Sequence and Series

TALENT & OLYMPIAD

Introduction

The ordered collection of objects is called sequence. The sequence having specified patterns is called progression. In this chapter besides discussing about the arithmetic progression, we will also discuss about the geometric progression and arithmetic-geometric progression. The various numbers occurring in the sequence is called term of the sequence. A sequence having finite number of terms is called finite sequence, where as the sequence having infinite number of terms is called infinite sequence.

The real sequence is that sequence whose range is a subset of the real number.

A series is defined as the expression denoting the sum of the terms of the sequence.

The sum is obtained after adding the terms of the sequence. If $a_1, a_2, a_3, ---, a_n$ is a sequence having n terms, then the sum of the series is given by,

$$\sum_{n=1}^{m} a_n = a_1 + a_2 + a_3 + ---- a_n$$

Arithmetic Progression (A.P.)

A sequence is said to be in A.P, if the difference between the consecutive terms is a constant. The difference between the consecutive terms of an AP is called **common difference** and any general term is called **nth term** of the sequence.

If $a_1, a_2, a_3, ---, a_n$ be the **nth terms** of the sequence in A.P., then nth terms of the sequence is given by $a_n = a + (n-1)d$,

Where **'a'** is the first term of the sequence
'd' is the common difference
'n' is the number of terms of the sequence.

Sum of N-Terms of the A.P.

If $a_1, a_2, a_3, ---, a_n$ be the n terms of the sequence in A.P., then the sum of n-terms of the sequence is given

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Arithmetic Mean

If 'a' and 'b' are any two terms of A.P., then the arithmetic mean is given by $AM = \frac{a+b}{2}$

Properties of AP

(a) If a constant is added or subtracted from each term of the AP sequence, then the resulting sequence is also in AP.

(b) If each term of an AP sequence is multiplied or divided by a constant, then the resulting sequence is also in AP.

Commonly Asked QUESTIONS

For a sequence in A.P., the sum of n terms of the sequence is $2n + 3n^2$, find the 20th term of the sequence:

(a) 121 (b) 112
(c) 119 (d) 124
(e) None of these

Answer: (c)

Explanation

We have, $S_n = 2n + 3n^2$

$S_1 = 5, S_2 = 16$

Thus common difference is

$d = S_2 - 2a = 16 - 10 = 6$

Hence the 20th term of the sequence is $= 6 \times 320 - 1 = 119$

Find the number of odd integers starting with 15 and will give the sum as 975.

(a) 25 (b) 15

(c) 20 (d) 30

(e) None of these

Answer: (a)

Explanation

Here we have a = 15 and common difference = 2 Since we know that, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

But $S_n = 975$

$\therefore \frac{n}{2}\left[2a + (n-1)d\right] = 975$

$\Rightarrow \frac{n}{2}\left[2 \times 15 + (n-1) \times 2\right] = 975$

$\Rightarrow n^2 + 14n - = 975 = 0$

Since number of terms cannot be negative, hence n = - 39 is rejected. Therefore n = 25.

If the 15th term of an AP is 45 and 20th term is 60, and then find the 30th term of the AP.

(a) 70 (b) 90

(c) 110 (d) 120

(e) None of these

Answer: (b)

Explanation

We have,

$A_{15} = A + 14d = 45$

$a_{20} = a + 19d = 60$

On solving the above equation we get,

a = 3 and d = 3

Therefore, $a_{30} = a + 29d$

$\Rightarrow a_{30} = 3 + 29 \times 3 = 90$

If the number of different cards of different colours Thomas has are in AP. If he has cards of seven different colours in the order of VIBGYOR such that third colour is four times the first colours and the sixth colours is 17, then the number of fifth colour cards is:

(a) 14 (b) 16

(c) 12 (d) 10

(e) None of these

Answer: (a)

The 10th common term between the series $3+7+11+---and\,1+6+11+---$ is given by:

(a) 175
(b) 155
(c) 187
(d) 191
(e) None of these

Answer: (d)

Geometric Progression (G.P.)

A sequence is said to be in G.P. if the ratio between the consecutive terms is constant. The sequence $a_1, a_2, a_3, ---, a_n$ is said to be in G.P. if the ratio of the consecutive term is a constant.

If **'r' is the common ratio**, then the nth term of the sequence is given by $a_n = a\ r^{n-1}$

The sum of **n terms** of the G.P. sequence is given by

$S_n = \frac{a(r^n - 1)}{r-1}\ If\ \ r > 1 and\ S_n = \frac{a(1-r^n)}{1-r}\ if\ r > 1$ **Sum to infinity** is a G.P. series is given by $S_\infty = \frac{a}{1-r}$.

Geometric Mean (G.M.)

If **'a' and 'b'** are any two terms of G.P., then the geometric mean is given by $GM = \sqrt{ab}$

Properties of GP

(a) If each term of GP is multiplied or divided by a constant, then the resulting sequence is also in GP.
(b) If each term of the GP is raised to the same power then the resulting sequence is also in GP.
(c) The reciprocal of each term of GP also results in GP.

Commonly Asked QUESTIONS

The sum of the series of the sequence given by - $\frac{2}{3}, -1, \frac{3}{2}, ...$ to 5 terms is given by:

(a) 0
(b) $\frac{55}{24}$
(c) $\frac{65}{34}$
(d) $\frac{65}{24}$
(e) None of these

Answer: (b)

Explanation

Since the above sequence is in GP, and first term is $a = \frac{2}{3}$ and common ratio $= r = -\frac{3}{2}$

The sum of n terms of GP is given by $S_n = \frac{a(1-r^n)}{1-r}$

$$\Rightarrow S_n \frac{\frac{2}{3}\left\{1-\left(-\frac{3}{2}\right)^5\right\}}{1-\left(-\frac{3}{2}\right)}$$

$$\Rightarrow S_n \frac{4}{15}\left\{1+\frac{243}{32}\right\}$$

$$\Rightarrow S_n = \frac{55}{24}$$

Robert and James were playing a game and Robert asks James to find the three numbers which are in GP such that their sum is 19 and their product is 216. The three numbers are.

(a) (4, 6, 9)
(b) (3, 6, 12)
(c) (5, 10, 20)
(d) (7, 14, 28)
(e) None of these

Answer: (a)

Explanation

Let 'a7 be the first term of the GP and V be the common ratio. Then the three terms which are in GP can be written $as \frac{a}{r}, a, ar..$

According to question,

$$\frac{a}{r} \times a \times ar = 21$$

$$\Rightarrow a^3 = 216$$

$$\Rightarrow a = 6$$

Also, $\frac{a}{r} + a + ar = 19$

$$\Rightarrow \frac{6}{r} + 6 + +6r = 19$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow \frac{3}{2} or \frac{2}{3}$$

There is a sequence of numbers such that they are in GP and sum to infinite of the number of terms of the sequence is 15 and the sum of their square is 45. The first term of the sequence is given by:

(a) 3
(b) 5
(c) 9
(d) 13
(e) None of these

Answer: (b)
Explanation

The sum of infinite number of terms in GP is given by $S_\infty = \frac{a}{1-r}$

$$\Rightarrow 15 = \frac{a}{1-r^2} ----(1)$$

Also the sum of square of the terms of the GP is given by,

$$S_\infty = \frac{a^2}{1-r^2}$$

$$\Rightarrow 45 = \frac{a^2}{1-r^2} ----(2)$$

Solving (1) and (2) we get,

a = 5

The three numbers in GP are such that their continued product is 216, and the sum of product in pairs is 156; then the three numbers are:

(a) (2, 4, 8) (b) (1, 5, 25)

(c) (3, 9, 27) (d) (2, 6, 18)

(e) None of these

Answer: (d)

If the three terms x, y, z are the p^{th}, q^{th} *and* r^{th} **terms of GP. Then the value of** $X^{q-r}Y^{r-p}Z^{p-q}$**.**

(a) 1 (b) 0

(c) 3 (d) 5

(e) None of these

Answer: (a)

Harmonic Progression (H.P.)

The sequence is said to be in H.P. If the reciprocal of its terms gives the A.P. It has got wide application in the field of geometry and theory of sound. The questions are generally solved by inverting the terms and using the property of arithmetic progression.

Three numbers a, b, c are said to be in HP if,

$$\frac{a}{c} = \frac{a-b}{a-c}$$

Harmonic Mean (HM)

If **'a' and 'b'** be any two terms, then their harmonic mean is given by $HM = \frac{2ab}{a+b}$.

Relation Between AM, GM, and HM

Since we know that,

$$AM = \frac{a+b}{2}, \; GM = \sqrt{ab} \text{ and } HM = \frac{2ab}{a+b}$$

Then,

$$AM \times HM = \frac{a+b}{2} \times \frac{2ab}{a \times b} = ab = G^2$$

$$AM \times HM = GM^2$$

Form the above relation we can say that **AM > GM** and **GM** is intermediate value between **AM** and **HM**, therefore **GM > HM**. Hence we can say that **AM > GM > HM**.
Also the relation between **A** and **G** is given by,

$$AM - GM = \frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}}$$

Commonly Asked QUESTIONS

The harmonic mean between two numbers is — and geometric mean is 12. The two numbers are:

(a) (3 & 20) (b) (2 & 12)
(c) (6 & 24) (d) (5 & 32)
(e) None of these

Answer: (c)
Explanation
Let the two number be ‘a’ and ‘b’.
Then, $HM = \frac{2ab}{a+b}$ and $GM = \sqrt{ab}$
Putting the value of HM and GM in the above relation we get,

$$\frac{48}{5} = \frac{2ab}{a+b} \text{ and } 12 = \sqrt{ab}$$

On solving these two equations we get, A = 6 & b = 24

If p^{th} term of HP is equal to q^{th} and the p, then (p + q) term of the series is.

(a) $\frac{pq}{p+q}$ (b) $\frac{p-q}{p+q}$
(c) $\frac{p-q}{pq}$ (d) $\frac{p+q}{pq}$
(e) None of these

Answer: (a)

Explanation
Let 'a' and 'd' be the first term and common difference of an AP,
Then the p^{th} and q^{th} term of the AP is $a_p(p-1)d \text{ and } a_p = a+(q-1)d$
For HP series the corresponding terms are,

$$\Rightarrow \frac{1}{pq} = a+(p-1)d \text{ and } \frac{1}{p} = a+(q-1)d$$

On solving the above equation we get,

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{pq}$$

Therefore, $(p+q)^{th}\ term = \frac{p+q}{pq}$

Hence $(p+q)^{th}$ of the HP is given by $\frac{pq}{p+q}$

For any two numbers the ratio of HM : GM is 12 :13, and then the ratio of the two numbers is given by:

(a) 3 : 8 (b) 2 : 5

(c) 4 : 9 (d) 5 : 7

(e) None of these

Answer: (c)

Explanation

Let the two number be 'a' and 'b'. Then,

$$HM = \frac{2ab}{a+b}\ and\ GM = \sqrt{ab}$$

$$\Rightarrow \frac{HM}{GM} = \frac{\frac{2ab}{a+b}}{\sqrt{ab}}$$

$$\Rightarrow \frac{12}{13} = \frac{2\sqrt{ab}}{a+b}$$

$$\Rightarrow \frac{13}{12} = \frac{a+b}{2\sqrt{ab}}$$

By componendo and dividendo, we get

$$\Rightarrow \frac{25}{1} = \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}$$

$$\Rightarrow \frac{25}{1} = \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}$$

$$\Rightarrow \frac{5}{1} = \frac{\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)}$$

$$\Rightarrow \frac{a}{b} = \frac{9}{4}$$

The number of bricks arranged in a complete pyramid on a square base of side 10 units is given by.

(a) 290 (b) 385

(c) 425 (d) 525

(e) None of these

Answer: (b)

Explanation

The relation used here is given by $S = n^2 + (n-1)^2 + (n-2)^2 + ----+1$

The number of shots arranged in a graveyard in the shape of pyramid whose base is in the form of equilateral triangle of side 8 units is given by.

(a) 100 (b) 140
(c) 120 (d) 64
(e) None of these

Answer: (c)

Explanation

The relation used here is given by,

The number of shot in each layer is $S = n + (n-1) + (n-2) + - - - - + 1$

Total number of shot is given by, $S = \frac{n(n+1)(n+2)}{6}$

You Must KNOW

- The "golden course" is a sequence of numbers first created by the Italian mathematician Leonardo di Pisa, or Pisano, known also under the name Fibonacci in 1202.
- The number $\left[\pi = \frac{(radii)\times 5}{2} + 0.5 - 1.618...\right]$ or Golden Ratio, is intimately related to Fibonacci numbers.

  The closed form of F_n is: $F_n = \frac{\{\pi - (1-\pi)\times n\}}{(radii)\times 5}$
- The convergence or divergence of an infinite series remains unaffected by the addition or removal of finite numbers.
- The convergence or divergence of an infinite series remains unaffected by multiplying each term by a finite number.
- In mathematics, the limit of Fibonacci series is called Golden Ratio. This ratio is approximately equal to 1.618.

SUMMARY

- The number arranged in any definite order is called sequence.
- If the sequence which contains finite number of terms is called finite sequence.
- A sequence is said to be in AP if the common difference between any two consecutive terms is a constant.
- A sequence is said to be in GP if the ratio between the consecutive term is a constant.
- A sequence is said to be in HP is the reciprocal of the terms of the sequence results in AP.

Self Evaluation TEST

Duration 10 Minutes

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1. **If 19 arithmetic mean is inserted between $\frac{1}{4}$ *and* $-9\frac{3}{4}$, then the 18th arithmetic mean is.**

(a) $-\frac{35}{4}$ (b) $-\frac{29}{4}$

(c) $-\frac{31}{4}$ (d) $-\frac{33}{4}$

(e) None of these

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2. **The sum of n terms of the series given by $\frac{2x^2-1}{x}, 4x-\frac{3}{x}, \frac{6x^2-5}{x}, ---$ is:**

(a) $\left[n(n+1)\times+\frac{n^2}{x}\right]$ (b) $\left[n(n-1)\times+\frac{n^2}{x}\right]$

(c) $\left[n(n-1)\times-\frac{n^2}{x}\right]$ (d) $\left[n(n-1)\times+-\frac{n^2}{x}\right]$

(e) None of these

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3. **The ratio of sum of n terms of two arithmetic progressions is (7n + 1) : (4n + 27) then find the ratio of their 12th terms.**

(a) $\frac{162}{119}$ (b) $\frac{16}{11}$

(c) $\frac{4}{3}$ (d) $\frac{14}{13}$

(e) None of these

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4. **If the sum of first seven terms is 49 and that of first seventeen terms is 289, then the sum of first 25 terms of the sequence is ____.**

(a) 144 (b) 256

(c) 324 (d) 625

(e) None of these

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5. **The two sets of numbers each containing three terms is in AP; such that the sum of each set is 15. The difference between the common differences of the numbers of two sets is 1 and the ratio of the product of two sets is 7 : 8. The sets of numbers in two sets are.**

(a) (4, 6, 8 & 5, 6, 7) (b) (3, 5, 7 & 4, 5, 6)

(c) (2, 4, 6 & 3, 4, 5) (d) (5, 7, 9 & 6, 7, 8) (e) None of these

6. **If** $\log_y b, b^{\frac{y}{2}}, \& \log_a y$ **are in Gp, then the value of y is:**

(a) $\log_b a + \log_a b$ (b) $\log_b a - \log_a b$

(c) $\log_b \log_a b$ (d) 1

(e) None of these

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7. **The value of** $(1^3 + 2^3 + --- + 20^3) - (1 + 2 + 3 + --- + 20)$ **is given by:**

(a) 47890 (b) 45890

(c) 48890 (d) 43890

(e) None of these

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8. **There are 30 potted flowering plants placed in a row from the gate to the entrance of the house. If each plant is separated by a distance of 5 m form each other and the gardener has to water the plants in the morning which is at a distance of 10 m from the first plant from the entrance of the house. If he starts from the entrance, then how much distance will the gardener has to walk to water all the plants?**

(a) 4795m (b) 4580m

(c) 4889m (d) 4385m

(e) None of these

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9. **A three digit number is such that if 400 is subtracted from it, then it results in a three digit number whose digits are in AP. If the digits of the first number are in GP, then the sum of these two numbers is:**

(a) 1479 (b) 1458

(c) 1462 (d) 1438

(e) None of these

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10. **The three numbers a, b, c, are such that they are in HP and their square is in AP, then the ratio of a : b : c is:**

(a) $1+\sqrt{3} : -2 : 1-\sqrt{3}$ (b) $1+\sqrt{3} : 4 : 1-\sqrt{3}$

(c) $1+\sqrt{3} : 4 : 1-2\sqrt{3}$ (d) 1+2^:2:1-2^3

(e) None of these

Answers – Self Evaluation Test																			
1.	A	**2.**	C	**3.**	A	**4.**	A	**5.**	B	**6.**	C	**7.**	D	**8.**	A	**9.**	C	**10.**	A

Self Evaluation Test
SOLUTIONS

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1. Let $A_1, A_{2,} ---, A_{19}$ the arithmetic mean between $\frac{1}{4}$ and $-9\frac{3}{4}$. such that the resulting sequence is in AP.

Then the first term of the sequence is, $a = \frac{1}{4}$ and $a = -9\frac{3}{4}$

$\Rightarrow a + 20d = -\frac{39}{4}$

On solving we get, $d = -\frac{1}{2}$

Therefore the required arithmetic mean is given by,

$A_{18} = a + 18 = -\frac{35}{4}$

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2. Since the above given sequence is in AP,

The first term of the sequence is $= a = \frac{2x^2 - 1}{x}$

The common difference is $= 2\left(x - \frac{1}{x}\right)$

$\therefore S_n = \frac{n}{2}\left[2\left(\frac{2x^2 - 1}{x}\right) + (n-1)2\left(x - \frac{1}{x}\right)\right]$

On solving we get, $S_n = \left[n(n+1)x - \frac{n^2}{x}\right]$

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3. Let the first term and common difference of the two series be a_1 , d and a_1, d_1 respectively.

Then, the ratio of their n terms are given by,

$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$

Putting n = 23, we get

$\Rightarrow \frac{2a_1 + (23-1)d_1}{2a_2 + (23-1)d_2} = \frac{7 \times 23 + 1}{4 \times 23 + 27}$

$\Rightarrow \frac{a_1 + 11d_1}{a_2 + 11d_2} = \frac{162}{119}$
