{RHS}$$

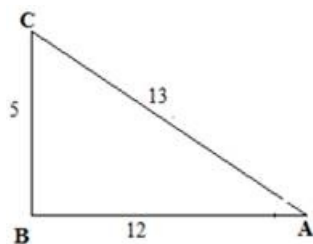
HENCE PROVED

Question: 27

In a ΔABC ,

Solution:

ΔABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AC^2 = BC^2 + AB^2$$

$$= AC^2 = (5)^2 + (12)^2$$

$$= AC^2 = 25 + 144$$

$$= AC^2 = 169$$

$$= (13)^2$$

$$\rightarrow AC = 13$$

$$\text{i. } \cos A = AB/AC = 12/13$$

$$\text{ii. } \operatorname{cosec} A = AC/BC = 13/5$$

iii. $\cos C = BC/AC = 5/13$

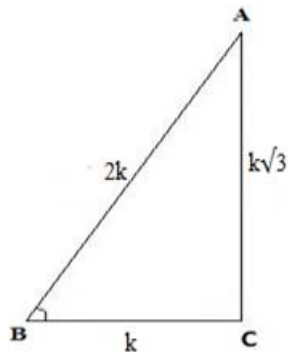
iv. $\operatorname{cosec} C = AC/AB = 13/12$

Question: 28

If $\sin \alpha = 1/2$, pr

Solution:

We have, $\sin \alpha = k/(2k) = BC/AB$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AB^2 = BC^2 + AC^2$$

$$= (2k)^2 = (k)^2 + AC^2$$

$$= 4k^2 = k^2 + AC^2$$

$$= AC^2 = 3k^2$$

$$\rightarrow AC = k\sqrt{3}$$

$$\therefore \cos \alpha = AC/AB = (k\sqrt{3})/(2k) = \sqrt{3}/2$$

$$\text{Then, } 3\cos \alpha - 4\cos^3 \alpha = 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$= 0$$

$$\text{LHS} = \text{RHS}$$

HENCE PROVED

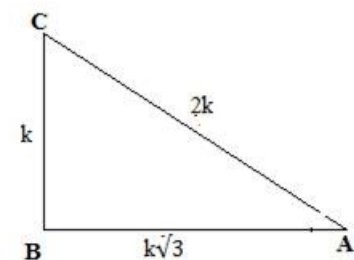
Question: 29

In a ΔABC ,

Solution:

We have, $\tan A = k/(k\sqrt{3}) = BC/AB$

ΔABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AC^2 = BC^2 + AB^2$$

$$= AC^2 = (k)^2 + (k\sqrt{3})^2$$

$$= AC^2 = k^2 + 3k^2$$

$$= AC^2 = 4k^2$$

$$\rightarrow AC = 2k$$

$$\therefore \sin A = BC/AC = k/(2k) = 1/2$$

$$\cos A = AB/AC = (k\sqrt{3})/(2k) = \sqrt{3}/2$$

$$\sin C = AB/AC = (k\sqrt{3})/(2k) = \sqrt{3}/2$$

$$\cos C = BC/AC = k/(2k) = 1/2$$

$$\text{i. } \sin A \cos C + \cos A \sin C = (1/2)(1/2) + (\sqrt{3}/2)(\sqrt{3}/2)$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 4/4$$

$$= 1$$

$$\therefore \text{RHS} = \text{LHS}$$

HENCE PROVED

$$\text{ii. } \cos A \cos C - \sin A \sin C = (1/2)(\sqrt{3}/2) - (1/2)(\sqrt{3}/2)$$

$$= (\sqrt{3}/4) - (\sqrt{3}/4)$$

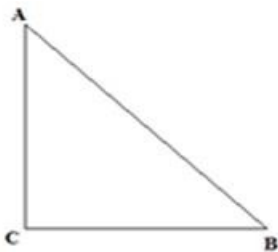
$$= 0$$

$$\therefore \text{RHS} = \text{LHS}$$

HENCE PROVED

Question: 30

If Consider $\triangle ABC$ to be a right - angled triangle.



$$\therefore \sin A = BC/AB$$

$$\sin B = AC/AB$$

Given that $\sin A = \sin B$

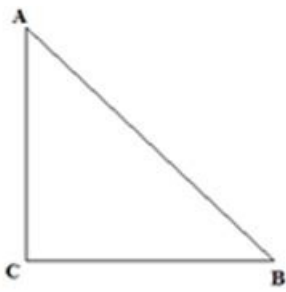
$$BC/AB = AC/AB$$

$$\rightarrow BC = AC$$

$\rightarrow \angle A = \angle B$ (In a triangle, angles opposite to equal sides are equal)

Question: 31

If Consider $\triangle ABC$ to be a right - angled triangle.



$$\therefore \tan A = BC/AC$$

$$\tan B = AC/BC$$

Given that $\tan A = \tan B$

$$BC/AC = AC/BC$$

$$BC^2 = AC^2$$

$$\rightarrow BC = AC$$

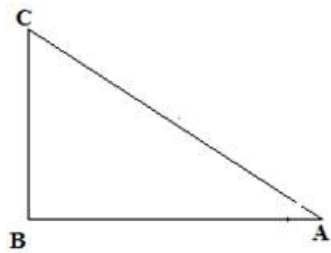
$\rightarrow \angle A = \angle B$ (In a triangle, angles opposite to equal sides are equal)

Question: 32

In a right $\triangle ABC$,

Solution:

Consider $\triangle ABC$ to be a right - angled triangle.



$$\tan A = 1 = BC/AB$$

$$\rightarrow BC = AB$$

$$\text{Also, } \tan A = 1 = \sin A / \cos A$$

$$\rightarrow \sin A = \cos A$$

By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore AC^2 = BC^2 + AB^2$$

$$= AC^2 = 2BC^2$$

$$= (AC/BC)^2 = 2$$

$$= AC/BC = \sqrt{2}$$

$$\rightarrow \operatorname{cosec} A = \sqrt{2}$$

$$\rightarrow \sin A = 1/\sqrt{2} = \cos A$$

$$2 \sin A \cos A = 2(1/\sqrt{2})(1/\sqrt{2})$$

$$= 2(1/2)$$

$$= 1$$

$$\therefore \text{LHS} = \text{RHS}$$

HENCE PROVED

Question: 33

In the given figure

Solution:

ΔPQR is a right-angled triangle

By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$$\therefore PR^2 = RQ^2 + PQ^2$$

$$= (x+2)^2 = x^2 + PQ^2$$

$$= x^2 + 4 + 4x = x^2 + PQ^2$$

$$PQ^2 = 4 + 4x$$

$$\rightarrow PQ = 2\sqrt{x+1}$$

$$\text{a. } \cot\phi = RQ/PQ = \frac{x}{2\sqrt{x+1}}$$

$$\therefore \sqrt{x+1} \cot\phi \sqrt{x+1} \times \frac{x}{2\sqrt{x+1}} =$$

$$= x/2$$

$$\text{b. } \tan\theta = RQ/PQ = \frac{x}{2\sqrt{x+1}}$$

$$\therefore \sqrt{x^3 + x^2} \tan\theta = \sqrt{x^3 + x^2} \times \frac{x}{2\sqrt{x+1}} =$$

$$= x\sqrt{x+1} \times \frac{x}{2\sqrt{x+1}}$$

$$= x^2/2$$

$$\text{c. } \cos\theta = PQ/PR = \frac{2\sqrt{x+1}}{x+2}$$

Question: 34

If $x = \operatorname{cosec} A + c$

Solution:

$$x + y = \operatorname{cosec} A + \cos A + \operatorname{cosec} A - \cos A$$

$$= 2\operatorname{cosec} A$$

$$x - y = \operatorname{cosec} A + \cos A - (\operatorname{cosec} A - \cos A)$$

$$= \operatorname{cosec} A + \cos A - \operatorname{cosec} A + \cos A$$

$$= 2\cos A$$

$$\text{Consider the LHS, } \left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 - 1 = \left(\frac{2}{2\operatorname{cosec} A}\right)^2 + \left(\frac{2\cos A}{2}\right)^2 - 1$$

$$= \sin^2 A + \cos^2 A - 1$$

$$= 1 - 1 \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= 0$$

$$= \text{RHS}$$

HENCE PROVED

Question: 35

If $x = \cot A + \cos$

Solution:

$$x - y = \cot A + \cos A - (\cot A - \cos A)$$

$$x - y = \cot A + \cos A - \cot A + \cos A$$

$$x - y = 2\cos A$$

$$x + y = \cot A + \cos A + \cot A - \cos A$$

$$x + y = 2\cot A$$

$$\text{Consider the LHS} = \left(\frac{x-y}{2}\right)^2 + \left(\frac{x-y}{x+y}\right)^2 = \left(\frac{2\cos A}{2}\right)^2 + \left(\frac{2\cos A}{2\cot A}\right)^2$$

$$= \cos^2 A + \sin^2 A$$

$$= 1 \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \text{RHS}$$

HENCE PROVED