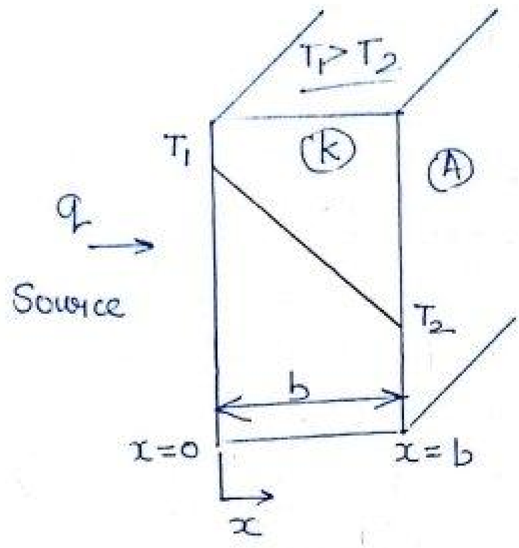
Conduction Heat transfer

Integration to Fourier's law of Conduction

Case(1):- Conduction Heat transfer through a slab



Assume: ① steady state Heat transfer Condition i.e. ($T \neq f$ time)

② one dimensional heat conduction

$T = f(x)$ only

At $x = 0 \Rightarrow T = T_1$

$x = b \Rightarrow T = T_2$

③ uniform 'k' of material and no Heat generation in slab.

$$q = -kA\frac{dT}{dx} \Rightarrow \int_{x=0}^{x=b} q\,dx = \int_{T_1}^{T_2} -kA\,dT \Rightarrow \boxed{q = \frac{kA(T_1 - T_2)}{b}}$$

* In slab, A will Not change in the direction of heat flow.

* To satisfy steady state conditions of slab

$$q \neq f(x) \quad \text{i.e.} \quad q_x = q_{x+dx}$$

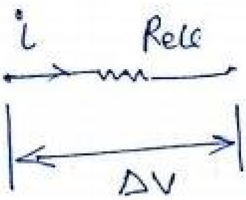
$$q = \frac{kA(T_1 - T_2)}{b}$$

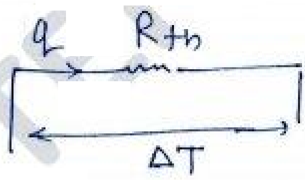
$$\therefore \frac{q}{A} = \frac{k(T_1 - T_2)}{b} = \text{Heat transfer Rate per unit area}$$

$$\text{Heat Flux } Q = \frac{q}{A} = \frac{k(T_1 - T_2)}{b} \ \text{watt/m}^2$$

Electrical Analogy :-

Electric	thermal
i Amp	q watt
ΔV (OR) Emf	ΔT (°C)
$R_{electric}(\Omega)$	$R_{thermal}$





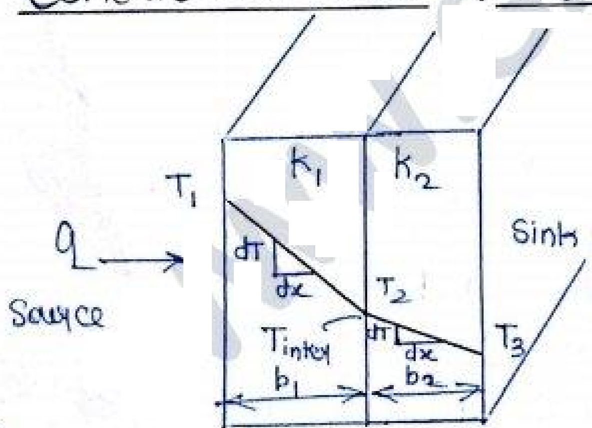
ohm's law

$$R_{ele} = \frac{\Delta V}{i}\ \Omega \qquad \boxed{R_{th} = \left(\frac{\Delta T}{q}\right)}\ K/\text{watt}$$

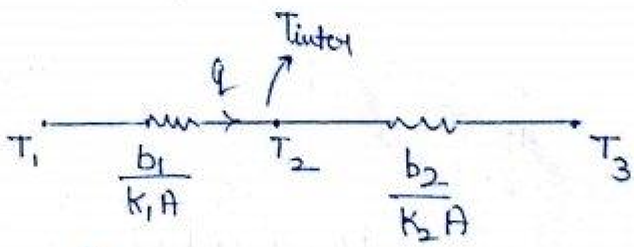
For ~~sig~~ single slab

$$\boxed{(R_{th})_{conduction} = \frac{(T_1 - T_2)}{q} = \frac{b}{KA}}\ K/\text{watt}$$

Conduction Heat transfer through Composite slab →



Assume :- steady state, one dimensional conduction H.T. through complete slab



⇒ $K_1 < K_2$ because $\left(\frac{dT}{dx}\right)_{in(I)} > \left(\frac{dT}{dx}\right)_{in(II)}$ (q & A being same through)

$\therefore$ Rate of Conduction Heat transfer through composite slab

$$\boxed{q = \frac{T_1 - T_3}{\frac{b_1}{k_1 A} + \frac{b_2}{k_2 A}}}$$

Heat flux = Per unit Area

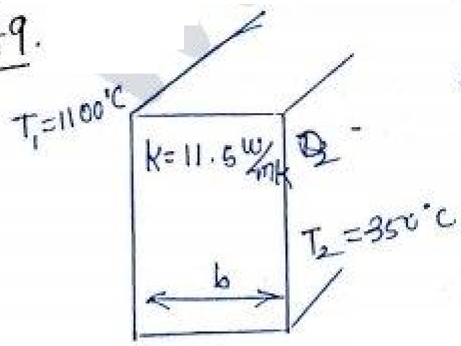
$$\boxed{q/A = \frac{T_1 - T_3}{\frac{b_1}{k_1} + \frac{b_2}{k_2}}}$$

Interface temp. (T_2)

$$\therefore \quad q = \frac{T_1 - T_2}{b_1/k_1 A} \Rightarrow T_2 = ?$$

$$q = \frac{T_2 - T_3}{b_2/k_2 A} \Rightarrow T_2 = ?$$

Q.49.

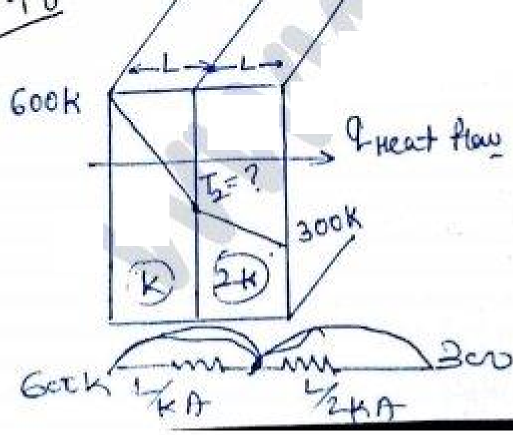


$q/A = 2500\ W/m^2 \qquad b = ?$

$$q/A = 2500 = \frac{(1100 - 300)}{(b)} \times 11.5$$

$$b = 0.345\ m$$

Q.48



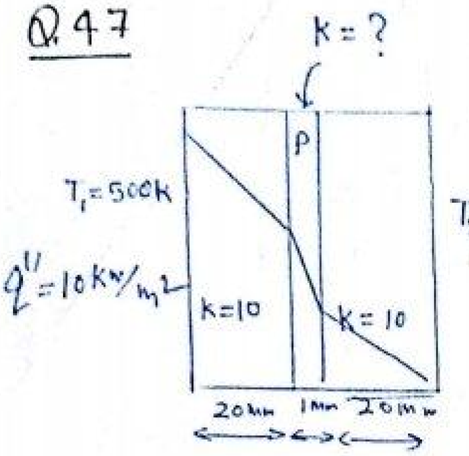
$$\frac{600 - T_2}{L/KA} = \frac{T_2 - 300}{L/2KA}$$

~~1200 - 2T_2 = T_2 - 300~~

$$600 - T_2 = 2T_2 - 600$$

$$T_2 = 400K$$

Q.47

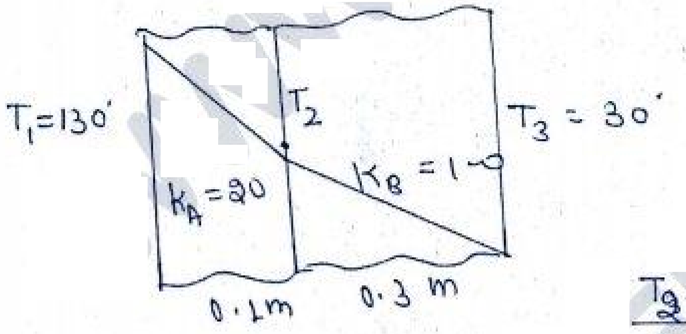


$$\frac{20}{10^4 \times A} \qquad \frac{1}{k \times 10^3 \times A} \qquad \frac{20}{10 \times 10^3 \times A}$$

$$\frac{q}{A} = \frac{500 - 360}{\frac{20}{10^4} + \frac{1}{1000k} + \frac{20}{10^4}} = 10^4$$

$$\Rightarrow \quad k = 0.1 \ W/mK$$

Q.51



$$\frac{0.1}{20A} \qquad \frac{0.3}{100A}$$

$$\frac{T_2 - T_3}{0.3/100A} = \frac{T_1 - T_2}{0.1/20A}$$

$$\Rightarrow \frac{100(T_2 - T_3)}{3} = \frac{20(130 - T_2)}{1}$$

$$\cancel{3000 + 100T_2 = 780 - 60T_2}$$

$$100(T_2 - 30) = 60(130 - T_2)$$

$$T_2 = 67.5$$

Q.5

$$\frac{T_1 - T_i}{2b/k_1} = \frac{T_i - T_2}{b/k_2} \qquad T_i = \frac{T_1 + T_2}{2}$$

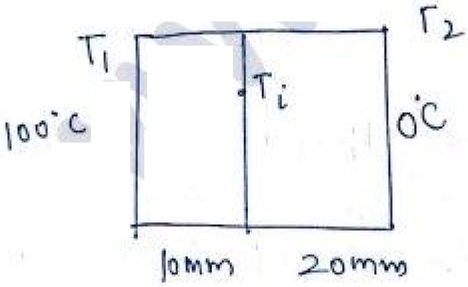
$$k_1 (T_1 - T_i) = (2T_i - 2T_2) k_2$$

$$k_1 \left(T_1 - \frac{T_1 + T_2}{2} \right) = 2k_2 \left(\frac{T_1 + T_2}{2} - \cancel{2} T_2 \right)$$

$$k_1 (T_1 - T_2) = 2k_2 (T_1 - T_2)$$

$$k_1 = 2k_2$$

Q.20



$$\frac{k_{Al}}{k_{iron}} = 3/1$$

$$\frac{T_1 - T_i}{10/k_{Al}} = \frac{T_i - T_2}{20/k_{iron}}$$

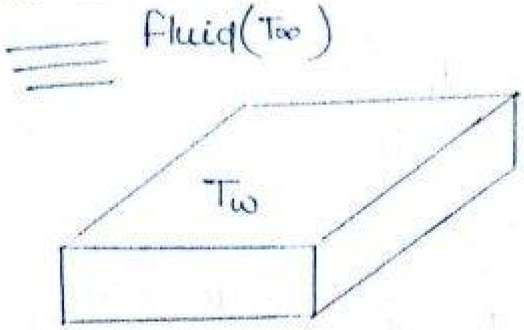
$$2(100 - T_i) = \frac{1}{3}(T_i - 0)$$

$$200 = \left(2 + \frac{1}{3}\right) T_i \qquad 2 + \frac{1}{3} = \frac{6+1}{3}$$

$$T_i = \frac{200 \times 3}{7}$$

$$T_i = 85.7\,^\circ C \quad \underline{ans}$$

Convection Thermal Resistance :-



A - Area of Contact

$$q_{conv} = hA\Delta T$$

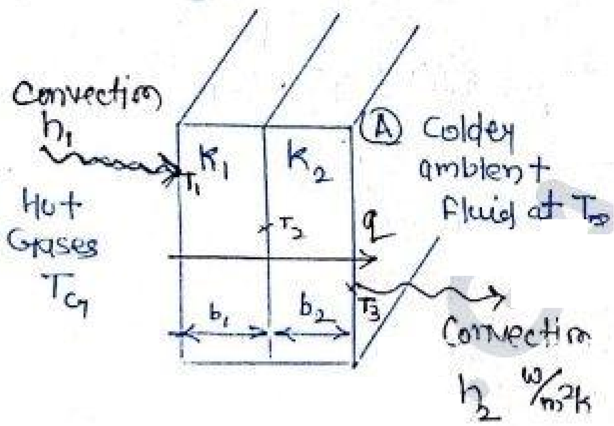
$$q_{conv.} = hA(T_w - T_\infty)$$

$$R_{th} = \frac{\Delta T \rightarrow \text{thermal potential}}{q \rightarrow \text{Heat current}}$$

Convective Mode

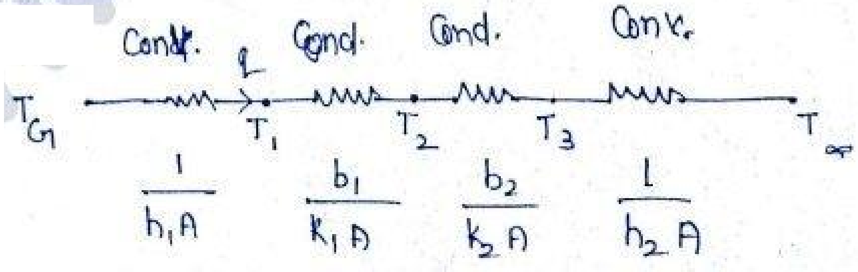
$$\therefore \boxed{(R_{th}) = \frac{T_w - T_\infty}{q} = \left(\frac{1}{hA}\right)_{Contact} \frac{kelvin}{watt}}$$

Conduction Convection Heat trasfer through a Composite slab



Assume :- steady state, 1-D, Conduction-Convection H.T. through composite slab.

Thermal Circuit



∴ Rate of Heat transfer between Hot Gases and ambiend fluid

$$\boxed{q = \frac{(T_G - T_\infty)}{\frac{1}{h_1 A} + \frac{b_1}{k_1 A} + \frac{b_2}{k_2 A} + \frac{1}{h_2 A}} \text{ watt}}$$

∴ q/A = H.T. Rate per unit area

$$\boxed{q/A = \text{Heat Flux} = \frac{(T_G - T_\infty)}{\frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}}} \text{ watt}/m^2 \quad -①$$

Defining overall Heat transfer coefficient U as a parameter which take into account all modes of heat transfer into a ~~sign~~ single entity i.e. from the equation :-

$$q = UA(T_G - T_\infty) \text{ watt}$$

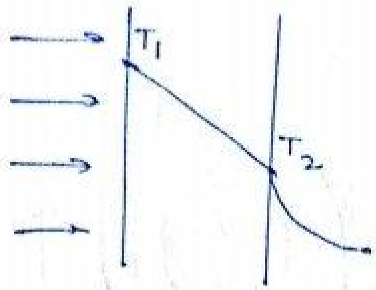
$$q = UA\Delta T \quad -② \quad q = \frac{\Delta T}{1/UA}$$

Comparing ① & ②

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}} \quad W/m^2K$$

* U & h have same unit.
* If U is More, Total R_{th} less ⇒ q More

Q.4.6



$b = 30\,mm$

$k = 15\ W/mK$

$q'' = 10^5\ W/m^2$

$h = 250\ W/m^2K$

$T_\infty = 25\,°C$

$$T_1 \xrightarrow{\frac{b}{kA}} T_2 \xrightarrow{\frac{1}{hA}} T_\infty$$

$$\Rightarrow \frac{1}{U} = \frac{b}{kA} + \frac{1}{hA} = R_{th}$$

$$R_{th} = \frac{0.030}{15} + \frac{1}{250} = 6 \times 10^{-3}$$

$$q = UA\Delta T$$

$$q'' = q/A = U(T_1 - T_\infty)$$

$$10^5 = \frac{1}{6\times10^{-3}}(T_1 - 25)$$

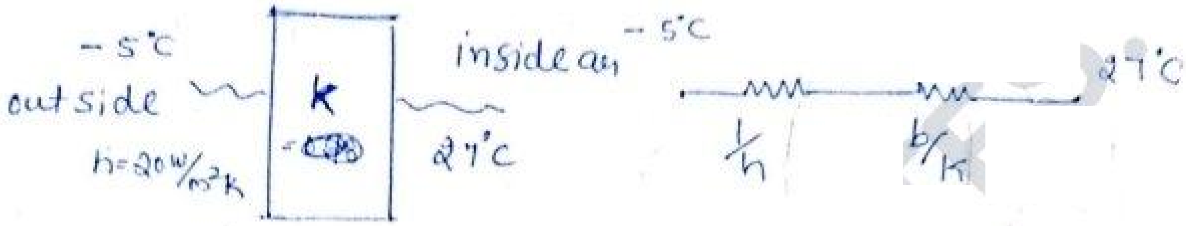
$$600 = T_1 - 25 \Rightarrow \underline{T_1 = 625\,°C}$$

$$\frac{T_2 - T_\infty}{1/h} = 10^5$$

$$T_2 = 25 + 400$$

$$T_2 = 425\,°C$$

Q.41



$$q/A = R_{th}\,(\Delta T)$$

$$R_{th} = \frac{1}{h} + \frac{b}{k}$$

$$R_{th} = \frac{1}{20} + \frac{0.18}{0.9}$$

$R_{th} =$

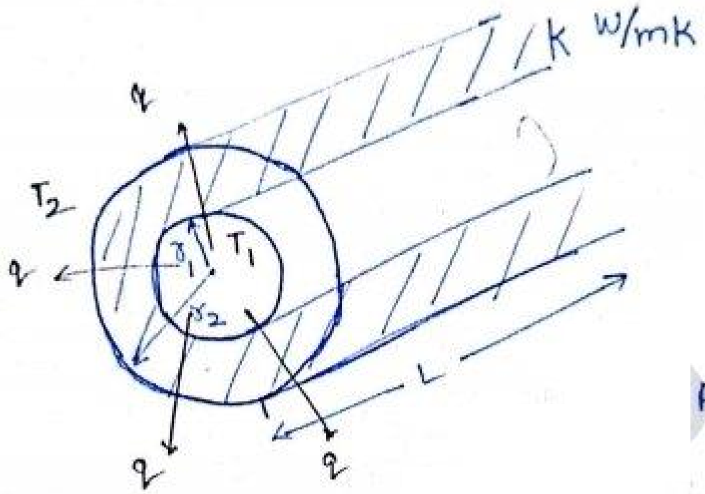
$$q'' = \frac{(27-(-5))}{\frac{1}{20} + \frac{0.18}{0.9}} = 128\ W/m^2$$

Q.26

$$\frac{1}{U} = \frac{1}{10} + \frac{1}{20} + \frac{0.05}{1} = 0.2$$

$U = 5\ W/m^2K$

Radial Conduction H.T. through hollow Cylinder :-



$T_1 > T_2$

$T = f(r)$

At $r = r_1 \Rightarrow T = T_1$

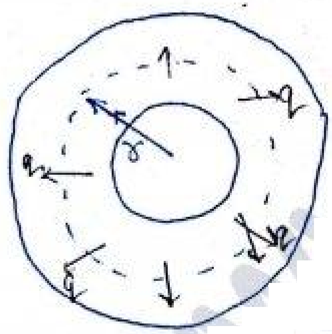
At $r = r_2 \Rightarrow T = T_2$

* Since temp Gradient are existing along radial direction, heat must conduct radially outward from inner cylinderical surface at T_1 to outer cylinderical surface at T_2.

* Unlike in case of slabs, here area of Conduction H.T. changes in the dirn of Heat Flow.

→ At any Radius 'r'

⇒ Area of Conduction H.T. $(A) = 2\pi r L$



Fourier's law of Conduction

⇒ Rate of Radial H.T. $= q = -kA\left(\frac{dT}{dx}\right)$ watt

⇒ $q = -k(2\pi r L)\,\frac{dT}{dr}$

Assume :- steady state
1-D (Radial H.T.)
Conduction

$T \neq f(\text{time})$

$$\int_{r_1}^{r_2} q \frac{dr}{r} = \int_{T_1}^{T_2} -2\pi k L \, dT$$

To satisfy steady state condition of Cylinder $q \neq f(r)$

i.e. $q_r = q_{r+dr}$

$$q \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi k L \int_{T_1}^{T_2} dT$$

$$q \ln\left(\frac{r_2}{r_1}\right) = -2\pi k L (T_2 - T_1) = 2\pi k L (T_1 - T_2)$$

∴ Rate of Radial Heat transfer

$$\boxed{q = \frac{2\pi k L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}} \text{ watt}$$

The corresponding conduction thermal resistance for

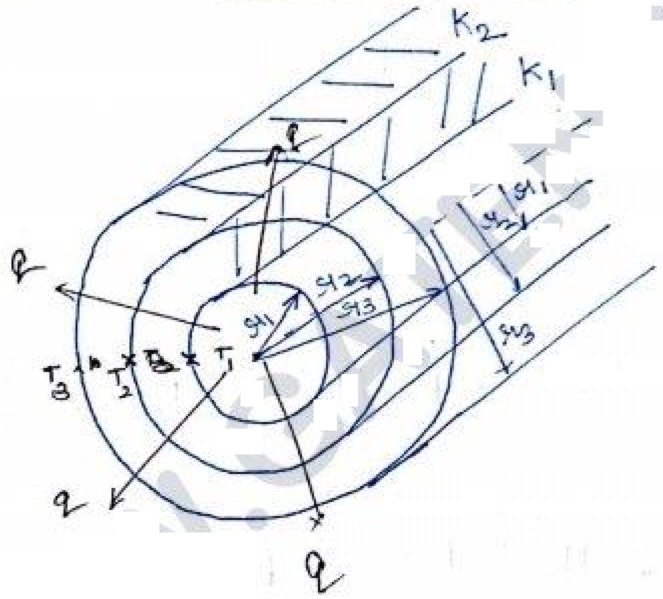
$T_1 \xrightarrow{q}$ ~~~ T_2

$$R_{th} = \frac{T_1 - T_2}{q} = \frac{\ln\left(r_2/r_1\right)}{2\pi k L} \text{ K/W}$$

$$\boxed{R_{th} = \frac{\ln\left(r_2/r_1\right)}{2\pi k L}} \text{ K/watt}$$

* If the thickness of the cylinder is very small and conductivity of cylinder is very high, than the above conduction Resistance almost zero.

Radial Conduction Heat transfer through a Composite Cylinder:-



$T_1 > T_2 > T_3$

* Since temp. Gradient are existing along radial direction Heat must be Conducting from the inner most cylindrical surface at T_1 to outer most cylindrical surface at T_3

Assume:- A steady state, 1-D (Radial) Conduction H.T.

Thermal Circuit:-

$$T_1 \quad \underset{\frac{\ln(r_2/r_1)}{2\pi K_1 L}}{\text{---}\!\!\wedge\!\!\wedge\!\!\wedge\!\!\text{---}} \quad T_2 \quad \underset{\frac{\ln(r_3/r_2)}{2\pi K_2 L}}{\text{---}\!\!\wedge\!\!\wedge\!\!\wedge\!\!\text{---}} \quad T_3$$

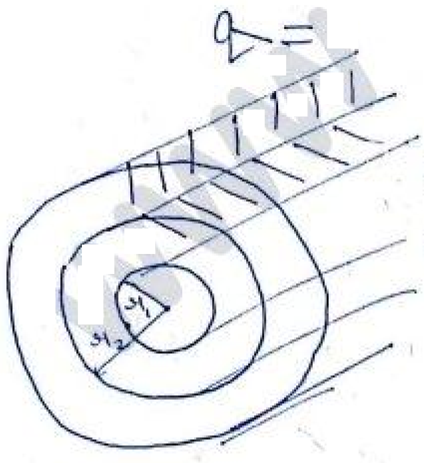
∴ Rate of Heat transfer

$$\boxed{q = \frac{T_1 - T_3}{\dfrac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L} + \dfrac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L}} \quad \text{watt}}$$

Q.2

$$\cancel{q = \frac{600}{\dfrac{\ln\left(5/2\right)}{2\times\pi\times 19\times L} + \dfrac{\ln\left(8/5\right)}{2\pi\times 0.2\times L}}}$$

Q2



$r_1 = 1$ cm

$r_2 = 2.5$ cm

$r_3 = 5.5$ cm

$\Delta T = 600\,°C$

$k_s = 19$ W/mK

$k_a = 0.2$ W/mK

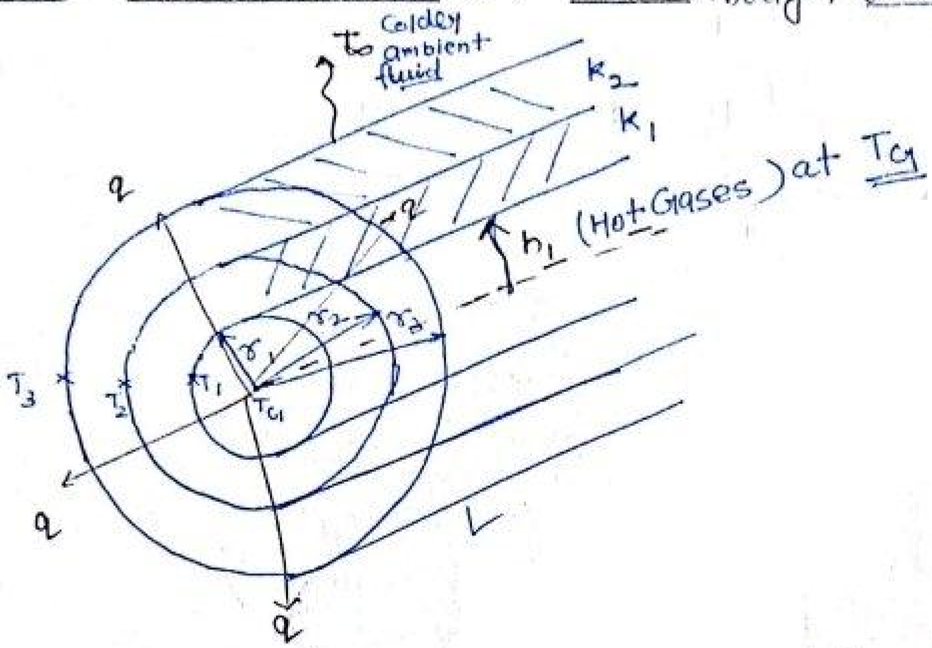
T_1 — $\dfrac{\ln\left(\frac{2.5}{1}\right)}{2\pi\times 19\times 1}$ — $\dfrac{\ln\left(\frac{5.5}{2.5}\right)}{2\pi\times 0.1\times 1}$ — T_2

Per Unit length

$$q = \frac{600}{\dfrac{\ln(2.5)}{2\pi\times 19} + \dfrac{\ln\left(\frac{5.5}{2.5}\right)}{2\pi\times 0.1}} = 944.72 \ \text{W/m}$$

$$\left\{\begin{array}{l} K_{steel} > K_{abs} \\ (R_{th})_{steel} <<< (R_{th})_{abs} \end{array}\right. \Rightarrow \underset{\simeq 10°C}{(\Delta T)_{\text{across steel}}} <<< \underset{\simeq 590°C}{(\Delta T)_{\text{across Abs}}}$$

Radial Conduction Convection H.T. through a Composite Cylinder



Assume :- steady state, 1-D, H.T. b/w Hot gasses and ambiend fluid

Thermal circuit

$$T_G \xrightarrow[\frac{1}{(h_1 2\pi r_1 L)}]{(Conv)_{inner}} T_1 \xrightarrow[\frac{\ln(r_2/r_1)}{2\pi K_1 L}]{Cond} T_2 \xrightarrow[\frac{\ln(r_3/r_2)}{2\pi K_2 L}]{Cond.} T_3 \xrightarrow[\frac{1}{(h_2 2\pi r_3 L)}]{(Conv)_{outer}} T_\infty$$

$$R_{th} = \left(\frac{1}{h_1 2\pi r_1 L}\right) + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L} + \frac{1}{(h_2 2\pi r_3 L)}$$

* Rate of Radial H.T. between Gas & fluid $\boxed{q = \frac{T_G - T_\infty}{R_{th}}}$ — (1)

* Defining overall heat transfer coefficient U_i that is based one the inside convection H.T. Area and overall H.T. Coefficient U_o i.e. based one the outside convection H.T. area

$$q = U_i A_i (T_G - T_\infty)$$

$$= U_o A_o (T_G - T_\infty)$$

$$\left.\begin{aligned} q &= U_i\, 2\pi r_1 L\, (T_G - T_\infty) \\ q &= U_o\, 2\pi r_2 L\, (T_G - T_\infty) \end{aligned}\right\} \quad \text{--} (2)$$

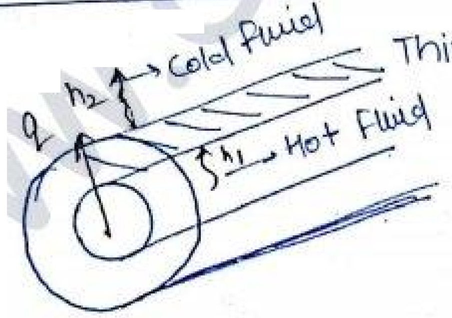
comparing (1) & (2), we Get

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{r_3}\frac{1}{h_2}}$$

$$\boxed{\frac{1}{U_o} = \frac{r_3}{r_1}\frac{1}{h_1} + \frac{r_3}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_2}}$$

* But Heat exchanger analysis whether LMTL method or effectiveness NTU Method U can be calculated as

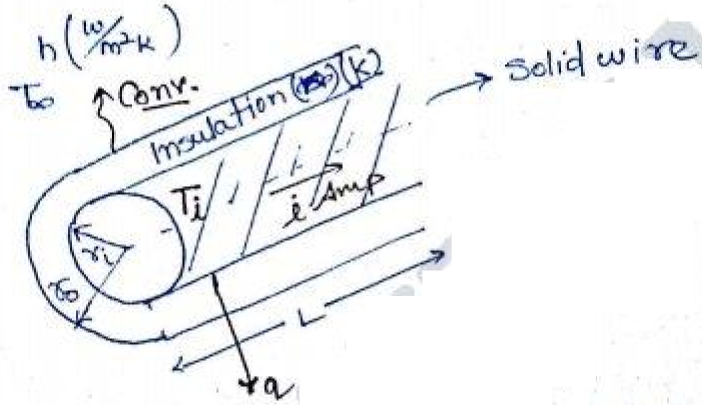
$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}$$ ⇒ Neglecting Conduction thermal Resistance



$r_1 \simeq r_2$ (very thin pipe)

Critical Radius of Insulation :-

For sufficiently thin wires putting the insulation around the wire or cable may result in increase of Heat transfer rate instead of decreasing it.



Consider a solid wire of Radius r_i inside which heat is being generated by passing electric power let an insulation having a thermal Conductivity (k) is being wrapped along wire upto the radius r_o the Heat Generated in the wire is Radially Conducted through the insulation and than from the surface of insulation Heat is Convected to the Ambiant Air T_∞ with a Convective heat transfer Coeffiecent h $watt/m^2K$

→ let T_i be the surface temprature of wire under steady state conditions

Drawing thermal circuit for radial H.T. b/w T_i and T_∞ is

Thermal Circuit

$$T_i \xrightarrow[\frac{\ln(r_o/r_i)}{2\pi kL}]{} \xrightarrow{q \text{ watt}} \xrightarrow[\frac{1}{h(2\pi r_o L)}]{} T_\infty$$

$\therefore$ Rate of Radial H.T. between wire and ambient

$$q = \frac{(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{h 2\pi r_o L}} = \frac{T_i - T_\infty}{(R_{th})_{total}}$$

→ Treating all other parameter i.e. r_i, L, k including h as constant now the q becomes a function r_o only.

$$q = f(r_o)$$

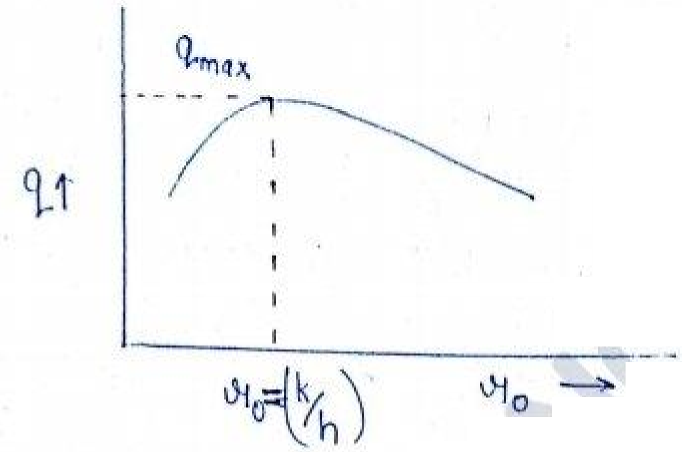
The value of r_o depands upon how much insulation warapped around the wire

for max^m H.T. Rate $\frac{dq}{dr_o} = 0$

$$\Rightarrow \frac{d}{dr_o}\left[\frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{2\pi r_o L h}}\right] = 0$$

$\Rightarrow$ $\boxed{r_o = \frac{k}{h}}$ This is called Critical Radius of insulation

Physical significance of Critical Radius of Insulation :—

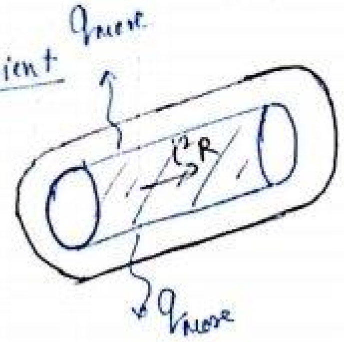


For sufficiently thin wires whose radius is lasser than critical radius of insulation, putting the insulation around the wire may result in increase of H.T. Rate instead of decreasing it. This happenes so because initially when more & more insulation is being wrapped around the wire, there is rapid decrease in convection thermal resistance as compared to little increase of conduction thermal resistance, the overall effect being decrease in total thermal resistance and increase of H.T. rate. This continues to happens upto critical radius of insulation beyond which any further insulation added shell decrease the Heat transfer rate.

* In case if the radius of the wire initialy taken is already more than critical radius of insulation, any Insulation wrapped around it shell directly decrease the H.T. rate.

Two Practicle Application Critical Radius

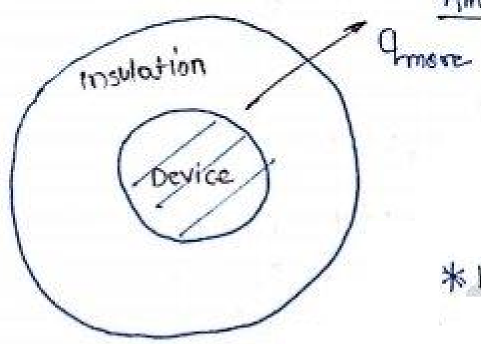
(1) Electric Power transmission Cables :-



As $T_{cable} \uparrow \Rightarrow$ Rate $\uparrow$

Insulation put up around the electric power transmission cable to increase the H.T. Rate between the cable and ambient so that the temp. of cable can be maintained low there by it's electric resistance can be maintained low Thus transmitting More electric power.

(2) Semiconductor Electronic/spherical Device :-



Ambient

$$\boxed{(r_{critical})_{Spherical} = \frac{2K}{h}}$$

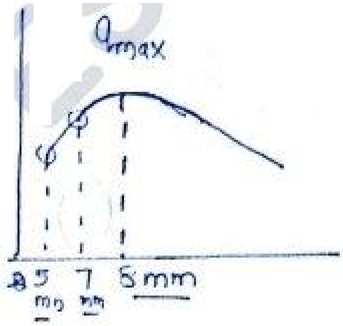
* In both example purpose is more H.T.

Q.40

$k = 0.08 \ W/mK$

$h = 10 \ W/m^2K$

$r_c = \frac{k}{h} = \frac{0.08}{1000} = 8 \ mm$



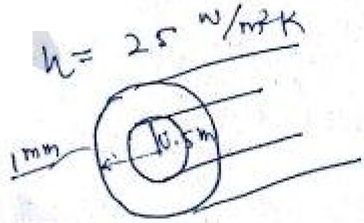
$D_i = 10mm$, $r_i = 5 \ mm$

$r_o = 2 + 5 = 7 \ mm$

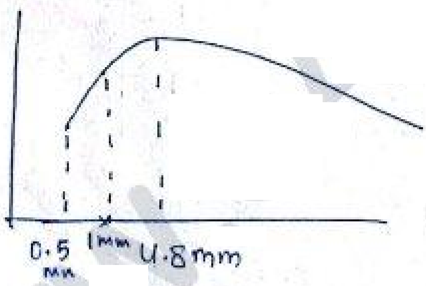
(c) Further addition of insulation first ↑ then ↓ up to 8mm

Q.50 $r_{critical} = \frac{k_{inst}}{h} = \left(\frac{0.1}{2}\right) m = 5 \ cm$

Q.43 $h = 25 \ W/m^2K$ $k = 0.12 \ W/mK$



$r_c = \frac{0.12}{25} = 4.8 \ mm$



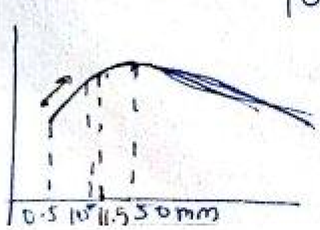
Further addition of plastic cover heat transfer will ↑

(32) $r_i = 0.5 \ mm$

$t = 1 mm$ $r_o = 1.5 \ mm$

$(r_o)_1 = 11.5$

$r_c = \frac{k}{h} = \frac{0.50}{1000} m = 50 \ mm$



As $q_{\uparrow \ from \ cable}$ ⇒ T_{cable} ↓

⇒ $R_{electric}$ ↓

⇒ Electric power carrying capacity ↑ (A)

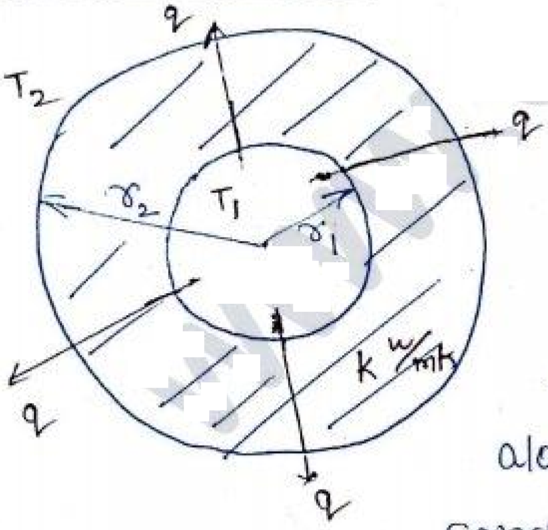
Q.30 $\left(r_c\right)_{Sp.} = 2 \times \frac{0.04}{1000} \text{ m} = 8 \text{ mm}$

$r_c = 8 \text{ mm}$

diameter = 16 mm

Q.29 c (provided the

Radial Conduction H.T. Through a hollow sphere:—



$T = f(r)$

At $r = r_1 \Rightarrow T = T_1$

At $r = r_2 \Rightarrow T = T_2$

* Since temp. Gradient are existing along the radial dir^n, heat must conduct radially outward from inner spherical surface at T_1 to outer spherical surface T_2.

like in case of Cylindrical heat transfer, here also area of Conduction H.T. changes in the direction of heat flow

At any Radius 'r'

Area of Conduction H.T. = $A = 4\pi r^2$

Fourier's law of conduction

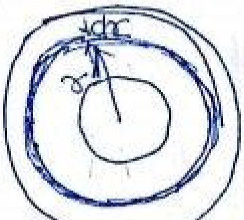
Rate of Radial heat transfer $q = -KA \frac{dT}{dr}$

$$\Rightarrow q = -k(4\pi r^2)\frac{dT}{dr} \quad \text{watt}$$

$$\int_{r_1}^{r_2} q \frac{dr}{r^2} = -\int_{T_1}^{T_2} 4\pi k \, dT$$

Assume :- steady state, 1-D Radial H.T.

To satisfy stead state condition of sphere



$q_r \neq f(r)$

$$q_r = q_{r+dr}$$

$$q\int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} dT$$

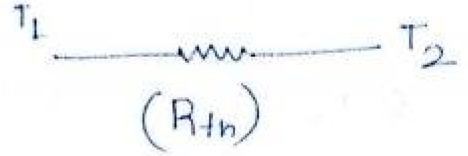
$$q\left[-\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi k(T_2 - T_1)$$

$$q\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi k(T_1 - T_2)$$

Rate of Radial H.T.

$$\boxed{q = \frac{4\pi k(T_1 - T_2) r_1 r_2}{(r_2 - r_1)}}$$

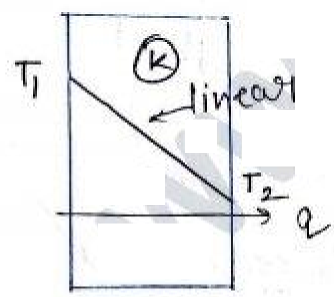
The corresponding thermal Resistance offered by Hollow sphere will be



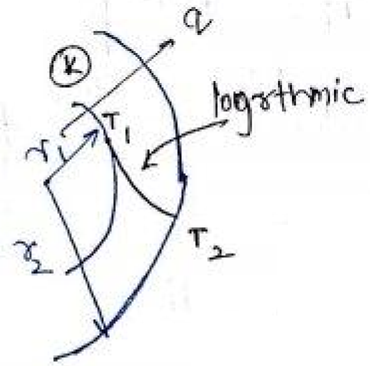
$$(R_{th})_{Cond} = \frac{T_1 - T_2}{q} = \frac{r_2 - r_1}{4\pi k r_1 r_2} \text{ K/watt}$$

$$\boxed{(R_{th})_{Cond} = \frac{r_2 - r_1}{4\pi k r_1 r_2}} \text{ K/watt}$$

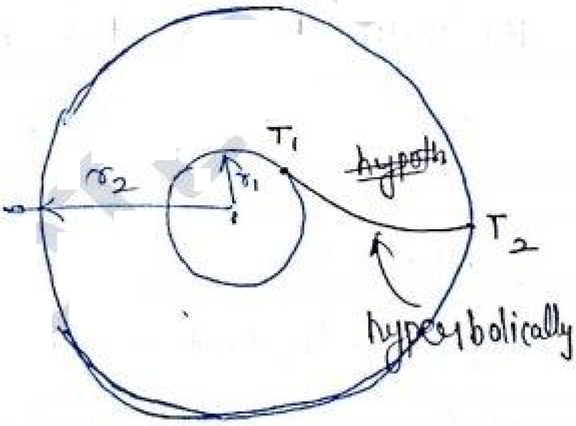
Slab



Hollow Cylinder

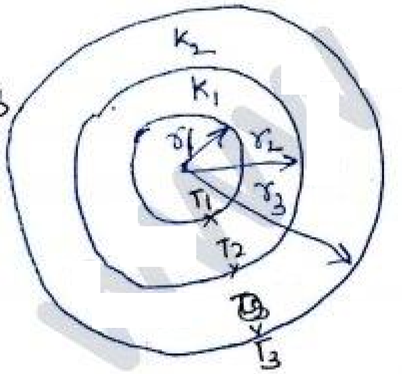


Hollow Sphere

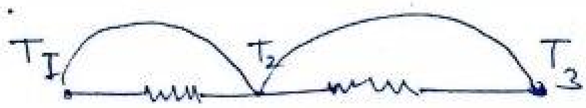


$R_{th} = \frac{b}{kA}$ K/watt $\quad R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L}$ K/watt $\quad R_{th} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$ K/watt

Q.23



$\frac{k_1}{k_2} = \frac{1}{2} \qquad \frac{d_i}{d_o} = 0.8$



$\frac{r_2 - r_1}{4\pi k_1 r_1 r_2} \qquad \frac{r_3 - r_2}{4\pi k_2 r_2 r_3}$

$$\frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

$$\frac{T_1 - T_2}{T_2 - T_3} = \frac{R_1}{R_2} = \frac{r_2 - r_1}{4\pi K_1 r_1 r_2} \times \frac{4\pi K_2 r_2 r_3}{r_3 - r_2}$$

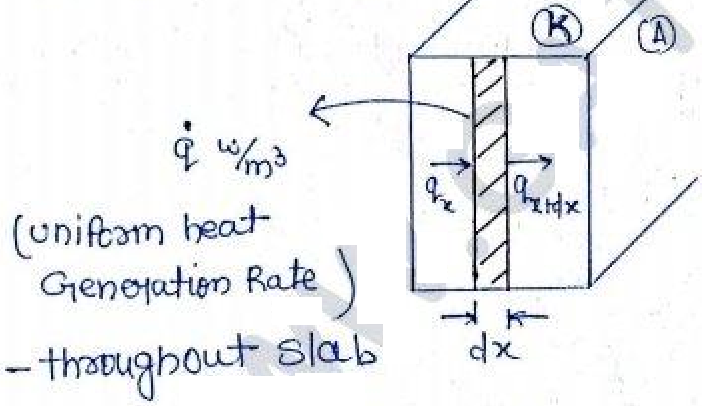
$$= \frac{r_3}{r_1} \times \frac{K_2}{K_1}$$

$$= \frac{10}{0.8} \times \frac{2}{2}$$

$$\frac{T_1 - T_2}{T_2 - T_3} = 2.5 \quad \text{Ans}$$

Generalised (3D, steady (OR) unsteady, with or without heat Generation) conduction equation:-

Heat can be generated inside a solid by either by exothermic chemical reaction or by thermo nuclear fission or by supplying electric power.



Consider a differential element of slab of length dx as shown in fig.

Let q_x = Heat conducted into the element

$$q_x = -KA\left(\frac{dT}{dx}\right)$$

q_{x+dx} = Heat conducted out of the element

$$q_{x+dx} = q_x + \frac{\partial}{\partial x}(q_x)\,dx$$

$\frac{\partial q_x}{\partial x}$ → Rate of change of heat transfer.

Heat generated in the element $= \dot{q} \times$ Volume of element
$= \dot{q} \times A\,dx$

writting the energy balance for x-dir^n conduction through the element we Get

$q_x + q_{gen} = q_{x+dx}$ + Rate of change of Internal energy of element w.r.t. time
I.E.

$$\cancel{q_x} + \dot{q}_x A\,dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + \frac{\partial}{\partial \tau}(mC_pT)$$

$$\dot{q}_x A\,dx = \frac{\partial}{\partial x}(q_x)\,dx + \frac{\partial}{\partial \tau}(\rho A\,dx\,C_pT)$$

where $\underline{m}$ is the mass of element
$m = \rho \times A\,dx$

C_p - because it is solid and solid have only one specific heat

$$\dot{q}_x \cancel{A\,dx} = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + \frac{\partial}{\partial \tau}(\rho \cancel{A\,dx}\,C_pT)$$

treating all fluid properties as constant we Get.

$$\boxed{K \frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p \left(\frac{\partial T}{\partial \tau}\right)}$$

writting the energy balance similarly for all the 3-Direction Conduction that are occuring along x, y & z dirn. simultaneously we Get.

$$K \frac{\partial^2 T}{\partial x^2} + K \frac{\partial^2 T}{\partial y^2} + K \frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial \tau}$$

$$\Rightarrow \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial \tau}$$

$$\nabla^2 T + \dot{q}/k = \frac{\rho C_p}{k} \frac{dT}{d\tau}$$

Thermal Diffusivity (α):– Defining a thermophysical property of material as ratio b/w thermal Conductivity of material ~~of~~ to thermal Capacity of material.

$$T.D. = \boxed{\alpha = \left(\frac{K}{\rho C_p}\right)} \; m^2/sec.$$

$K = W/mk$

$\rho = kg/m^3$

$C_p = J/kgK$

ρC_p – heat capacity (OR) Heat storage ability

Note:- Thermal diffusivity of a material tells about the ability of the material to allow the heat energy to get diffused or pass through the medium more rapidaly.

* Higher the conductivity of the material and lesser its heat capacity (storage ability) more the value of thermal diffusivity:

$$\boxed{\alpha_{gases} > \alpha_{liquids}} \qquad K_{air} < K_{water}$$

$$\alpha = \frac{k}{\rho C_p} \qquad \alpha_{air} > \alpha_{water} \qquad (\rho C_p)_{air} <<< (\rho C_p)_{water}$$

⇒ So now

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\underbrace{\left(\frac{\partial T}{\partial \tau}\right)}_{\text{Rate of Cooling/Heating}}} \quad (K/sec)$$

⇒ Generalized Heat conduction equation

Q.14

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{dT}{dt}\right)$$

$$\frac{dT}{dt} \underset{\text{proportion}}{\propto} \frac{d^2 T}{dx^2}$$

Because α is a property

$$\alpha = \text{Constant}$$

Q.27

$$T = 4 - 10x + 20x^2 + 10x^3$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial}{\partial x}\left(0 - 10 + 40x + 30x^2\right)\right)$$

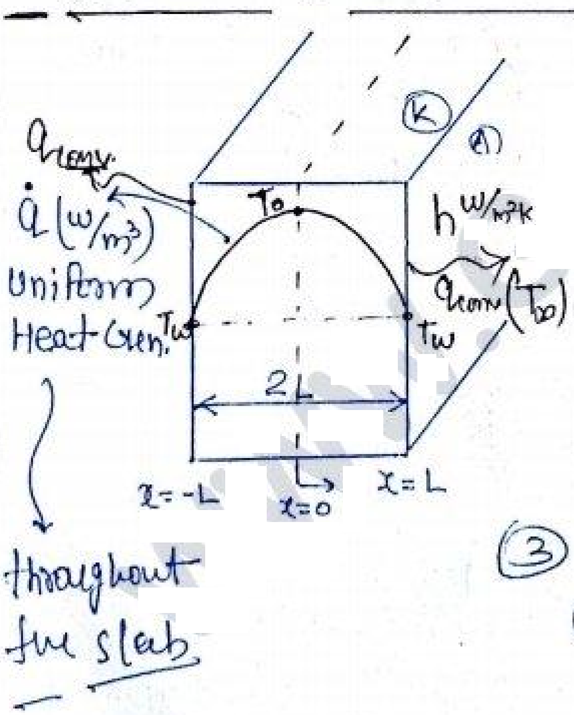
$$\frac{\partial T}{\partial t} = \alpha\left(40 + 60x\right)$$

$$\left.\frac{\partial T}{\partial t}\right|_{x=1m} = 2\times10^{-3}\ \frac{m^2}{hr}\left(40 + 60\times1\right)\ ^\circ C/m^2$$

$$\frac{\partial T}{\partial t} = 2\times10^{-3}\times100$$

$$\frac{\partial T}{\partial t} = 0.2\ ^\circ C/hr$$

Heat Generation In a slab:-



Objective:- To Get temprature distribution i.e. $T = f(x)$ with the slab.

Assume:-
1. steady state H.T. Condition i.e. $T \neq f(\text{time})$
2. 1-D Heat Conduction i.e. $T = f(x)$ only
3. Constant 'k' of material and uniform heat Generation Rate i.e. $\dot{q}$ = Const.

To maintain steady state condition of slab while Generating heat, all the heat Generated in slab must be convected to a fluid either from one side of slab or from both the side.

Now consider Generalised Heat Conduction eq^n

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{\,0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{\,0} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\cancel{\left(\frac{\partial T}{\partial t}\right)}^{\,0}$$

$$\frac{\partial^2 T}{\partial x^2} = -\dot{q}/k$$

$$\frac{\partial T}{\partial x} = -\dot{q}/k \; x + C_1$$

$$T = -\dot{q}/k \cdot \frac{x^2}{2} + C_1 x + C_2$$

C_1 and C_2 are constant of integration that are to be obtain ~~by~~ from boundary Conditions (BC's)

One special boundary condition is

At $x = +L$ & $x = -L \Rightarrow T = T_\omega$

→ i.e. both side of the slab are subjected to the same temprature.

→ This can be possible only if both sides of the slab are expose to same fluid at same temp with the same h ~~fluid at the same temp~~

to satisfy this boundary condition

$$C_1 = 0$$

So
$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_2$$

→ The temp. slab is max^m when $\frac{dT}{dx} = 0$

$$\Rightarrow \quad 0 = -\frac{\dot{q}}{k}x + \cancel{C_1}^{\,0}$$

$$\Rightarrow \quad x = 0$$

∴ we see max. temp at the mid plane of slab.

Note
→ In case if both side of the slab are at diff temp. than C_1 could not be zero, which means max^n temp will not occur at mid plane or centre line.

let max^m temp. of slab be T_0

So at $x = 0$, $T = T_0$

$$C_2 = T_0$$

∴ The temp distribution within the slab is :-

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + T_0$$

$$\boxed{T_0 - T = \frac{\dot{q}x^2}{2k}} \quad \text{— (1)}$$

→ parabolic temp. distribution

At $x = +L$ (OR) $x = -L \Rightarrow T = T_w$

$$\Rightarrow T_0 - T_w = \frac{\dot{q} L^2}{2k} \quad -(2)$$

$$\frac{(1)}{(2)} \quad \frac{T_0 - T}{T_0 - T_w} = \left(\frac{x}{L}\right)^2$$

Non dimensional Format of temp. distribution

$$T_0 \text{ (or) } T_{max} = T_w + \frac{\dot{q} L^2}{2k}$$

at centre line

* The side wall temp T_w can be obtained from energy balance equation of for steady state condition of slab

Heat Generated in the slab = Heat convected from both sides of the slab

$$\dot{q} \times \text{Vol. of slab} = 2 \times \underbrace{h \times A (T_w - T_\infty)}_{\text{one side}}$$

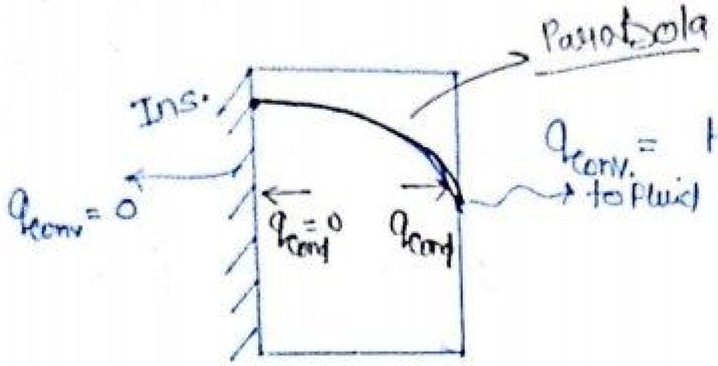
$$\dot{q} \times A \times 2L = 2 \times h \times A (T_w - T_\infty)$$

$$\boxed{T_w = \frac{\dot{q} L}{h} + T_\infty}$$

So $\quad T_{max} = T_0 = \dfrac{\dot{q} L^2}{2k} + \dfrac{\dot{q} L}{h} + T_\infty$

The other extreme case is one side of slab is perfectly insulated (adiabatic surface)

* The other extreme case is one side of the slab is perfectly insulated (adiabatic wall)



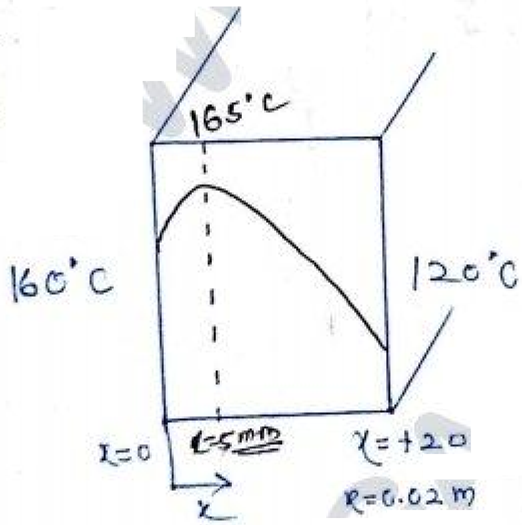
$q_{conv.}$ = Heat Generated in the slab.

$\therefore$ Heat conducted at the insulated surface = 0

$\Rightarrow \quad -KA\left(\frac{dT}{dx}\right) = 0$ at the insulated surface

$\Rightarrow \left(\frac{dT}{dx}\right) = 0$ at the insulated surface

<u>Q.17</u>



$L = 20$ mm

$\dot{q} = 80 MW/m^3 = 80\times10^6\ W/m^3$

$T_1 = 160°C$, $T_2 = 120°C$

$K = 200\ W/mK$

Since steady state, one-dimensional conduction with uniform heat generation

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{K}x + C_1$$

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1 x + C_2$$

B.C.'s at $x = 0 \quad T = 160^\circ C$

$x = 0.02 \quad T = 120^\circ C$

So $\quad C_2 = 160$

$\Rightarrow \quad x = 0.02, \quad T = 120^\circ C$

$$120 = -\frac{80 \times 10^6}{200} \times \frac{(0.02)^2}{2} + C_1(0.02) + 160$$

$$C_1 = 2000$$

So $\quad T = -\frac{\dot{q}}{k}\frac{x^2}{2} + 2000x + 160$

T_{max} $\quad \frac{dT}{dx} = 0 \Rightarrow -\frac{80 \times 10^6}{200} \times x + 2000 = 0$

$$x = \frac{2000 \times 200}{80 \times 10^6}$$

$$x = \frac{4}{800} = \frac{1}{200} \text{ m}$$

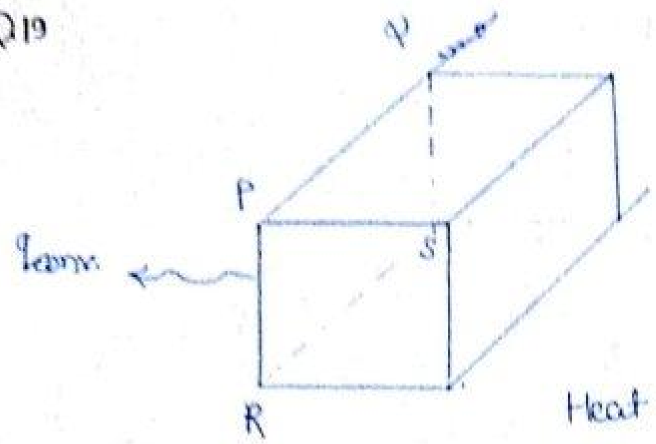
$$\boxed{x = 5 \text{ mm}}$$ Ans

T_{max} is Put $(x = 0.005 \text{ m})$

$$T_{max} = -\frac{80 \times 10^6}{200}\left(\frac{0.005^2}{2}\right) + 2000(0.005) + 160$$

$$T_{max} = 165^\circ C$$ Ans

Q19



for any steady state condition of slab writing the energy balance.

Heat Generated in the slab = Area Heat convected from PQRS to ambiant fluid

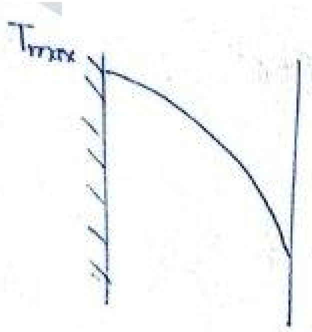
$$\left(\dot{q} \times \text{Vol. of Slab}\right) = h A_{PQRS}\left(T_{PQRS} - T_{\infty}\right)$$

$$100 \times 2 \times 1 \times 2 \text{ watt} = (2 \times 2)(10)(T - 30)$$

$$10 = T - 30$$

$$T = 40^{\circ}C$$

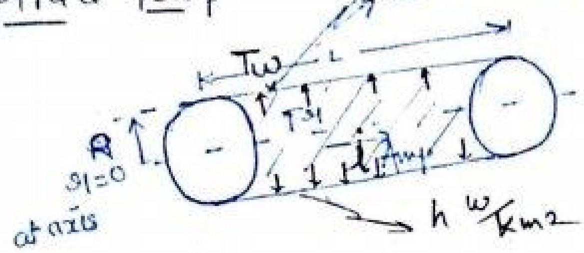
28



T_{max} $x = 0$

Heat Generated In a Cylinder :-

T_w - surface temp



R - Radius
L - length

$$\text{Uniform Heat Generation Rate} = \dot{q} = \frac{i^2 R_{ele}}{\text{Vol. of Cylinder}} \quad \text{watt}/m^3$$

$$\dot{q} = \frac{i^2 R_{ele}}{\pi R^2 L} \qquad R_{ele} = \frac{\rho L}{A} \ \Omega$$

$$\dot{q} = \frac{i^2 R_{ele}}{\pi R^2 L} \quad \text{watt}/m^3$$

Objective :- To get temprature distribution within the Rod i.e. $T = f(r)$

Assume ① steady state H.T. Condition $T \neq f(\text{time})$

To maintain the steady state Condition of Rod while generating heat All the heat generated in rod must be convected to a fluid from the surface of a rod with a given h.

② 1-D Heat Conduction i.e. $T = f(r)$ only.

③ Uniform heat generation Rate and constant 'k' of Material $\dot{q}$ = Const

k = Const.

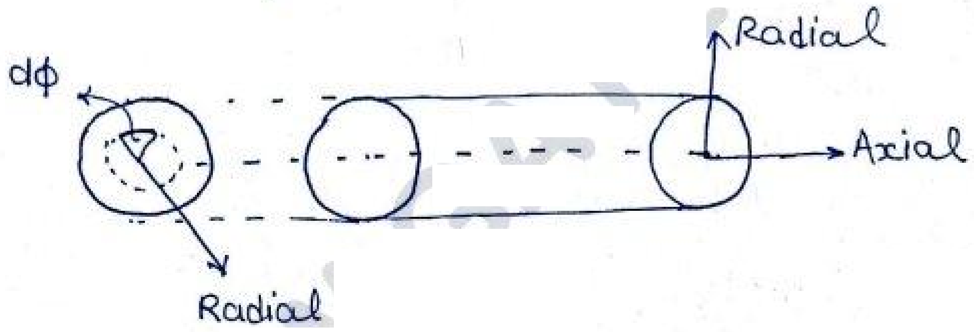
The generalized Heat Conduction equation in Cylindrical Cordinate System is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi} + \frac{\partial^2 T}{\partial z^2} + \dot{q}/k = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

r = radial Cordinate

z = Axial Cordinate

ϕ = Azimuthal Cordinate



in 1-D, steady state

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi}}}_{1D}^{0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{1D}^{0} + \dot{q}/k = \underbrace{\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}}_{steady}^{0}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\dot{q}/k$$

$$\Rightarrow \quad r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} = -\dot{q}/k \, r$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \qquad \frac{d}{dr}\left(\underset{u}{r}\,\underset{v}{\frac{\partial T}{\partial r}}\right) = -\frac{\dot{q}r}{k}$$

$$d(uv) = u\,dv + v\,du$$

Integrating w.r.t. to r

$$r\frac{dT}{dr} = -\frac{\dot{q}r^2}{2k} + C_1$$

$$\therefore \quad \frac{dT}{dr} = -\frac{\dot{q}r}{2k} + \frac{C_1}{r} \qquad -(1)$$

$$T = -\frac{\dot{q}r^2}{4k} + C_1 \ln r + C_2 \qquad (1a)$$

* C_1 & C_2 are constant of integration that are to be obtained from Boundary Condition's

* one - boundary condition is:-

  At $r = R$ (i.e. at the periphery of Rod

  $\Rightarrow T = T_w$

* The second boundary Condition is:-

  For steady state condition of Rod, writting the energy balance

Heat Generated in the Rod = Heat Conduction ~~Convected~~ Radially at the surface
= Heat Convected from the surface of Rod to fluid

$$q \times \pi R^2 L = -k(2\pi RL)\left(\frac{dT}{dr}\right)_{at\ r=R}$$

$$\therefore \left(\frac{dT}{dr}\right)_{\text{at } r=R} = -\frac{\dot{q}R}{2K} \quad \text{--- (2)}$$

from eq^n (1)

$$\left(\frac{dT}{dr}\right)_{\text{at } r=R} = -\frac{\dot{q}R}{2K} + \frac{C_1}{R} \quad \text{--- (1)}$$

from (1) & (2)

$$C_1 = 0$$

* C_1 can also be said as zero because logrithim function become ∞ when $r = 0$ .ie. axis of the rod but the temp. at the axis of the rod can not be ∞ (infinite)

The temp. of the Rod is max when

$$\frac{dT}{dr} = 0$$

so

$$0 = -\frac{\dot{q}r}{2K} + \cancel{\frac{C_1}{r}}^{\,0}$$

$$\Rightarrow \quad r = 0 \quad \text{(Axis of the Rod)}$$

$\therefore$ we see max. temp. at the axis of Rod

let the max. temp. of Rod be T_0

$\therefore$ At $r = 0$ (Axis) $\Rightarrow$ $T = T_0$

from eq (19) $\Rightarrow$ $T_0 = C_2$

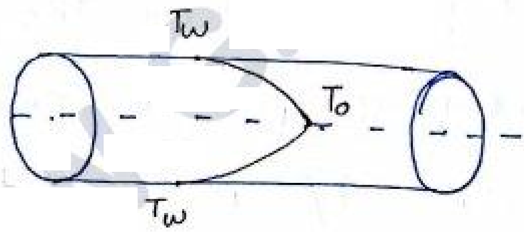
Therefore the temp. distribution within the rod

$$\boxed{T = -\frac{\dot{q} r^2}{4k} + T_0}$$

$$\therefore \quad T - T_0 = -\frac{q r^2}{4k}$$

$$\therefore \quad \boxed{T_0 - T = \frac{q r^2}{4k}} \quad -③$$

Parabolic temp. distribution



At $r = R \rightarrow T = T_w$ (surface Temp.)

$$T_0 - T_w = \frac{q R^2}{4k} \qquad -④$$

from ③ & ④

$$\boxed{\frac{T_0 - T}{T_0 - T_w} = \left(\frac{r}{R}\right)^2}$$

Non-dimensional format of temp. distribution

* The surface temp. T_w can be obtained from energy balance equation for steady state condition of Rod

Heat generated in the Rod = Heat conducted from the surface of Rod to Fluid

$$\Rightarrow \quad \dot{q} \times \pi R^2 L = h(2\pi R L)(T_w - T_\infty)$$

$$\therefore \quad T_w = \frac{\dot{q}R}{2h} + T_\infty$$

subsitituting in eq^n ④

$$T_{0\,(OR)}\, T_{max} = \frac{\dot{q}R^2}{4K} + \frac{\dot{q}R}{2h} + T_\infty$$

At axis

* if the $\dot{q}$ Value are very high and if the h Values are very low than the T_0 Value may increase anormously which may Finally Result in Melting of Rod/wire itself. and the melting begains at the axis of rod.

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$$\dot{q} \times (\pi R^2 L) = h(2\pi R L)(T_w - T_\infty)$$

$$\boxed{T_w = T_\infty + \frac{\dot{q}R}{2h}}$$