1. Sets and functions

Exercise 1.1

1. Question

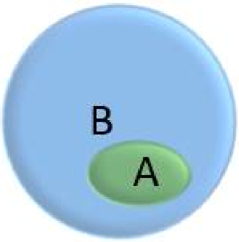
If A ⊂ B, then show that A ⋃ B = B (use Venn diagram).

Answer

The statement (A ⊂ B) indicates that set A is a subset of set B which mean that all values of set A are in set B.

A union operation between two sets generate a new set which contain all the elements of the two sets.

But from the given statement it’s clear that A is a subset of B so the union operation between these two will give a set same as set B so a Venn diagram to represent (A ⋃ B) can be drawn as



A ⋃ B

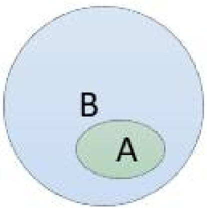
2. Question

If A ⊂ B, then find A ⋂ B and A \ B (use Venn diagram).

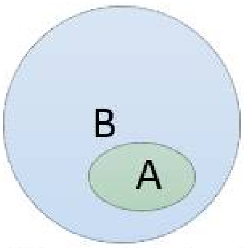
Answer

Form (A ⊂ B) it’s clear that the set A is a subset of B which means all the elements of set A are in B.

A intersection operation between two sets generate a new set which contain all the common elements of the two sets. So the intersection of both the sets will give the common elements of sets A and B and from the given statement we know that A is a subset of B so the intersection will give all the elements of set A. So Venn diagram to represent (A ⋂ B) can be drawn as



A∩B



Not possible to draw

A difference or compliment operation between set A and B will give all the elements which are only a part of set A and are not belong to set B. So it’s not possible to find the elements which are only the part of set A but not a part of B because A is a subset of B so all the elements of A are in B.

3. Question

Let P = {a, b, c}, Q = {g, h, x, y} and R = {a, e, f, s}. Find the following:

(i) P \ R

(ii) Q + R

(iii) R \ (P + Q)

Answer

(i) The set which will be obtained from P \ R is called as a complement set. It indicates the elements which are only a part of the set P and doesn't belong to R.

Mathematically,

P \ R = $\{x|x \in p$ but $x \notin R\}$ $\{x|x \in p$ but $x \notin R\}$

We are given

P = {a, b, c},

R = {a, e, f, s}

So using the given values

P \ R = {a, b, c} \ {a, e, f, s}

= {b, c}

(ii) The above statement can also be written as ($Q \cap R$). An intersection operation between two sets gives a set which contain the elements which are common to both the sets.

We are given

Q = {g, h, x, y}

R = {a, e, f, s}

So using the value given

($Q \cap R$) = {g, h, x, y} $\cap$ {a, e, f, s}

= Ø

It's called as a disjoint set as both the sets Q and R doesn't have any elements in common.

(iii) Before proceeding we will split the entire operation into two halves. We know that the statement (P + Q) can be written as ($P \cap Q$) and it will give a set of all common elements

We are given that

P = {a, b, c},

Q = {g, h, x, y}

R = {a, e, f, s}

So using the values given

($P \cap Q$) = {a, b, c} $\cap$ {g, h, x, y}

= Ø [disjoint sets as there are no common elements]

Now the statement R \ (P + Q) shows a difference operation of two sets. The difference operation will give elements which are a part of set R only and doesn't exist in ($P \cap Q$).

So using the values given

R \ (P + Q) = R\ ($P \cap Q$)

= {a, e, f, s} \ Ø

= {a, e, f, s} (.ans)

4. Question

If A = {4,6,7,8,9}, B = {2,4,6} and C = {1,2,3,4,5,6}, then find

(i) $A \cup (B \cap C)$

(ii) $A \cap (B \cup C)$

(iii) $A \setminus (C \setminus B)$

Answer

(i) Here first we will find the elements of the set ($B \cap C$). As we now the intersection operation will give a set containing the common elements of both the sets. After that we will use union operation between A and the set of elements obtained from intersection operation of B and C to find the next set of element. We know that a union operation between two sets gives all the element present in sets.

We are given that A = {4, 6, 7, 8, 9},

B = {2, 4, 6}

C = {1, 2, 3, 4, 5, 6}

So

$(B \cap C) = \{2, 4, 6\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{2, 4, 6\}$

Now,

$A \cup (B \cap C) = \{4, 6, 7, 8, 9\} \cup \{2, 4, 6\}$

$= \{2, 4, 6, 7, 8, 9\}$

(ii) Here first we will find the union set i.e. ($B \cup C$) it will give a new set of elements which are common to both B and C after that we will find the intersection between set A and the set obtained from the union operation of B and C

We are given that A = {4, 6, 7, 8, 9},

B = {2, 4, 6}

C = {1, 2, 3, 4, 5, 6}

So $(B \cup C) = \{2, 4, 6\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

Now $A \cap (B \cup C) = \{4, 6, 7, 8, 9\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{4, 6\}$

(iii) Here we will solve the problem in two part. First we will find the difference set $C \setminus B$ which gives the element that are only a part of set C and doesn't exist in set B after that we will find the difference between set A and the element obtained from $(C \setminus B)$.

We are given that A = {4, 6, 7, 8, 9},

B = {2, 4, 6}

C = {1, 2, 3, 4, 5, 6}

So $C \setminus B = \{1, 2, 3, 4, 5, 6\} \setminus \{2, 4, 6\}$

$= \{1, 3, 5\}$

Now $A \setminus (C \setminus B) = \{4, 6, 7, 8, 9\} \setminus \{1, 3, 5\}$

$= \{4, 6, 7, 8, 9\}$

5. Question

Given A = {a,x,y,r,s},B = {1,3,5,7, -10}, verify the commutative property of set union.

Answer

The commutative property of set union says that for two given sets the value of their union are commutative in nature.

Mathematically, if A and B are two sets with a number of elements then with the help of commutative property of set theory we can conclude that

$A \bigcup B = B \bigcup A$

Here it's given that

A = {a, x, y, r, s},

B = {1, 3, 5, 7, -10}

So using the data given

$A \bigcup B$ = {a, x, y, r, s} $\bigcup$ {1, 3, 5, 7, -10}

= {a, x, y, r, s, 1, 3, 5, 7, -10}……………………..(1)

$B \bigcup A$ = {1, 3, 5, 7, -10} $\bigcup$ {a, x, y, r, s}

= {1, 3, 5, 7, -10, a, x, y, r, s}

= {a, x, y, r, s, 1, 3, 5, 7, -10}……………………(2)

So from (1) an (2) it's clear that the set union operation is commutative in nature i.e. $A \bigcup B = B \bigcup A$ (proved)

6. Question

Verify the commutative property of set intersection for

A = {m, n, o, 2, 3, 4, 7} and B = {2, 5, 3, - 2, m, n, o, p}.

Answer

The commutative property of set intersection says that for two given sets the value of their intersection are commutative in nature.

Mathematically, if A and B are two sets with a number of elements then with the help of commutative property of set theory we can conclude that

$A \bigcap B = B \bigcap A$

It's given that

A = {m, n, o, 2, 3, 4, 7}

B = {2, 5, 3, - 2, m, n, o, p}.

So using given data

$A \bigcap B$ = {m, n, o, 2, 3, 4, 7} $\bigcap$ {2, 5, 3, - 2, m, n, o, p}.

= {m, n, o, 2, 3}……………. (1)

Similarly

$B \bigcap A$ = {2, 5, 3, - 2, m, n, o, p} $\bigcap$ {m, n, o, 2, 3, 4, 7}

= {2, 3, m, n, o}

= {m, n, o, 2, 3}………………. (2)

So from (1) an (2) it's clear that the set intersection operation is commutative in nature i.e. $A \bigcap B = B \bigcap A$ (proved)

7. Question

For A = {x | x is a prime factor of 42}, B = {x | 5 < x $\leq$ 12, $x \in \mathbb{N}$ } and C = {1, 4, 5, 6}, verify A ∪ (B ∪ C) = (A ∪ B) ∪ C.

Answer

From the above statements we can find the elements of the set.

So,

A = {2, 3, 7}

B = {6, 7, 8, 9, 10, 11, 12}

C = {1, 4, 5, 6}

*Prime numbers are the numbers which can only be divided by 1 or the number itself.

*Factors are the number which are multiplied together to get a new number so the term prime factorization may be defined as the finding the prime numbers which are multiplied together to give the source word.

The elements of B are natural number and they lies between 6 and 12.

Here we have to verify A ⋃ (B ⋃ C) = (A ⋃ B) ⋃ C. It's the associative union property of set theory.

L.H.S.

A ⋃ (B ⋃ C)

Here first we will split the entire operation and have to find elements of (B⋃ C) using the given data then we will find the union of set A and the set obtained from the union of B and C

So (B ⋃ C) = {6, 7, 8, 9, 10, 11, 12}⋃ {1, 4, 5, 6}

= {6, 7, 8, 9, 10, 11, 12, 1, 4, 5}

Now

A ⋃ (B ⋃ C) = {2, 3, 7}⋃ {6, 7, 8, 9, 10, 11, 12, 1, 4, 5}

= {2, 3, 7, 6, 8, 9, 10, 11, 12, 1, 4, 5}

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}………….(1)

R.H.S

(A ⋃ B) ⋃ C

Here first we have to find the value of (A⋃ B) from the given data then we will find the union between the set obtained from (B ⋃ C)

And C

So (A ⋃ B) = {2, 3, 7}⋃ {6, 7, 8, 9, 10, 11, 12}

= {2, 3, 7, 6, 8, 9, 10, 11, 12}

Now

(A ⋃ B) ⋃ C = {2, 3, 7, 6, 8, 9, 10, 11, 12}⋃ {1, 4, 5, 6}

= {2, 3, 7, 6, 8, 9, 10, 11, 12, 1, 4, 5}

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}………….(2)

So from (1) & (2) it's clear that L.H.S. = R.H.S which verifies that

A ⋃ (B ⋃ C) = (A ⋃ B) ⋃ C.

8. Question

Given P = {a, b, c, d, e}, Q = {a, e, i, o, u} and R = {a, c, e, g}. Verify the associative property of set intersection.

Answer

The associative property of set intersection says that for three given sets the intersection operation between them, are associative in nature. Here for the three sets P, Q, R the associative property of set intersection can be represented as

$P \cap (Q \cap R) = (P \cap Q) \cap R$

So using the data given we have to prove the above statement.

Data given,

P = {a, b, c, d, e}

Q = {a, e, i, o, u}

R = {a, c, e, g}

L.H.S

For ease of solving we will split the statement $P \cap (Q \cap R)$ in two part where first we use the intersection operation between Q and R then another intersection will be done between P and values obtained from intersection operation between Q and R.

So, $(Q \cap R)$ = {a, e, i, o, u} ⋂ {a, c, e, g}

= {a, e}

Now, $P \cap (Q \cap R)$ = {a, b, c, d, e} ⋂ {a, e}

= {a, e}…….(1)

R.H.S.

For ease of solving we will split the statement $(P \cap Q) \cap R$ in two part where first we use the intersection operation between P and R then another intersection will be done between values obtained from intersection operation between P and Q and values of R.

So $(P \cap Q)$ = {a, b, c, d, e} ⋂ {a, e, i, o, u}

= {a, e}

Now $(P \cap Q) \cap R$ = {a, e} ⋂ {a, c, e, g}

= {a, e}……………… (2)

From statement (1) and (2) it’s clear that L.H.S is equal to R.H.S which proved that $P\cap (Q \cap R) = (P \cap Q) \cap R$.

9. Question

For A = {5,10,15, 20}; B = {6,10,12,18,24} and C = {7,10,12,14,21,28},| verify whether A \ (B \ C) = (A \ B) \ C. Justify your answer.

Answer

The given statement A \ (B \ C) = (A \ B) \ C can be rewritten as

A \ (B\C) = (A \ B) \ C.

Here we have to compare the elements obtained from the difference or compliment of two sets and check whether they are associative in nature or not.

Given data

A = {5, 10, 15, 20};

B = {6, 10, 12, 18, 24}

C = {7, 10, 12, 14, 21, 28}

L.H.S

For easy solving we can spit the statement A \ (B\C) into two halves where first we will find the difference between B and C after we will find difference between set A and the result obtained from the difference of B and C

So (B\C) = {6, 10, 12, 18, 24} \ {7, 10, 12, 14, 21, 28}

= {6, 18, 24}

A \ (B\C) = {5, 10, 15, 20} \ {6, 18, 24}

= {5, 10, 15, 20}………… (i)

R.H.S

(A \ B) \ C again we will split this statement in two to find the difference result. First of all we will find the difference between A and B and after that another difference operation will be done between the results obtained from (A \ B) and C

Using the data given

(A \ B) = {5, 10, 15, 20} \ {6, 10, 12, 18, 24}

= {5, 10, 15, 20}

(A \ B) \ C = {5, 10, 15, 20} \ {7, 10, 12, 14, 21, 28}

= {5, 15, 20}………… (ii)

From (i) and (ii) it’s clear that L.H.S. and R.H.S aren’t same so

A \ (B\C) = (A \ B) \ C is a false statement.

i.e A \ (B\C) ≠ (A \ B) \ C

10. Question

Let A = {- 5, - 3, - 2, - 1}, B = {- 2, - 1,0}, and C = {- 6, - 4, - 2}. Find A \ (B \ C) and (A \ B) \ C. What can we conclude about set difference operation?

Answer

The statement A \ (B \ C) can be rewritten as A (B \ C). Here we have find the compliment values of the two statements and compare the results.

That means we have to check the associative properties of compliment or difference operation

That is we have to check A (B \ C) = (A \ B) \ C or not

Here we are given

A = {- 5, - 3, - 2, - 1},

B = {- 2, - 1,0}

C = {- 6, - 4, - 2}

L.H.S

Here for easy solving we will split the statement A \ (B \ C) into two parts. First we find the difference between set B and C then the difference between set A and the result obtained form (B\C).

So using the data given

(B\C) = {- 2, - 1, 0} \ {- 6, - 4, - 2}

= {-1, 0}

A \ (B \ C) = {- 5, - 3, - 2, - 1} \ {-1, 0}

= {–5, –3, –2}………….(i)

R.H.S.

Here for easy solving we will split the statement (A \ B) \ C into two parts. First we find the difference between set A and B then again the difference is to be found between result obtained form (A\B) and C.

So

(A \ B) = {- 5, - 3, - 2, - 1} \ {- 2, - 1, 0}

= {-5,-3}

Now (A \ B) \ C = {-5,-3} \ {- 6, - 4, - 2}

= {-5,-3}...............(ii)

So from (i) and (ii) it’s clear that

L.H.S ≠ R.H.S

I.e. A (B \ C) ≠ (A \ B) \ C

So from the above statement it’s clear that the difference or the compliment operation of set is not associative.

11 A. Question

For A = {- 3,- 1, 0, 4,6,8,10}, B = {- 1,- 2, 3,4,5,6} and C = {- 1, 2,3,4,5,7}, show that

(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(iii) Verify (i) using Venn diagram

Answer

(i) A ⋃ (B ⋃ C) = (A ⋃ B) ⋃ (A ⋃ C)

L.H.S.

The statement A ⋃ (B ⋃ C) can be split into two parts for ease of solving. So using the given data first we will find the union of B and C then again we have to find the union between A and the results obtained from the union of B and C.

Data given

A = {- 3,-1, 0, 4, 6, 8, 10},

B = {- 1,-2, 3, 4, 5, 6}

C = {- 1, 2, 3, 4, 5, 7}

So (B ⋃ C) = {-1,-2, 3, 4, 5, 6} ⋃ {-1, 2, 3, 4, 5, 7}

= {-1, -2, 3, 4, 5, 6, 7}

Now A ⋃ (B ⋃ C) = {-3, -1, 0, 4, 6, 8, 10} ⋃ {-1,-2, 3, 4, 5, 6, 7}

= {-3, -2, -1, 0, 3, 4, 5, 6, 7, 8, 10}.......(i)

R.H.S.

The statement (A ⋃ B) ⋃ (A ⋃ C) can be split into three parts for ease of solving. First we will find the union between A, B and A, C then again we will do union operation between the results obtained from the union of A, B and A, C

So (A ⋃ B) = {-3, -1, 0, 4, 6, 8, 10} ⋃ {-1, -2, 3, 4, 5, 6}

= {-3, -2, -1, 0, 3, 4, 5, 6, 8, 10}

(A ⋃ C) = {-3, -1, 0, 4, 6, 8, 10} ⋃ {-1, 2, 3, 4, 5, 7}

= {-3, -1, 0, 3, 4, 5, 7, 6, 8, 10}

(A ⋃ B) ⋃ (A ⋃ C) = {-3, -2, -1, 0, 4, 5, 6, 8, 10} ⋃

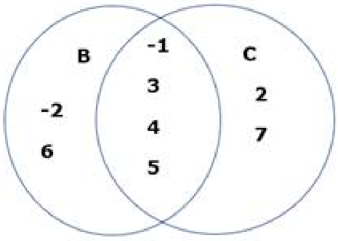
{-3, -1, 0, 3, 4, 5, 7, 6, 8, 10}

= {-3, -2, -1, 0, 3, 4, 5, 6, 7, 8, 10}....(ii)

From statement (i) and (ii) its verified that L.H.S equals to R.H.S

I.e. A ⋃ (B ⋃ C) = (A ⋃ B) ⋃ (A ⋃ C)

Venn diagram for A ⋃ (B ⋃ C) = (A ⋃ B) ⋃ (A ⋃ C)



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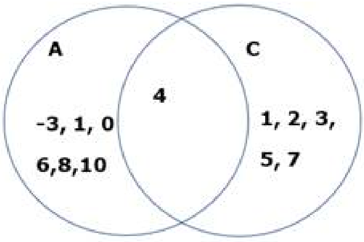

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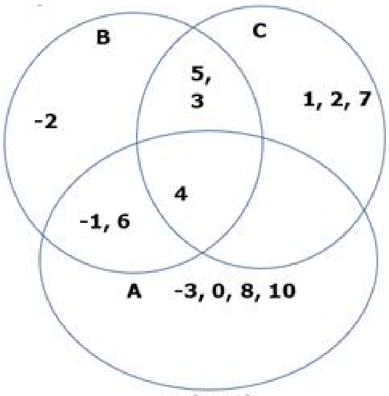

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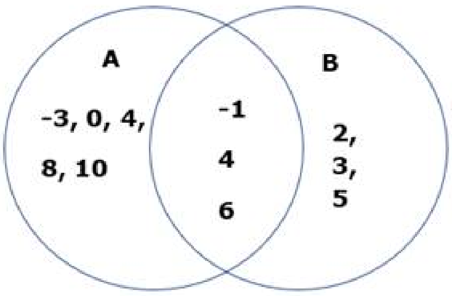

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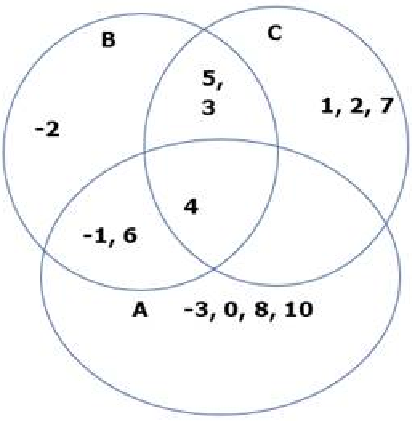


(A U B) U (A U C)

11 B. Question

For A = {- 3,- 1, 0, 4,6,8,10}, B = {- 1,- 2, 3,4,5,6} and C = {- 1, 2,3,4,5,7}, show that

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

(iv) Verify (ii) using Venn diagram.

Answer

(ii) A ⋂ (B ⋂ C) = (A ⋂ B) ⋂ (A ⋂ C)

L.H.S.

The statement A ⋂ (B ⋂ C) can be split into two parts for ease of solving. So using the given data first we will find the intersection of B and C then again we have to find the intersection between A and the results obtained from the intersection of B and C

Data given

A = {-3,-1, 0, 4, 6, 8, 10}

B = {-1,-2, 3 , 4, 5, 6}

C = {-1, 2, 3, 4, 5, 7}

So (B ⋂ C) = {- 1, - 2, 3, 4, 5, 6} ⋂ {- 1, 2, 3, 4, 5, 7}

= {-1,-2, 3, 4, 5}

Now A ⋂ (B ⋂ C) = {- 3, - 1, 0, 4, 6, 8, 10} ⋂ {-1,-2, 3, 4, 5}

= {-1, 4}……………………. (i)

R.H.S

The statement (A ⋂ B) ⋂ (A ⋂ C) can be split into three parts for ease of solving. First we will find the intersection between A, B and A, C then again we will do intersection operation between the results obtained from the intersection of A, B and A, C.

(A ⋂ B) = {- 3, - 1, 0, 4, 6, 8, 10} ⋂ {- 1, - 2, 3, 4, 5, 6}

= {-1, 4, 6}

(A ⋂ C) = {- 3, - 1, 0, 4, 6, 8, 10} ⋂ {- 1, 2, 3, 4, 5, 7}

= {-1, 4}

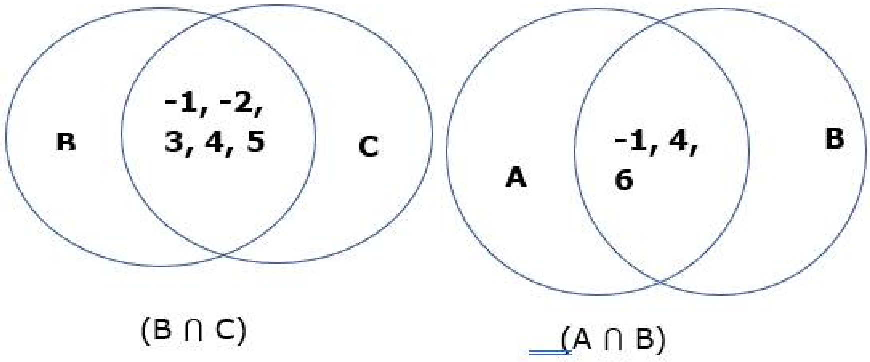
(A ⋂ B) ⋂ (A ⋂ C) = {-1, 4, 6} ⋂ {-1, 4}

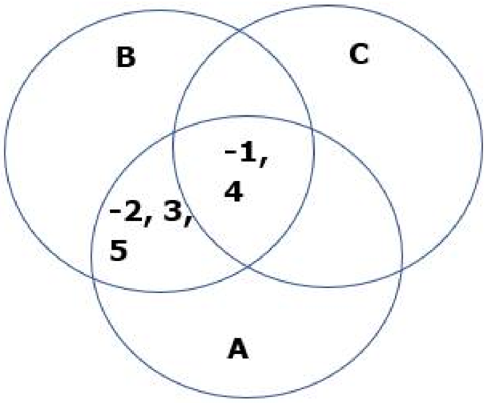
= {-1, 4}…………………….. (ii)

From statement (i) and (ii) its verified that L.H.S equals to R.H.S

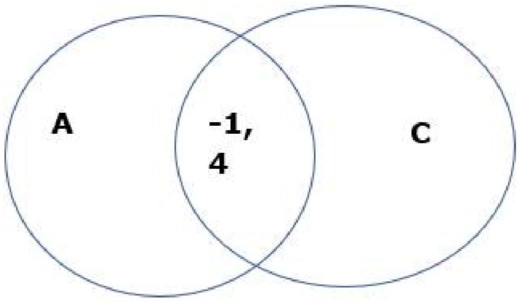
I.e. A ⋂ (B ⋂ C) = (A ⋂ B) ⋂ (A ⋂ C)

Venn diagram for A ⋂ (B ⋂ C) = (A ⋂ B) ⋂ (A ⋂ C)

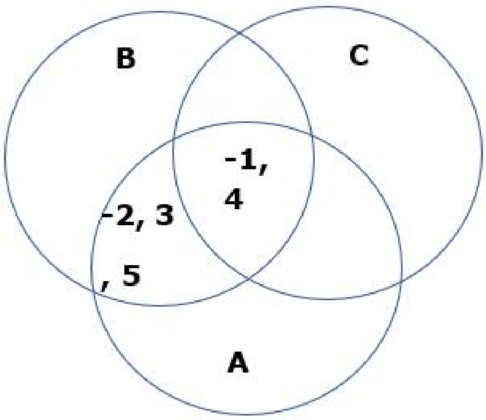




A ∩ (B ∩ C)



(A ∩ C)



(A ∩ B) ∩ (A ∩ C)

Exercise 1.2

1. Question

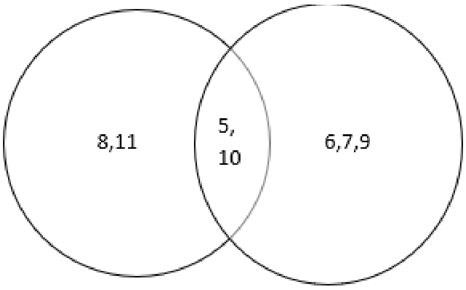
Represent the following using Venn diagrams

(i) U = {5,6,7,8, ...13}, A = {5,8,10,11}, and B = {5,6,7,9,10}

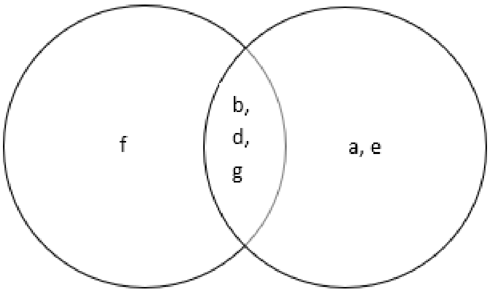
(ii) U = {a, b, c, d, e, f, g, h}, M = {b, d, f, g}, and N = {a, b, d, e, g}

Answer

(i) Given A = {5, 8, 10, 11} and B = {5, 6, 7, 9, 10}

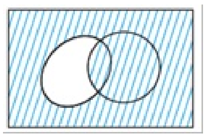


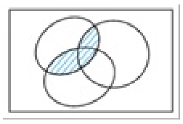
(ii) Given M = {b, d, f, g} and N = {a, b, d, e, g}

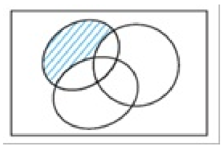


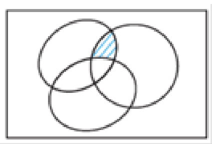
2. Question

Write a description of each shaded area. Use symbols U, A, B, C,â,∩, ’ and \ as necessary

(i) 

(ii) 

(iii) 

(iv) 

Answer

(i) A' U (A⋂B) or (A/B)'

(ii) (A⋂B) U (A⋂C)

(iii) A / (B U C)

(iv) (A ⋂ B) / C

3. Question

Draw Venn diagram of three sets A,B and C illustrating the following:

(i) A ∩ B ∩ C

(ii) A and B are disjoint but both are subsets of C

(iii) A ∩ (B\C)

(iv) (B ∪ C)\A

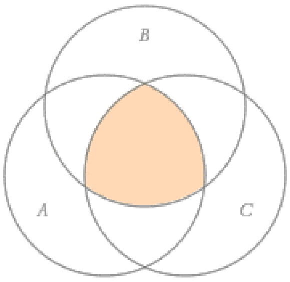
(v) A∪(B ∪ C)

(vi) C ∪ (B\A)

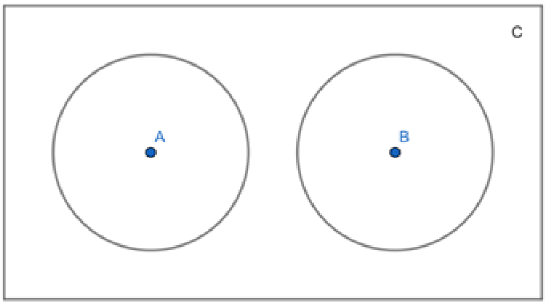
(vii) C ∩ (B ∪ A)

Answer

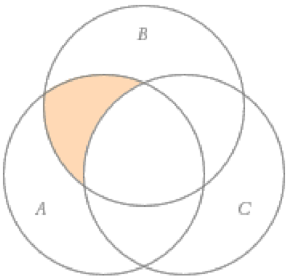
(i) A ⋂ B ⋂ C



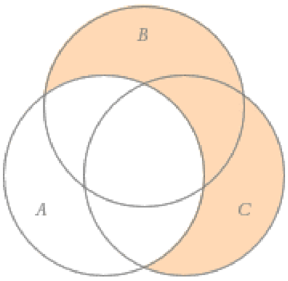
(ii) A and B are disjoint but both are subsets of C



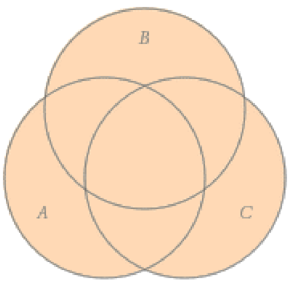
(iii) A ⋂ (B \ C)



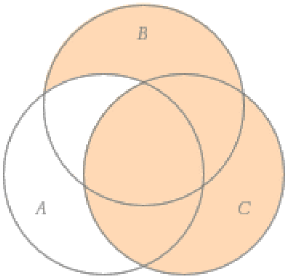
(iv) (B ⋃ C) \ A



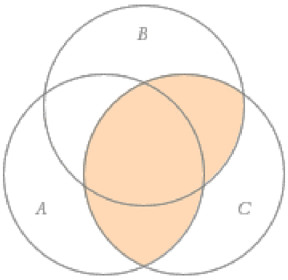
(v) A ⋃ (B ⋃ C)



(vi) C ⋃ (B \ A)



(vii) C ⋂ (B ⋃ A)



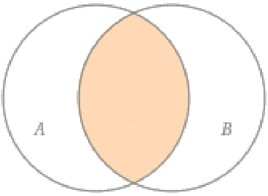
4. Question

Use Venn diagram to verify (A ∩ B) ∪ (A\B) = A.

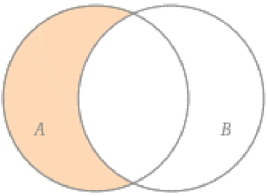
Answer

LHS:

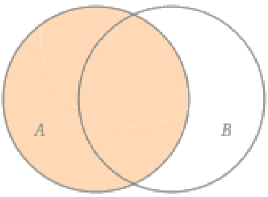
A ⋂ B



A \ B

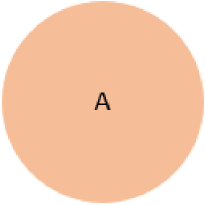


(A ⋂ B) ⋃ (A \ B)



RHS:

A



∴ LHS = RHS

Hence, (A ⋂ B) ⋃ (A \ B) = A

5. Question

Let U = {4, 8, 12, 16, 20, 24, 28}, A = {8, 16, 24} and B = {4, 16, 20, 28}. Find (A∪ B)' and (A ∩ B)'

Answer

(i) Consider A ⋃ B,

⇒ A ⋃ B = {8, 16, 24} ⋃ {4, 16, 20, 28} = {4, 8, 16, 20, 24, 28}

Now, (A ⋃ B)' = U \ {4, 8, 16, 20, 24, 28} = {12}

(ii) Consider A ⋂ B,

⇒ A ⋂ B = {8, 16, 24} ⋂ {4, 16, 20, 28} = {16}

Now, (A ⋃ B)' = U \ {16} = {4, 8, 12, 20, 24, 28}

6. Question

Given that U = {a, b, c, d, e, f, g, h}, A = {a, b, f, g}, and B = {a, b, c}, verify De Morgan's laws of complementation.

Answer

We know that De Morgan's laws of contemplation are:

(a) (A ⋃ B)' = A' ⋂ B'

(b) (A ⋂ B)' = A' ⋃ B'

Case (a):

First, we shall verify (a). To do this, we consider

A ⋃ B = {a, b, f, g} ⋃ {a, b, c} = {a, b, c, f, g}

⇒ (A ⋃ B)' = U \ {a, b, c, f, g} = {d, e, h} … (1)

Now, A' = U \ A = {c, d, e, h}

⇒ B' = U \ B = {d, e, f, g, h}

Then, A' ⋂ B' = {c, d, e, h} ⋂ {d, e, f, g, h} = {d, e, h} … (2)

From (1) and (2), it follows that (A ⋃ B)' = A' ⋂ B'.

Case (b):

Next, we shall verify (b). To do this, we consider

A ⋂ B = {a, b, f, g} ⋂ {a, b, c} = {a, b}

⇒ (A ⋂ B)' = U \ {a, b} = {c, d, e, f, g, h} … (3)

Now, A' = U \ A = {c, d, e, h}

⇒ B' = U \ B = {d, e, f, g, h}

Then, A' ⋃ B' = {c, d, e, h} ⋃ {d, e, f, g, h} = {c, d, e, f, g, h} … (4)

From (3) and (4), it follows that (A ⋂ B)' = A' ⋃ B'.

Hence, De Morgan’s laws of contemplation are verified.

7. Question

Verify De Morgan’s laws for set difference using the sets given below:

A = {1, 3, 5, 7, 9, 11, 13, 15}, B = {1, 2, 5, 7} and C = {3, 9, 10, 12, 13}.

Answer

We know that De Morgan’s laws for set difference are:

(a) A \ (B ⋃ C) = (A \ B) ⋂ (A \ C)

(b) A \ (B ⋂ C) = (A \ B) ⋃ (A \ C)

Case (a):

First, we shall verify (a). To do this consider,

B ⋃ C = {1, 2, 5, 7} ⋃ {3, 9, 10, 12, 13}

= {1, 2, 3, 5, 7, 9, 10, 12, 13}

⇒ A \ (B ⋃ C) = A \ {1, 2, 3, 5, 7, 9, 10, 12, 13}

= {11, 15} … (1)

Now, A \ B = {1, 3, 5, 7, 9, 11, 13, 15} \ {1, 2, 5, 7}

= {3, 9, 11, 13, 15}

And A \ C = {1, 3, 5, 7, 9, 11, 13, 15} \ {3, 9, 10, 12, 13}

= {1, 5, 7, 11, 15}

Then, (A \ B) ⋂ (A \ C) = {3, 9, 11, 13, 15} ⋂ {1, 5, 7, 11, 15}

= {11, 15} … (2)

From (1) and (2), it follows that A \ (B ⋃ C) = (A \ B) ⋂ (A \ C).

Case (b):

Next, we shall verify (b). To do this consider,

B ⋂ C = {1, 2, 5, 7} ⋂ {3, 9, 10, 12, 13}

= ϕ

⇒ A \ (B ⋂ C) = A \ ϕ

= {1, 3, 5, 7, 9, 11, 13, 15} … (3)

Now, A \ B = {1, 3, 5, 7, 9, 11, 13, 15} \ {1, 2, 5, 7}

= {3, 9, 11, 13, 15}

And A \ C = {1, 3, 5, 7, 9, 11, 13, 15} \ {3, 9, 10, 12, 13}

= {1, 5, 7, 11, 15}

Then, (A \ B) ⋃ (A \ C) = {3, 9, 11, 13, 15} ⋂ {1, 5, 7, 11, 15}

= {1, 3, 5, 7, 9, 11, 13, 15} … (4)

From (3) and (4), it follows that A \ (B ⋂ C) = (A \ B) ⋃ (A \ C).

Hence, De Morgan’s laws of set difference are verified.

8. Question

Let A **=** {10, 15, 20, 25, 30, 35, 40, 45, 50}**,** B **=** {1, 5, 10, 15, 20, 30} and C **=** {7, 8, 15, 20, 35, 45, 48}**.** Verify A \ (B ⋂ C) = (A \ B) ⋃ (A \ C).

Answer

To do this consider,

B ⋂ C = {1, 5, 10, 15, 20, 30} ⋂ {7, 8, 15, 20, 35, 45, 48}

= {15, 20}

⇒ A \ (B ⋂ C) = A \ {15, 20}

= {10, 25, 30, 35, 40, 45, 50} … (1)

Now, A \ B = {10, 15, 20, 25, 30, 35, 40, 45, 50} \ {1, 5, 10, 15, 20, 30}

= {25, 35, 40, 45, 50}

And A \ C = {10, 15, 20, 25, 30, 35, 40, 45, 50} \ {7, 8, 15, 20, 35, 45, 48}

= {10, 25, 30, 40, 50}

Then, (A \ B) ⋃ (A \ C) = {25, 35, 40, 45, 50} ⋂ {10, 25, 30, 40, 50}

= {10, 25, 30, 35, 40, 45, 50} … (2)

From (1) and (2), it follows that A \ (B ⋂ C) = (A \ B) ⋃ (A \ C).

Hence, verified.

9 A. Question

Using Venn diagram, verify whether the following are true:

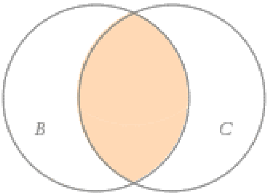
A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Answer

A ⋃ (B ⋂ C) = (A ⋃ B) ⋂ (A ⋃ C)

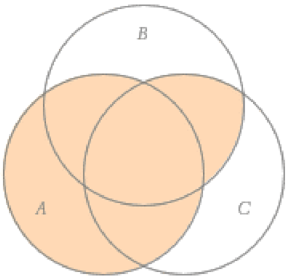
LHS:

Consider B ⋂ C,



Then

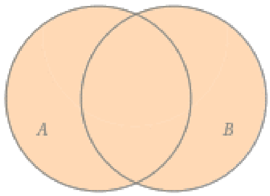
A ⋃ (B ⋂ C)



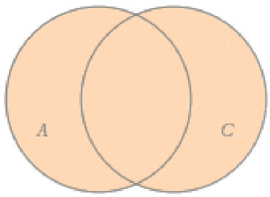
= LHS

RHS:

Consider A ⋃ B,

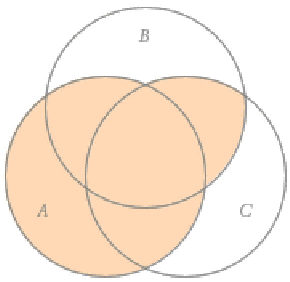


Then, consider A ⋃ C,



Then,

(A ⋃ B) ⋂ (A ⋃ C)



= RHS

∴ LHS = RHS

Hence verified.

9 B. Question

Using Venn diagram, verify whether the following are true:

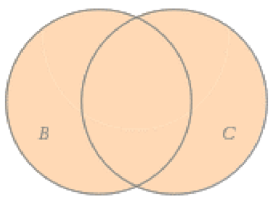
A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Answer

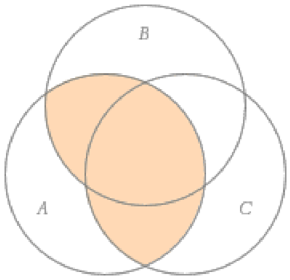
A ⋂ (B ⋃ C) = (A ⋂ B) ⋃ (A ⋂ C)

LHS:

Consider B ⋃ C,



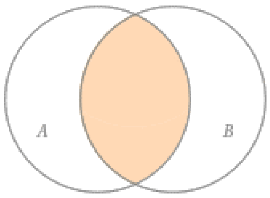
Then, A ⋂ (B ⋃ C),



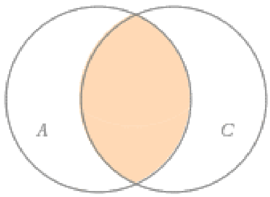
= LHS

RHS:

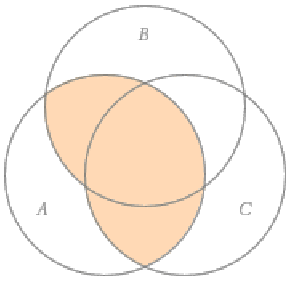
Consider A ⋂ B,



Then, consider A ⋂ C,



Then, (A ⋂ B) ⋃ (A ⋂ C)



= RHS

∴ LHS = RHS

Hence proved.

9 C. Question

Using Venn diagram, verify whether the following are true:

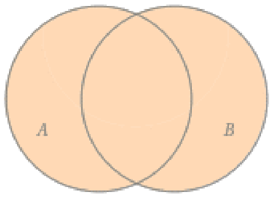
(A ∪ B)’ = A’ ∩ B’

Answer

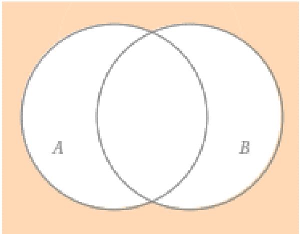
(A ⋃ B)’ = A’ ⋂ B’

LHS:

Consider A ⋃ B,



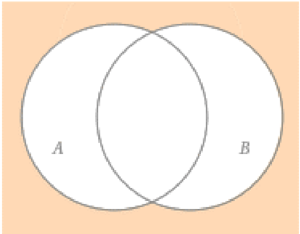
Then, (A ⋃ B)’,



= LHS

RHS:

A’ ⋂ B’



= RHS

∴ LHS = RHS

Hence proved.

9 D. Question

Using Venn diagram, verify whether the following are true:

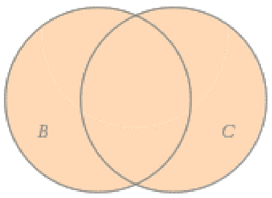
A \ (B ∪ C) = (A \ B) ∩ (A \ C)

Answer

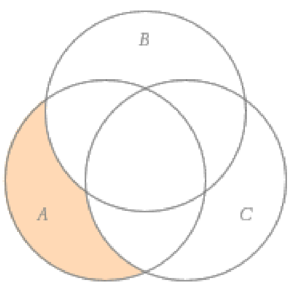
A \ (B ⋃ C) = (A \ B) ⋂ (A \ C)

LHS:

Consider B ⋃ C,



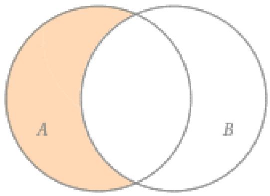
Then, A \ (B ⋃ C)



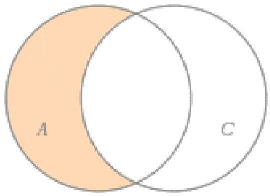
= LHS

RHS:

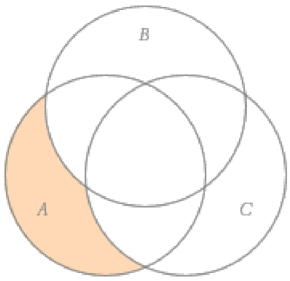
Consider A \ B,



Then, consider A \ C,



Then, (A \ B) ⋂ (A \ C)



= RHS

∴ LHS = RHS

Hence proved.

Exercise 1.3

1. Question

If A and B are two sets and U is the universal set such that n(∪) = 700, n(A) = 200, n(B) = 300 and n (A ∩ B) = 100,find n(A’ ∩ B’).

Answer

Here we are provided with the cardinality of two sets A and B with and a universal set U and the cardinality of intersection of both the sets A and B is also provided.

The cardinality of the sets are as follows,

n(U) = 700

n(A) = 200

n(B) = 300

n(A ∩ B) = 100

From De Morgan’s Law we know that if U is an universal set and it contain two set named A and B then,

<u>n(A’ ⋂ B’) = n(A ⋃ B)’</u> …(I)

To find the value of n(A' ∩ B') we first we have to find the value of (A U B)'.

We know that when the cardinality of two sets and the cardinality of their intersection is given we can find the cardinality of their union using the formula given below,

$\underline{n(A \cup B) = n(A) + n(B) - n(A \cup B)}$

When we have the cardinality of union of two set the cardinality of their union difference (A U B)' can be found using the formula below

$\underline{n(A \cup B)' = n(U) - n(A \cup B)}$

*(A U B)' show the elements which are only the part of the universal set U and doesn’t exist in (A U B).

So using formula and putting values,

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$= 200 + 300 - 100$

$= 500 - 100$

$= 400$

and $n(A \cup B)' = n(U) - n(A \cup B)$

$= 700 - 400$

$= 300$

So putting these values in eq^n (i) we will find

$n(A' \cap B') = n(A \cup B)'$

$n(A' \cap B') = 300$

2. Question

Given n(A) = 285, n(B) = 195, n(∪) = 500, n(A ∪ B) = 410, find n(A’ ∩ B’).

Answer

Here we are provided with the cardinality of two sets A and B with and a universal set U and the cardinality of union of both the sets A and B is also provided.

The cardinality of the sets are as follows,

$n(U) = 500$

$n(A) = 285$

$n(B) = 195$

$n(A \cup B) = 410$

From De Morgan’s Law we know that if U is an universal set and it contain two set named A and B then,

$n(A' \cap B') = n(A \cup B)'$......(i)

To find the value of $n(A' \cap B')$ we first we have to find the value of $(A \cup B)'$.

We know that when the cardinality of union of two sets is given we can find the cardinality of their union difference by using the formula given below, The cardinality of $(A \cup B)'$ is given by

$\underline{n(A \cup B)' = n(U) - n(A \cup B)}$

So using the formula and data given

$n(A \cup B)' = n(U) - n(A \cup B)$

$= 510 - 410$

$= 90$

Now from De Morgan’s law,

$n(A' \cap B') = n(A \cup B)'$

$= 90$

3. Question

For any three sets A, B and C if n(A) = 17 n(B) = 17, n(C) = 17, n (A ∩ B) = 7 n(B ∩ C) = 6, n(A ∩ C) = 5 and n(A ∩ B ∩ C) = 2, find n(A ∪ B ∪ C).

Answer

Here we are provided with the cardinality of three set A, B, C along with the cardinality of their intersection and those values are as follows

n(A) = 17

n(B) = 17

n(C) = 17

n(A ∩ B) = 7

n (B ∩ C) = 6

(A ∩ C) = 5

Here we have to find the value of n(A U B U C)

We know that when we have the cardinality of three known sets with the cardinality of their intersections and we have to find the cardinality of A U B U C we can use the formula

n(A U B U C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

…… (i)

So using the formula putting the values we will find

n(A U B U C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

= 17 + 17 + 17 – 7 – 6 – 5 + 2

= 53 – 18 + 2

= 55 – 20

= 35

So n(A U B U C) = 35

4 A. Question

Verify n(A ∪ B ∪ C) = n(A) + n(B) + n(C) –n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C) for the sets given below:

A = {4,5,6}, B = {5,6,7,8} and C = {6,7,8,9}

Answer

Here we are given that

A = {4, 5, 6}

B = {5, 6, 7, 8}

C = {6, 7, 8, 9}

Here we have to verify the eq^{n} given below

n(A U B U C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

Here we have to find the cardinality of all the sets and their intersection. So basing on the data given we can find the values for the eq^{n} .

(A ⋂ B ⋂ C) will give elements which are common to all the three sets while (A ⋃ B ⋃ C) will give combination of all distinct elements

A = {4, 5, 6}

B = {5, 6, 7, 8}

C = {6, 7, 8, 9}

(A ∩ B) = {4, 5, 6} ∩ {5, 6, 7, 8} = {5, 6}

(B ∩ C) = {5, 6, 7, 8} ∩ {6, 7, 8, 9} = {6, 7, 8}

(A ∩ C) = {4, 5, 6} ∩ {6, 7, 8, 9} = {6}

(A ∩ B ∩ C) = {4, 5, 6} ∩ {5, 6, 7, 8} ∩ {6, 7, 8, 9} = {6}

(A ⋃ B ⋃ C) = {4, 5, 6} ⋃ {5, 6, 7, 8} ⋃ {6, 7, 8, 9}

= {4, 5, 6, 7, 8, 9}

So from the above expression we can find the cardinality of all the values,

n(A) = 3

n(B) = 4

n(C) = 4

n(A∩B) = 2

n(B∩C) = 3

n(A∩C) = 1

n(A ∩ B ∩ C) = 1

n(A ⋃ B ⋃ C) = 6

now using the formula,

n(A U B U C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

⇒ 6 = 3 + 4 + 4 – 2 – 3 – 1 + 1

⇒ 6 = 6 hence verified

4 B. Question

Verify n(A ∪ B ∪ C) = n(A) + n(B) + n(C) –n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C) for the sets given below:

A = {a,b,c,d,e}, B = {x,y,z} and C = {a,e,x}

Answer

Here we are given that

A = {a, b, c, d, e}

B = {x, y, z}

C = {a, e, x}

Here we have to verify the eq^n given below

n(AUBUC) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(A∩C) + n(A∩B∩C)

Here we have to find the cardinality of all the sets and their intersection. So basing on the data given we can find the values for the eq^n .

We are given that,

A = {a, b, c, d, e}

B = {x, y, z}

C = {a, e, x}

$(A \cap B) = \{a, b, c, d, e\} \cap \{x, y, z\} = \{\Phi\}$

∵ it's a Disjoint set and it has no element

$(B \cap C) = \{x, y, z\} \cap \{a, e, x\} = \{x\}$

$(A \cap C) = \{a, b, c, d, e\} \cap \{a, e, x\} = \{a, e\}$

$(A \cap B \cap C) = \{a, b, c, d, e\} \cap \{x, y, z\} \cap \{a, e, x\} = \{\Phi\}$

∵ it's a Disjoint set as $A \cap B$ has no element

$(A \cup B \cup C) = \{a, b, c, d, e\} \cup \{x, y, z\} \cup \{a, e, x\}$

$= \{a, b, c, d, e, x, y, z\}$

So from the above expression we can find the cardinality of all the values,

$n(A) = 5$

$n(B) = 3$

$n(C) = 3$

$n(A \cap B) = 0$

$n(B \cap C) = 1$

$n(A \cap C) = 2$

$n(A \cap B \cap C) = 0$

$n(A \cup B \cup C) = 8$

now using the formula and putting values obtained,

$n(AUBUC) = n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(A \cap C) + n(A \cap B \cap C)$

$\Rightarrow 8 = 5 + 3 + 3 – 0 – 1 – 2 + 0$

$\Rightarrow 8 = 8$ hence verified

5. Question

In a college, 60 students enrolled in chemistry, 40 in physics, 30 in biology, 15 in chemistry and physics, 10 in physics and biology, 5 in biology and chemistry. No one enrolled in all the three. Find how many are enrolled in at least one of the subjects.

Answer

Let's consider that student enrolled in chemistry, physics and Biology be C, P, B respectively

We are given the number of students enrolled per subject so can write,

Number of students enrolled in chemistry $n(C) = 60$

Number of students enrolled in Physics $n(P) = 40$

Number of students enrolled in biology $n(B) = 30$

Number of students enrolled in chemistry and Physics $n(C \cap P) = 15$

Number of students enrolled in Physics and Biology $n(P \cap B) = 10$

Number of students enrolled in biology and chemistry $n(B \cap C) = 5$

Number of students enrolled in Physics Biology and

Chemistry $n(P \cap B \cap C) = 0$

Here to find the number of student enrolled in at least on subjects first we have to find the of sum student with all three subject, with two subjects only and with only one subject

To find the no of student enrolled to a single subject only we have to find the differential value of each subjects.

So,

Student enrolled in Physics only

$n(P') = n(P) - n(C \cap P) - n(P \cap B) - n(P \cap B \cap C)$

$= 40 - 15 - 10 - 0 = 15$

Student enrolled in Chemistry only,

$n(C') = n(C) - n(C \cap P) - n(C \cap B) - n(P \cap B \cap C)$

$= 60 - 15 - 5 - 0$

$= 40$

Student enrolled in Biology only,

$n(B') = n(B) - n(P \cap B) - n(B \cap C) - n(P \cap B \cap C)$

$= 30 - 10 - 5 - 0$

$= 15$

So number of student enroll for at least one subject,

$n(P \cup B \cup C) = n(P') + n(B') + n(C') - n(P \cap B) - n(B \cap C) - n(P \cap C) + n(P \cap B \cap C)$

$= 15 + 40 + 15 + 15 + 10 + 5 + 0$

$= 100$

6. Question

In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also, 32% speak English and Tamil, 13% speak Tamil and Hindi and 10% speak English and Hindi, find the percentage of people who can speak all the three languages.

Answer

Let T, E and H be the people speaking Tamil, English and Hindi respectively.

so we are given

Total percentage of people = 100%

Percentage of people speaking Tamil = n(T) = 85%

Percentage of people speaking English = n(E) = 40%

Percentage of people speaking Hindi = n(H) = 20 %

Percentage of people speaking Tamil and Hindi = $n(T \cap H)$ = 13%

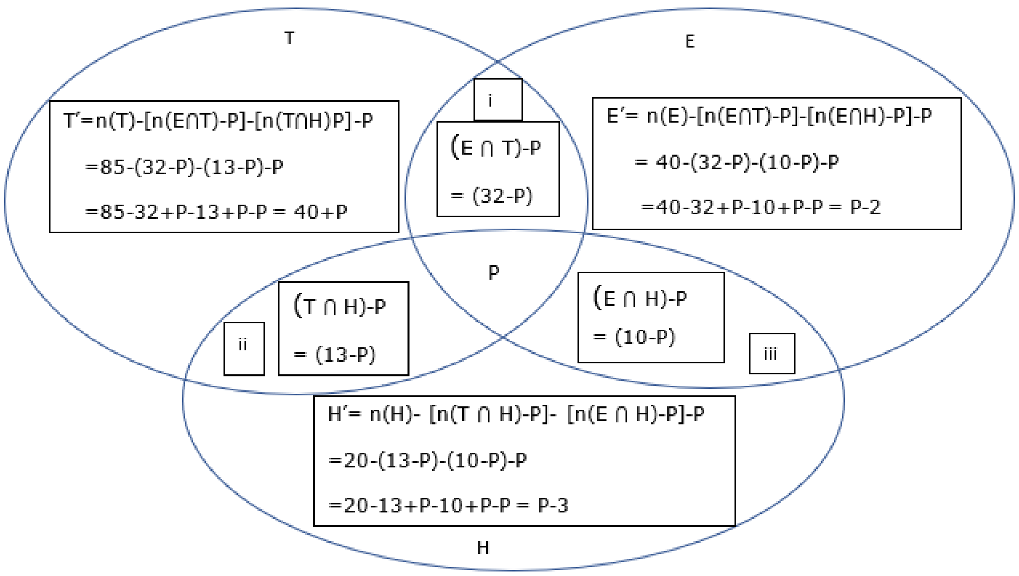
Percentage of people speaking English and Hindi = $n(E \cap H)$ = 10%

Percentage of people speaking English and Tamil = $n(E \cap T)$ = 32%

Percentage of people speaking Tamil, English and Hindi = $n(T \cap E \cap H)$

Assume that Percentage of people speaking Tamil, English and Hindi be P.

We can solve this by using Venn diagram,



The above Venn diagram gives the following,

People speaking all the three language = P

People speaking English and Hindi and no tamil = (E ⋂ H ⋂ T') = 10 – P

People speaking Tamil and Hindi and no English = (T ⋂ H ⋂ E') = 13 – P

People speaking English and Tamil and no Hindi = (T ⋂ E ⋂ H') = 32 – P

Only Hindi speaking people = H' = P – 3

Only English speaking people = E' = P – 2

Only Tamil speaking people = T' = 40 + P

We know that adding all the region of the Venn diagram will give the value of total number of element involved.

So, using all the data we have we can find the following

Total people = T' + H' + E' + (E⋂ H ⋂ T') + (T ⋂ H ⋂ E') + (T ⋂ E ⋂ H') + P

⇒100 = (40 + P) + (P – 3) + (P – 2) + (10 – P) + (13 – P) + (32 – P) + P

⇒ 100 = 40 + P + P – 3 + P – 2 + 10 – P + 13 – P + P

Eliminating elements of opposite signs

⇒100 = 90 + P

⇒P = 100 – 90

⇒P = 10

∴ From the above its clear that only 10% people can speak all the three language.

7. Question

An advertising agency finds that, of its 170 clients, 115 use Television, 110 use Radio and 130 use Magazines. Also, 85 use Television and Magazines, 75 use Television and Radio, 95 use Radio and Magazines, 70 use all the three. Draw Venn diagram to represent these data. Find

(i) how many use only Radio?

(ii) how many use only Television?

(iii) how many use Television and magazine but not radio?

Answer

Lets Let T,R and M are the people who use Television, Radio and Magazines respectively.

Number of people who use Television n(T) = 115

Number of people who use Radio n(R) = 110

Number of people who use Magazine n(M) = 130

Number of people who use Television and Magazines, n(T ∩ M) = 85

Number of people who use Television and Radio, n(T ∩ R) = 75

Number of people who use Radio and Magazine, n(R ∩ M) = 95

Number of people who use all the three n(T ∩ R ∩ M) = 70

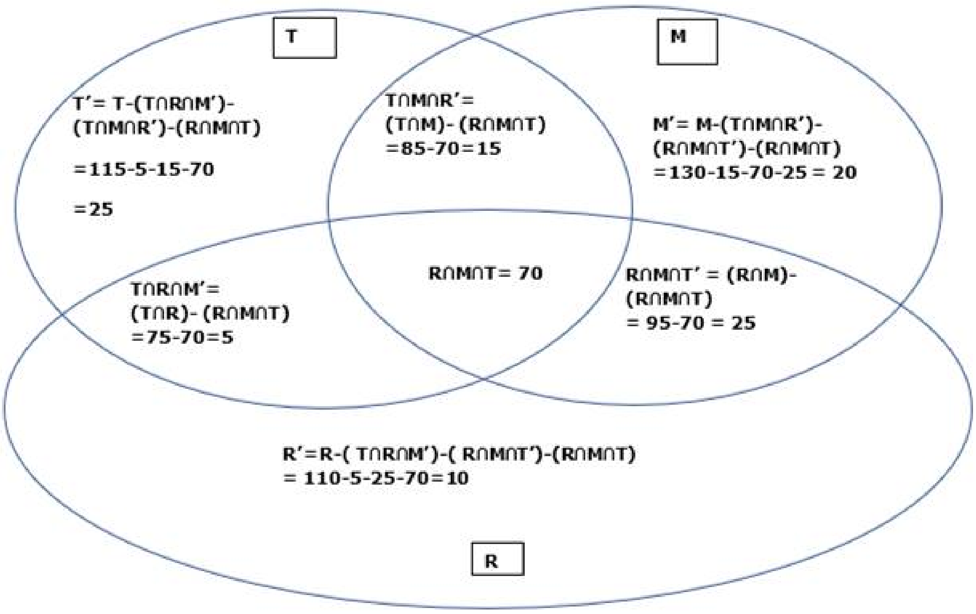
Here we have to find,

Number of People using Radio only = n(R')

Number of People using Television only = n(T')

Number of People using Television & magazine but not radio = n(T∩M∩R')

Using the data given we can draw a Venn diagram,



So from the Venn diagram it's clear that

(i) Number of People using Radio only = R' = 10

(ii) Number of people using television only = T' = 25

(iii) Number of People using Television & magazine but not radio = n(T∩M∩R') = 15

8. Question

In a school of 4000 students, 2000 know French, 3000 know Tamil and 500 know Hindi, 1500 know French and Tamil, 300 know French and Hindi, 200 know Tamil and Hindi and 50 know all the three languages.

(i) How many do not know any of the three languages?

(ii) How many know at least one language?

(iii) How many know only two languages?

Answer

Let total student be U and students who know Tamil, Hindi and French be T, H, F respectively.

We know that,

No of student, U = 4000

No students who know Tamil = T = 3000

No students who know French = F = 2000

No students who know Hindi = H = 500

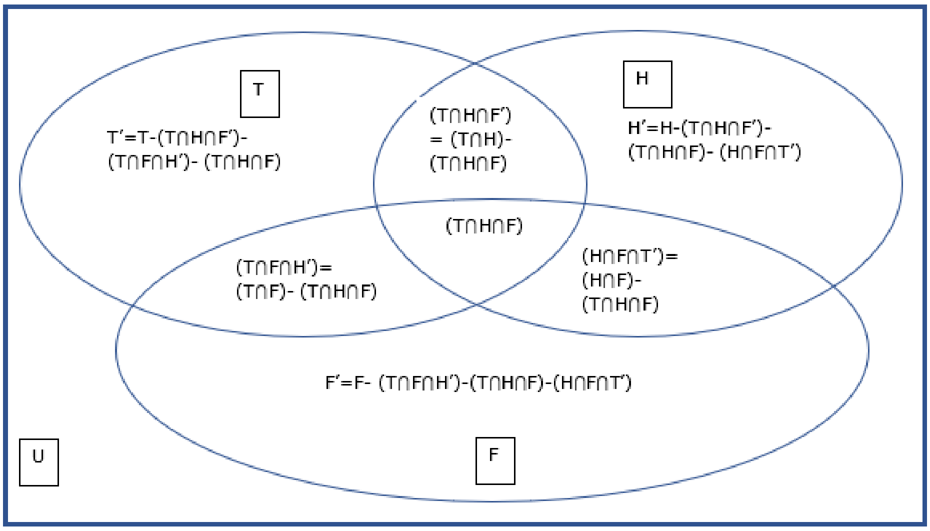
No students who know French and Tamil = $(F \cap T) = 1500$

No students who know Tamil and Hindi = $(T \cap H) = 200$

No students who know Hindi and French = $(H \cap F) = 300$

No students who know Tamil, Hindi and french = $(T \cap H \cap F) = 50$

it can be represented in a venn diagram



Student who knows only French and Tamil,

$(T \cap F \cap H') = [(T \cap F) - (T \cap H \cap F)] = 1500 - 50 = 1450$

Student who knows only Hindi and Tamil,

$(T \cap H \cap F') = (T \cap H) - (T \cap H \cap F) = 200 - 50 = 150$

Student who knows only French and Hindi,

$(H \cap F \cap T') = (F \cap H) - (T \cap H \cap F) = 300 - 50 = 250$

Student who knows only French, $F' = F - (H \cap F \cap T') - (T \cap H \cap F) - (T \cap F \cap H')$

$= 2000 - 1450 - 250 - 50 = 250$

Student who knows only Tamil, $T' = T - (T \cap H \cap F') - (T \cap F \cap H') - (T \cap H \cap F)$

$= 3000 - 150 - 1450 - 50 = 1350$

Student who knows only Hindi, $H' = H - (T \cap H \cap F') - (T \cap H \cap F) - (H \cap F \cap T')$

$= 500 - 150 - 50 - 250 = 50$

(i) So no of student who don't any of the three languages are the students who lies outside the set of T, H, F denoted as $(F \cup H \cup T)'$.

from cardinality of sets we know that

$n(F \cup H \cup T) = n(F) + n(H) + n(T) - n(F \cap H) - n(H \cap T) - n(F \cap T) + n(F \cap H \cap T)$

putting the values

$n(F \cup H \cup T) = 2000 + 500 + 3000 - 300 - 200 - 1500 + 50$

$= 3550$

So no of student who don’t any of the three languages $(F \bigcup H \bigcup T)'$ given as

$n(F \cup H \cup T)' = n(U) - n(F \cup H \cup T)$

$= 4000 - 3550$

$= 450$

Hence 450 no of student who don’t any of the three languages.

(ii) At least one language means the student must know minimum of one language while at max it can be 3. So its clear that here we have to find the total number of student who knows the language which is denoted as $n(F \bigcup H \bigcup T)$

Using cardinality of set theory

$n(F \cup H \cup T) = n(F) + n(H) + n(T) - n(F \cap H) - n(H \cap T) - n(F \cap T) + n(F \cap H \cap T)$

putting the values

$n(F \bigcup H \bigcup T) = 2000 + 500 + 3000 - 300 - 200 - 1500 + 50$

$= 3550$

So 3550 number of student knows at least one language.

(iii) Here we have to find the number of student who knows only two language.

So

Student who knows only two language =

$= (T \bigcap F \bigcap H') + (T \bigcap H \bigcap F') + (H \bigcap F \bigcap T')$

$= 1450 + 150 + 250$

$= 1850$

So 1850 number student knows two languages only.

9. Question

In a village of 120 families, 93 families use firewood for cooking, 63 families use kerosene, 45 families use cooking gas, 45 families use firewood and kerosene, 24 families use kerosene and cooking gas, 27 families use cooking gas and firewood. Find how many use firewood, kerosene and cooking gas.

Answer

Here let’s consider that F, G, K be the families using Firewood, Gas, Kerosene for cooking respectively and no of families be U

It’s given that

No of families = $n(U) = 120$

Families using firewood for cooking = $n(F) = 93$

Families using Gas for cooking = $n(G) = 45$

Families using kerosene for cooking = $n(K) = 63$

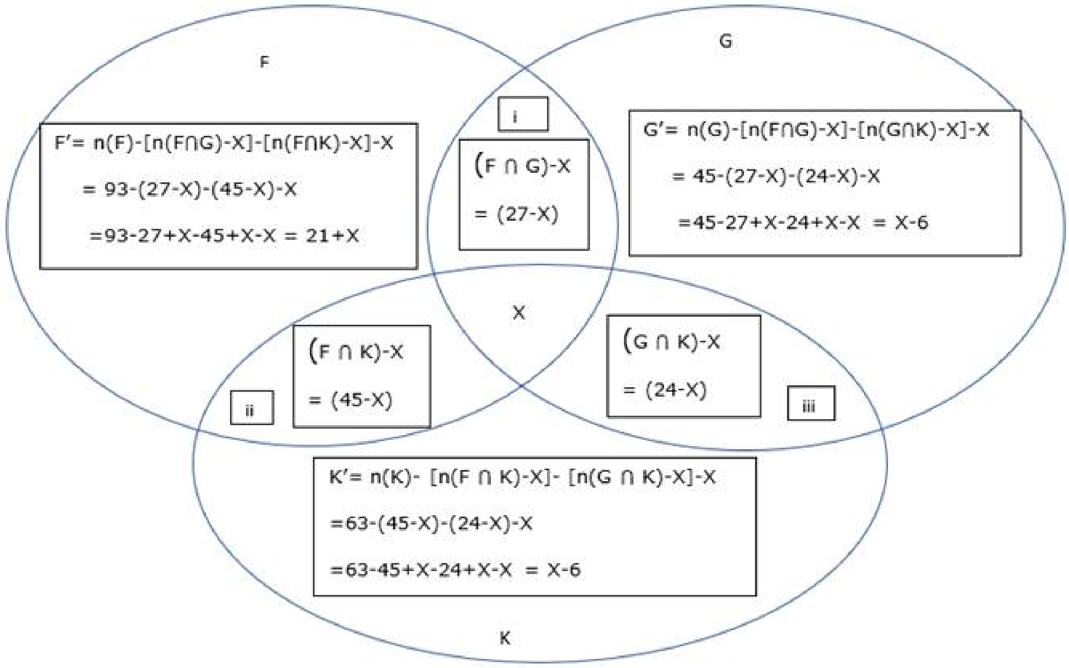
Families using firewood and kerosene for cooking = $n(F \bigcap K) = 45$

Families using kerosene and gas for cooking = $n(K \bigcap G) = 24$

Families using gas and firewood for cooking = $n(G \bigcap F) = 27$

Families using gas,kerosene and firewood for cooking = $n(G \bigcap K \bigcap F)$

Let Families using gas, kerosene and firewood for cooking = X



So from the above Venn diagram we get the following data,

People using only Gas, G' = X – 6

People using only Kerosene K' = X – 6

People using only firewood, F' = 21 + X

People using only firewood and gas = (F ⋂ G) – X

People using only gas and kerosene = (G ⋂ K) – X = (24 – X)

People using only firewood and kerosene = (F ⋂ K) – X = (45 – X)

We know that adding all the region of the Venn diagram will give the value of total number of element involved.

So, using all the data we have we can find the following

No of families = no of families using one fuel + no of families using two

Fuels + no of families using all the three fuels

Using the data obtained in the above expression we will find,

U = F' + G' + K' + (F ⋂ G ⋂ K') + (G ⋂ K ⋂ F') + (F ⋂ K ⋂ G') + X

U = F' + G' + K' + (F ⋂ G) – X + (G ⋂ K) – X + (F ⋂ K) – X + X

PUTTING ALL THE VALUES

⇒ U = 21 + X + X – 6 + X – 6 + 27 – X + 24 – X + 45 – X + X

⇒ 120 = 105 – X

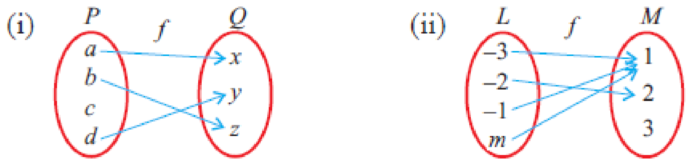
⇒ X = 15

Hence 15 people use all the three kind of fuel.

Exercise 1.4

1. Question

State whether each of the following arrow diagrams define a function or not. Justify your answer.



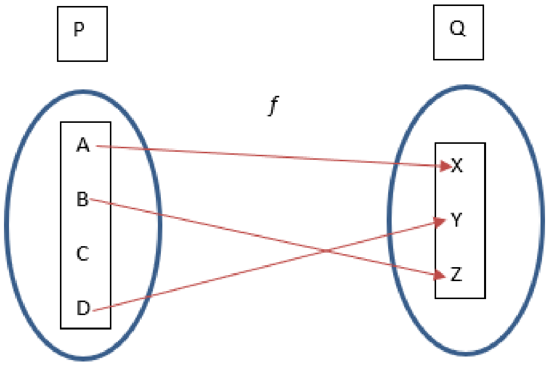
Answer

If P and Q are two relation then the two most important conditions for the relation to be treated as a function are as follows,

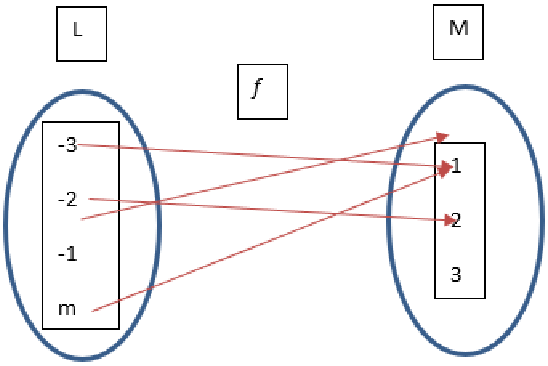
● Each element of P must have a unique image in Q.

● Each element of P has to be mapped with only one element in Q

If either of the above conditions is not satisfied, then the relation can't be consider as a function.



(i) The figure shows mapping from P to Q, the element "C" in P does not have any image in Q. Since, the above given relation does not meet the conditions for relation and functions mentioned above so it is not a function.



(ii) In the above mapping from L to m, each element of L has an image in M and the number of image for each element is one. Since, the above given relation satisfies all the conditions of relation and function mentioned above, so it is a function.

2. Question

For the given function F = { (1, 3), (2, 5), (4, 7), (5, 9), (3, 1) }, write the domain and range.

Answer

We know that for any two non-empty sets A and B. A function f from A to B may be defined as a rule that assigns each element of set A to tha of set B in a unique way (∀ x ∈ A and y ∈ B).

We denote y = f(x) which means mean y is a function of x. so the set A is called the domain of the function and set B is called the co-domain of the function. y is called as the image of x under f and x is called a preimage of y. Range may be defined as the set of all image and it's a subset of co-domain.

We can represent the function in set as described below

f = {(a,b) / a ∈ A and b ∈ B}

Where A is the domain and B is the pre-domain.

So as per data given,

F = {(1, 3), (2, 5), (4, 7), (5, 9), (3, 1)}

Here

Domain = {1, 2, 3, 4, 5}

Range = {1, 3, 5, 7, 9}

3. Question

Let A = { 10, 11, 12, 13, 14 }; B = { 0, 1, 2, 3, 5 } and fi : A→ B , i = 1,2,3.

State the type of function for the following (give reason):

(i) f_1 = { (10, 1), (11, 2), (12, 3), (13, 5), (14, 3) }

(ii) f_2 = { (10, 1), (11, 1), (12, 1), (13, 1), (14, 1) }

(iii) f_3 = { (10, 0), (11, 1), (12, 2), (13, 3), (14, 5) }

Answer

There are several kind of function such as,

One-one function:

If f : A → B is a function then its treated as an one to one function if no element of B is associated with more than one element of A . A one-one function is also called an injective function.

Onto function:

A function f:A→B is called as an onto function if every element in B has a pre-image in A.

One-one and onto function:

A function f:A→B is called as an One-one and onto function if f maps all distinct elements of A with all distinct images in B and all element in B is an image of some element in A.

Constant function:

A function f:A→B is called as Constant function if all distinct elements of A has a single image in B.

Here we are given

A = { 10, 11, 12, 13, 14 }

B = { 0, 1, 2, 3, 5 }

fi:A → B , where i = 1,2,3.

(i) Here both 12 and 14 have same image i.e. 3 and there is no preimage of 0 of B in A, So it’s clear that the function given is neither an one to one nor onto.

(ii) Here it’s clear that for all element of A there is only one image in B i.e. 1.we can also write that for f_2:A→ B, $f_2(x) = 1$, ∀ x ∈ 1. So the above function is a constant function.

(iii) Here range of f_3 = {0,1,2,3,4,5} = B

It’s an one-one and onto function as all element in A have a distinct and non-repetitive image in B

4. Question

If X = {1, 2, 3, 4, 5 }, Y = { 1, 3, 5, 7, 9 } determine which of the following relations from X to Y are functions? Give reason for your answer. If it is a function, state its type.

(i) R_1 = {(x, y)| y = x + 2, x∈ X , y ∈ Y }

(ii) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}

(iii) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}

(iv) R_4 = {(1, 3), (2, 5), (4, 7), (5, 9), (3, 1)}

Answer

Here we are given

X = {1, 2, 3, 4, 5}, Y = { 1, 3, 5, 7, 9 }

(i) R_1 = {(x, y)| y = x + 2, x∈ X , y ∈ Y }

So using the value we can write the function in set form

R_1 = {(1,3),(3,5),(5,7)}

From the above form it's clear that as the elements of X doesn't have a unique image in Y so it's not a function.

(ii) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}

Here R_2 is a function as all the element of X has a distinct image in Y. As 1 and 2 of X is related to 1 of Y and 3, 4 of X is related to 3 of Y so this function is onto function.

(iii) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}

Here in this relation 1 of X is related to 1 and 3 of Y which clearly contradict the definition of a function which says that every elements of a domain should have at most one image. So it's clear that it's not a function.

(iv) R_4 = {(1, 3), (2, 5), (4, 7), (5, 9), (3, 1)}

The above expression is a onto and one-one function as all the elements of X has a unique image in Y and no two elements of X have same image in Y. these functions are also termed as bijective function.

5. Question

If R = {(a, - 2), (- 5, b), (8, c), (d, - 1)} represents the identity function, find the values of a, b, c and d.

Answer

Identity function is a function given by f(x) = x.Given function: R = {(a, - 2), (- 5, b), (8, c), (d, - 1)}This means,f(a) = -2f(-5) = bf(8) = cf(d) = -1Now, from the definition, f(x) = xtherefore,a = -2-5 = b, b = -58 = c, c = 8d = -1So a = -2, b = -5, c = 8 and d = -1

6. Question

A = {–2, –1, 1, 2 } and $f=\left\{\left(x,\frac{1}{x}\right):x\in A\right\}$. Write down the range of f. Is f a function from A to A?

Answer

From the elements of A we can write the function in set order

So f = $\{\left(-1,-\frac{1}{2}\right),(-1,-1),(1,1),(2,\frac{1}{2})\}$

Here from the above expression the range of the function is given as

Range of f = $(\frac{1}{2},-1,1,\frac{1}{2})$

As the element of the range are not in the set A so clearly it's not a function from A to A.

7. Question

Let f = {(2, 7), (3, 4), (7, 9), (–1, 6), (0, 2), (5, 3) } be a function from A = { –1, 0, 2, 3, 5, 7 } to B = { 2, 3, 4, 6, 7, 9}.

Answer

Here we are given that

f = {(2, 7), (3, 4), (7, 9), (–1, 6), (0, 2), (5, 3)}

A = { –1, 0, 2, 3, 5, 7 }

B = { 2, 3, 4, 6, 7, 9 } where f:A→ B

Here from the above expression the range of the function can be given as

Range of f = {7, 4, 9, 6, 2, 3} = B

So it clear that it is an onto function, again the mapping shows that every element of A is mapped to an unique element of B and no two elements of A is linked with a single element of B so it’s a one-one function too.

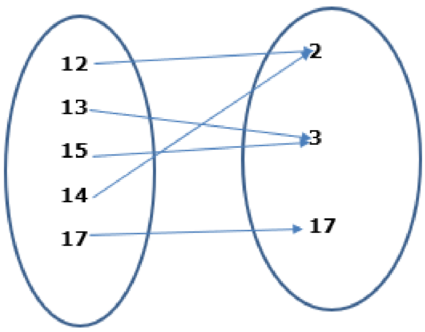
Hence its clear that the expression is a one-one and onto function

8. Question

Write the pre-images of 2 and 3 in the function

f = { (12, 2), (13, 3), (15, 3), (14, 2), (17, 17) }.

Answer



So from the image we can see that

Pre-image of 2 are 12 and 14

Pre-image of 3 are 13 and 15

9. Question

The following table represents a function from A = { 5, 6, 8, 10 } to B = { 19, 15, 9, 11 } where $f(x)$ = 2x - 1. Find the values of a and b.

x	5	6	8	10
$f(x)$	a	11	b	19

Answer

It’s given that f(x) = 2x-1

So f(5) = 2 × 5 -1

= 10-1

= 9

∴ a = 5

Similarly f(8) = 2 × 8 -1

= 16-1

= 15

∴ b = 15

So the values of a = 9 and b = 15

10. Question

Let A = {5, 6, 7, 8 }; B = { –11, 4, 7, –10,–7, –9,–13 } and f = {(x,y) : y = 3 - 2x, x∈ A, y ∈ B }|

(i) Write down the elements of f. (ii) What is the co-domain?

(iii) What is the range? (iv) Identify the type of function.

Answer

Here we are given

A = {5, 6, 7, 8 }

B = {–11, 4, 7, –10,–7, –9,–13 }

f = {(x,y) : y = 3 - 2x, x∈ A, y∈ B }

so we can find

f = {(x,y) : y = 3 - 2x, x∈ A, y∈ B }

f(5) = 3- 2×5 = 3-10 = -7

f(6) = 3 – 2×6 = 3-12 = -9

f(7) = 3 – 2×7 = 3-14 = -11

f(8) = 3 – 2×8 = 3- 16 = -13

(i) the elements of f are

From the above expression we get that

f = {(5,-7),(6,-9),(7,-11),(8,-13)}

(ii) co-domain

The co-domain is B and it can be given as

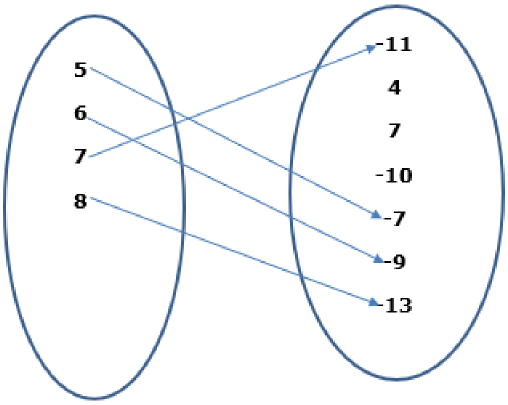
Codomain B = {-11,4, 7,-10,-7,-9,-13}

(iii) Range

Range may be defined as the image of A in B present in the function.

Range of f = {-7,-9,-11,-13}

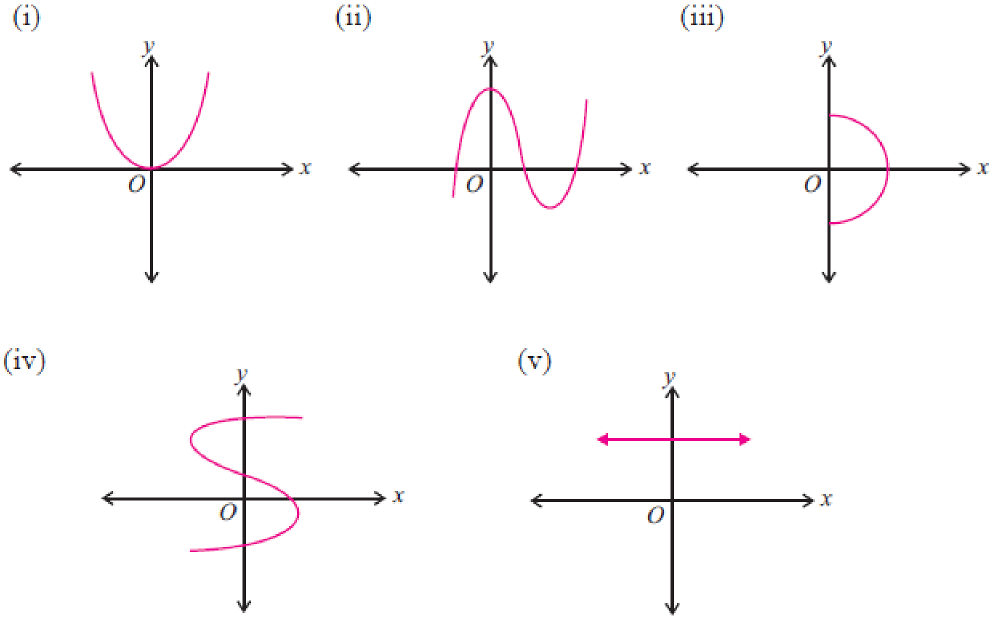
(iv) type of function



From the above data and the diagram it’s clear that the given function is a one-one function as all the element A has a unique and single image in B. it’s not a onto function as the range is not same as the co-domain .

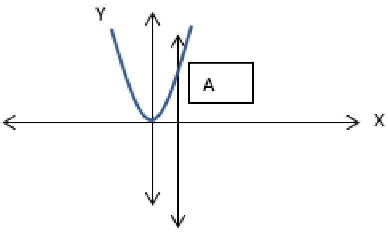
11. Question

State whether the following graphs represent a function. Give reason for your answer.



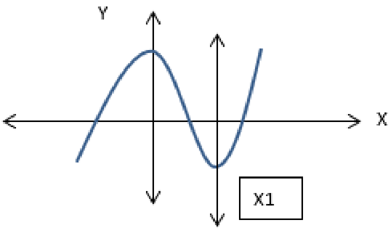
Answer

(i)



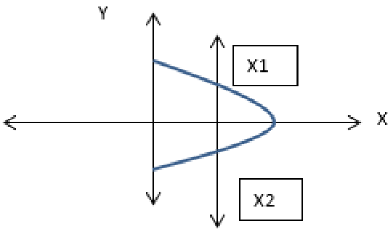
The above graph represents a function as we know that a graph represents a function only when the curve in the graph cuts the vertical axis only at a single point A.Here the curve cuts the vertical line at point X1 so it's a function.

(ii)



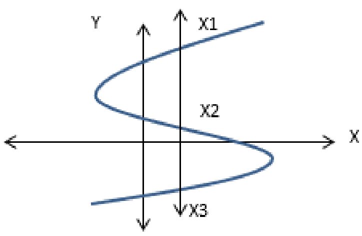
The above graph represents a function as a vertical line cuts the curve at X1 point.

(iii)

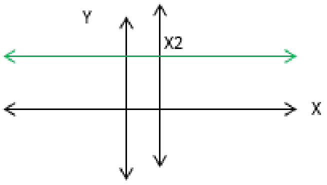


The above graoh doesn't represents a function as the vertical line and the curve intersects ech other at two points X1 and X2.

(iv)



The above figure doesn’t represent a function as the vertical line cuts the curve at three different points,.

(v) 

The graph is are presentation of a function as the straight line is cut at a single point X2 by the vertical line.

12. Question

Represent the function f = { (–1, 2), (– 3, 1), (–5, 6), (– 4, 3) } as

(i) a table (ii) an arrow diagram

Answer

We are given

f = { (–1, 2), (– 3, 1), (–5, 6), (– 4, 3) }

(i) a table

Representing the given data in tabular form

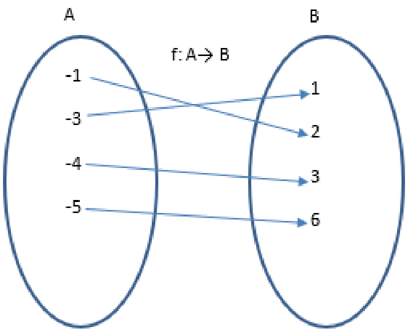
X	-1	-3	-5	-4
f(x)	2	1	6	3

(ii) an arrow diagram

we are given

A = {-1, -3, -5, -4}

B = {2, 1, 6, 3}



13. Question

Let A = { 6, 9, 15, 18, 21 }; B = { 1, 2, 4, 5, 6 } and f : A→ B be defined by

$f(x) = \frac{x-3}{3}$. Represent f by

(i) an arrow diagram

(ii) a set of ordered pairs

(iii) a table

(iv) a graph.

Answer

Given A = { 6, 9, 15, 18, 21 }

B = { 1, 2, 4, 5, 6 } and f : A⟶B

And $f(x) = \frac{x-3}{3}$

We can find the domain

$$f(6) = \frac{6-3}{3} = \frac{3}{3} = 1$$

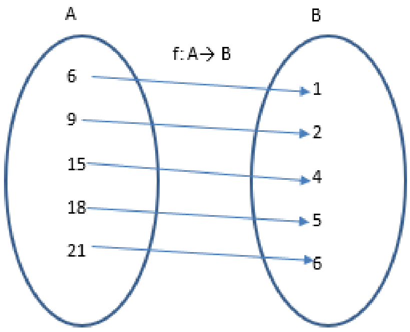
$$f(9) = \frac{9-3}{3} = \frac{6}{3} = 2$$

$$f(15) = \frac{15-3}{3} = \frac{12}{3} = 4$$

$$f(18) = \frac{18-3}{3} = \frac{15}{3} = 5$$

$$f(21) = \frac{21-3}{3} = \frac{18}{3} = 6$$

(i) an arrow diagram



(ii) a set of ordered pairs

From the given data and function value we can write the function as set of order pair as follows

f(x) = {(6,1),(9,2),(15,4),(18,5),(21,6)}

(iii) a table

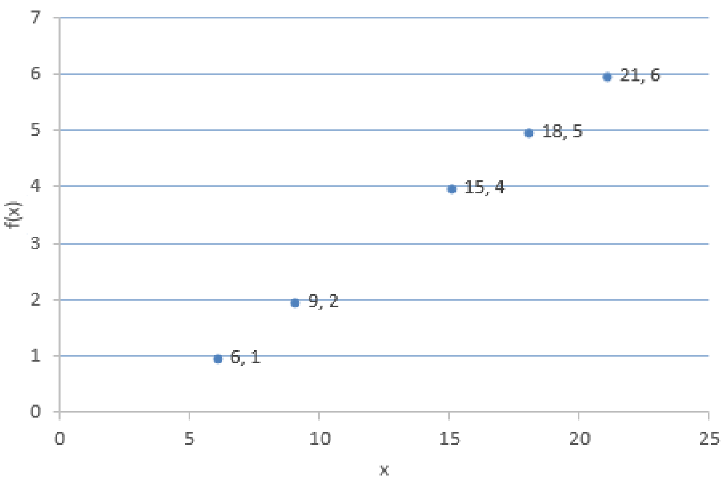
From the data we can draw the tabular representation taking X and f(X)

X	6	9	15	18	21
f(x)	1	2	4	5	6

(iv) a graph.

We can represent the function values in graph as follows

f(x) = {(6,1),(9,2),(15,4),(18,5),(21,6)}



14. Question

Let A = {4, 6, 8, 10 } and B = { 3, 4, 5, 6, 7 }. If f : A→ B is defined by $f(x)=\frac{1}{2}x+1$ then represent f by

(i) an arrow diagram

(ii) a set of ordered pairs

(iii) a table.

Answer

given A = {4, 6, 8, 10 }

B = { 3, 4, 5, 6, 7 }.

f : A⟶B is defined by $f(x) = \frac{1}{2}x + 1$

so we can find the domain

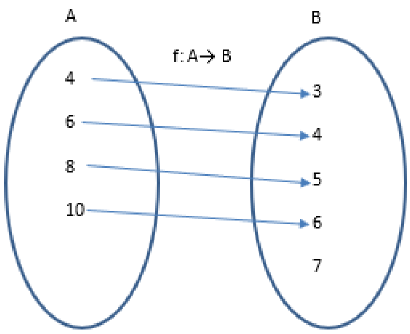
$$f(4) = \frac{1}{2}\times 4 + 1 = 2 + 1 = 3$$

$$f(6) = \frac{1}{2}\times 6 + 1 = 4$$

$$f(8) = \frac{1}{2}\times 8 + 1 = 5$$

$$f(10) = \frac{1}{2}\times 10 + 1 = 6$$

(i) an arrow diagram



(ii) a set of ordered pairs

Basing on the available data we can write the function as asset of order pair as follows

f(x) = {(4,3),(6,4),(8,5),(10,6)}

(iii) a table.

Using the data we have we can draw the tabel

X	4	6	8	10
f(x)	3	4	5	6

15. Question

A function $f:[-3,7)\to\mathbb{R}$ is defined as follows

$$f(x) = \begin{cases} 4x^2 - 1; -3 \le x < 2 \\ 3x - 2; 2 \le x \le 4 \\ 2x - 3; 4 < x < 7 \end{cases}$$

Find (i) f(5) + f(6) (ii) f(1)- f(-3)

(iii) f(-2h)- f(4) (iv) $\frac{f(3)+f(-1)}{2f(6)-f(-1)}$

Answer

We are given,

f: [-3,7) → ℝ

And the function given is

$$f(x) = \begin{cases} 4x^2 - 1;\ -3 \le x < 2 \\ 3x - 2;\ 2 \le x \le 4 \\ 2x - 3;\ 4 < x < 7 \end{cases}$$

(i) f(5) + f(6)

Here it is clear that 5 and 6 lies between 4 and 7 so we have to use the function,

f(x) = 2x - 3; 4 < x < 7

So,

f(5) = 2×5 - 3 = 10 - 3 = 7

f(6) = 2×6 - 1 = 12 - 3 = 9

∴ f(5) + f(6) = 7 + 9 = 16

(ii) f(1) - f(-3)

here its clear that 1 and -3 lies between the range of -3 ≤ x < 2 so we use the function

f(x) = $4x^2$ - 1

⇒ f(1) = $4(1)^2$-1 = 4-1 = 3

⇒ f(-3) = $4(-3)^2$-1 = 4×9 - 1 = 35

So f(1) - f(-3)

= 3 - 35

= -32

(iii) f(-2) - f(4)

As, x = -2

f(x) = $4x^2$ -1; -3 ≤ x <2

⇒ f(-2) = 4×$(-2)^2$-1 = 4×4 - 1 = 16 - 1 = 15

For, x = 4

f(x) = 3x-2; 2 ≤ x ≤ 4

⇒ f(4) = 3 × 4 - 2 = 12 - 2 = 10

⇒ f(-2)- f(4) = 15-10 = 5

(iv) $\frac{f(3)+f(-1)}{2f(6)-f(1)}$

Here the entire values ranges from -1 to 6 so the range will lie between -3≤x<2, 2≤ x<4 and 4<x<7

So

f(3) = 3x - 2; 2≤ x<4 = 3 × 3 – 2 = 9 - 2 = 7

f(-1) = $4x^2$ - 1: -3≤x<2 = $4(-1)^2$-1 = 4-1 = 3

f(6) = 2x - 3; 4<x<7 = 2 × 6-3 = 12-3 = 9

⇒ 2f(6) = 2 × 9 = 18

f(1) = $4x^2$ - 1: -3≤x<2 = $4(1)^2$ - 1 = 4 - 1 = 3So the function

$$\frac{f(3) + f(-1)}{2f(6) - f(1)}$$

$$= \frac{7 + 3}{18 - 3}$$

$$= \frac{10}{15}$$

$$= \frac{2}{3}$$

16. Question

A function $f:[-7,6) \to \mathbb{R}$ is defined as follows

$$f(x) = \begin{Bmatrix} x^2+2x+1; -7 \le x < -5 \\ x+5; -5 \le x \le 2 \\ x-1; 2 < x < 6 \end{Bmatrix}$$

(i) 2f(-4) + 3f(2)

(ii) f (- 7) - f (- 3)

(iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$

Answer

We are given,

f: [-7,6) → ℝ

[-7,6) = {x ∈ A: -7≤x<6}

= {-7, -6, -5, -4,-3, -2, -1, 0, 1, 2, 3, 4, 5}

And the function given are

$$f(x) = \begin{Bmatrix} x^2 + 2x + 1; -7 \le x < -5 \\ x + 5; -5 \le x \le 2 \\ x - 1; 2 < x < 6 \end{Bmatrix}$$

(i) 2f(-4) + 3f(2)

Here the function lies between -5<x<2

So

f(x) = x + 5

f(-4) = -4 + 5

= 1

f(2) = 2 + 5

= 7

∴ 2f(-4) = 2 × 1 = 2

∴ 3f(2) = 3× 7 = 21

So 2f(-4) + 3f(2)

= 2 + 21 = 23

(ii) f (- 7) - f (- 3)

here the function lies between $-7 \le x < -5$ and $-5 \le x < 2$

so $f(x) = x^2 + 2x + 1$

⇒ $f(-7) = (-7)^2 + 2\times(-7) + 1$

= 49-14 + 1

= 36

Similarly f(x) = x + 5

⇒ f(-3) = -3 + 5

= 2

∴ f (- 7) - f (- 3)

= 36 -2

= 34

(iii) $\frac{4f(-3)+2f(4)}{f(-6)-3f(1)}$.

For the above statement the function lies between

$-7 \le x < -5$ for f(-6)

$-5 \le x < 2$ for for f(-3) and f(1)

$2 < x < 6$ for f(4)

at $f(x) = x^2 + 2x + 1$; $-7 \le x < -5$

⇒ $f(-6) = (-6)^2 + 2(-6) + 1$

= 36-12 + 1 = 25

At f(x) = x + 5; $-5 \le x < 2$ for

⇒ f(-3) = -3 + 5 = 2

⇒ 4f(-3) = 4 × 2 = 8

Similarly,

f(1) = 1 + 5 = 6

⇒ 3f(1) = 3 × 6 = 18

At f(x) = x-1; $2 < x < 6$

⇒ f(4) = 4-1 = 3

⇒ 2f(4) = 2× 3 = 6

So the function given

$$\frac{4f(-3)\ +\ 2f(4)}{f(-6)-3f(1)}$$

$$=\frac{8\ +\ 6}{25-18}$$

$$=\ 2$$