

# CHAPTER-13

## PROBABILITY

### 1 mark questions

- 1) If  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$  then find  $P(A|B)$ . (U)
- 2) If  $P(E) = 0.6$ ,  $P(F) = 0.3$  and  $P(E \cap F) = 0.2$  then find  $P(E|F)$ . (U)
- 3) If  $P(B) = 0.5$  and  $P(A \cap B) = 0.32$  then find  $P(A|B)$ . (U)
- 4) If  $P(A) = \frac{1}{2}$  and  $P(B) = 0$  then find  $P(A|B)$  if exists. (K)
- 5) If  $P(A) = 0$  and  $P(B) = \frac{1}{2}$ , then find  $P(A/B)$  if exists. (U)
- 6) If A and B are two events such that  $P(A) \neq 0$ . Find  $P(B/A)$ , if A is a sub set of B. (U)
- 7) If  $2P(A) = P(B) = \frac{5}{13}$  and  $P(A|B) = \frac{2}{5}$  then find  $P(A \cap B)$ . (U)
- 8) If  $P(A) = 0.8$ ,  $P(B) = 0.5$ ,  $P(B|A) = 0.4$  then find  $P(A \cap B)$ . (U)
- 9) If A and B are two events such that  $P(A) \neq 0$ . Find  $P(B/A)$ , if  $A \cap B = \emptyset$ . (K)
- 10) If  $P(A) = 0.3$  and  $P(B) = 0.4$  find  $P(A/B)$ , if A and B are mutually exclusive events. (K)
- 11) If  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$  then find  $P(A|B)$ . (U)
- 12) If  $P(A) = \frac{7}{13}$ ,  $P(B) = \frac{9}{13}$  and  $P(A \cap B) = \frac{4}{13}$  then find  $P(A^c|B)$ . (K)
- 13) If  $P(A) = \frac{6}{11}$  and  $P(B/A) = \frac{2}{3}$ , then find  $P(A \cap B)$ . (U)
- 14) If A and B are two events such that  $P(A) + P(B) - P(A \cap B) = P(A)$ . Find  $P(A/B)$ . (U)
- 15) If  $P(A|B) = P(B|A)$  then prove that  $P(A) = P(B)$ . (K)
- 16) If A is a subset of B then prove that  $P(B|A) = 1$ . (K)
- 17) If  $P(A) \neq 0$ , prove that  $P(A|A) = 1$ . (K)
- 18) Define independent events. (K)
- 19) If  $P(A) = 0.3$  and  $P(B) = 0.4$  find  $P(A/B)$ , if A and B are independent events. (U)
- 20) Given that the event A and B are such that  $P(A) = \frac{1}{2}$ ,  $P(A \cap B) = \frac{3}{5}$  and  $P(B) = k$ , find k if A and B are independent. (U)
- 21) If  $P(A) = \frac{3}{5}$ ,  $P(B) = \frac{1}{5}$  then find  $P(A \cap B)$  if A & B are independent events. (U)
- 22) Let E and F be two events such that  $P(E) = \frac{3}{5}$ ,  $P(F) = \frac{3}{10}$   $P(E \cap F) = \frac{1}{5}$ . Are E and F independent events. (U)
- 23) If A and B are independent events with  $P(A) = 0.3$ ,  $P(B) = 0.4$  then find  $P(\text{not A and not B})$ . (U)
- 24) If A and B are independent events such that  $P(A) = 0.3$  and  $P(B) = 0.6$ , find  $P(A \text{ and } B)$ . (U)
- 25) If A and B are independent events such that  $P(A) = 0.3$  and  $P(B) = 0.6$ , find  $P(A \text{ and not B})$ . (U)
- 26) If A and B are independent events with  $P(A) = 0.3$ ,  $P(B) = 0.4$  then find  $P(A|B)$ . (U)
- 27) Define Theorem of total probability. (K)
- 28) An urn contains 5 red and 2 black balls, two balls are randomly selected. Let x represents the number of black balls. What are the possible values of x. (K)
- 29) An urn contains 4 green and 2 black balls. Two balls are randomly selected. Let X represents the number of black balls, what are the possible values of X? (K)

30) If  $x$  represent the difference between the number of heads and number of tails obtained when a coin is tossed six times, what are the possible values of  $X$ ? **(K)**

31) If  $X$  denote the sum of the numbers obtained when two fair dice are rolled. What are possible values of  $X$ . **(K)**

32) A coin is tossed 3 times let  $x$  represents the number of tails. What are the possible value of  $x$ . **(K)**

33) Probability distribution of  $x$  is

$X$	0	1	2	3	4
$P(X)$	0	$k$	$2k$	$2k$	$k$

Where  $k$  is constant, then find  $k$ . **(U)**

34) Probability distribution of  $x$  is

$X$	0	1	2	3	4
$P(X)$	0.1	$k$	$2k$	$2k$	$k$

Find  $k$ . **(U)**

35) The random variable has a probability distribution  $P(x)$  has following form where  $k$  is

$$\text{constant. } P(x) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 0 & \text{is other wise.} \end{cases} \quad \text{Find value of } k. \text{ (U)}$$

36) Is the given distribution of  $X$  is a Probability distribution of random variable  $X$ .

$X$	0	1	2	3	4
$P(X)$	0.1	0.5	0.2	0.1	0.3

**(U)**

37) Give the reason for the following distribution of  $X$  is not a Probability distribution of random variable  $X$ .

$x$	0	1	2	3	4
$P(x_1)$	0.1	0.5	0.2	-0.1	0.3

**(U)**

## Two mark questions

- If  $2P(A) = P(B) = \frac{5}{13}$  and  $P(A|B) = \frac{2}{5}$  then find  $P(A \cup B)$ . **(U)**
- Prove that  $P(A^c|B) = 1 - P(A|B)$ . **(U)**
- If  $A$  and  $B$  are independent events with  $P(A) = 0.3, P(B) = 0.4$  then find  $P(A \cup B)$ . **(U)**
- If  $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$  and  $P(A \cup B) = \frac{7}{11}$  then find  $P(B|A)$ . **(U)**
- $A$  and  $B$  are an events such that  $P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5}, P(B) = q$ , then find  $q$  if  $A$  and  $B$  are independent. **(U)**
- Let  $A$  and  $B$  are two events such that  $P(A) = \frac{1}{4}, P(A|B) = \frac{1}{2}$  and  $P(B|A) = \frac{2}{3}$  then find  $P(B) = ?$  **(U)**
- A fair die is rolled consider an events  $E = \{2, 4, 6\}, F = \{1, 2\}$  then find  $P(E|F)$ . **(K)**

8. A fair die is rolled. Consider events  $E = \{1, 3, 5\}$  and  $F = \{2, 3, 5\}$ , find  $P(F/E)$ . **(K)**
9. A couple has two children. Find the probability that both children are males if it is known that at least one of the children is male. **(K)**
10. Mother, Father and son line up at random for a family picture, find  $P(E/F)$ . If E: son on one end, F: father in middle. **(K)**
11. Consider an experiment of tossing two fair coins simultaneously. Find the probability that both are heads. Given that at least one of them is head. **(K)**
12. A couple has 2 children find the probability that both are female if it is known that elder child is female. **(K)**
13. Given that the 2 number appear in on throwing two dices are different. Find the probability of an event the sum of the number is 4. **(K)**
14. Find the conditional probability of obtaining the sum 8 given that the red die resulted is a number less than 4. **(K)**
15. In a hostel 60% of students read Hindi newspaper, 40% of students read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random then, If she reads Hindi newspaper find the probability that she also reads English newspapers. **(K)**
16. A coin is tossed 3 times then find  $P(E|F)$ , where E : Head on third toss and F : Head on first two tosses. **(U)**
17. A coin is tossed 3 times then find  $P(E|F)$ , where E : at least two heads and F : at most two heads. **(U)**
18. A black and red dice are rolled. Find the conditional probability of obtaining the sum greater than 9. Given that black die resulted as 5. **(U)**
19. A black and red dice are rolled. Find the conditional probability of obtaining the sum greater than 9. Given that black die resulted as 5. **(U)**
20. If A and B are independent events, then prove that A and  $B'$  are also independent. **(K)**
21. If A and B are independent events, then prove that  $A'$  and B are also independent. **(K)**
22. If A and B are independent events, then prove that  $A'$  and  $B'$  are also independent. **(K)**
23. If A and B are two independent events then prove that the probability of occurrence of at least one of A and B is given by  $1 - P(A') \cdot P(B')$ . **(K)**
24. If  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\text{not } A \text{ or not } B) = \frac{1}{4}$  then state whether A and B are independent. **(U)**
25. If A and B are two events such that  $P(A) = \frac{1}{4}$  and  $P(B) = \frac{1}{2}$ . Then  $P(A \cap B) = \frac{1}{8}$  then Find  $P(\text{not } A \text{ and not } B)$ . **(U)**
26. Find the probability of getting even prime number on each die, when a pair of dice is rolled. **(U)**
27. Two cards are drawn random without replacement from a pack of 52 playing cards. Find the probability that both are black cards. **(U)**
28. Two cards are drawn successfully with replacement from a pack 52 cards find the probability distribution of number of ace cards. **(U)**
29. A Urn contains 10 black and 5 white balls, 2 balls are drawn one after the other without replacement. What is the probability that both drawn balls are black. **(U)**
30. Three cards drawn successively without replacement from a pack of 52 well shuffled cards. What is the probability that 1<sup>st</sup> two cards are king and 3<sup>rd</sup> card drawn is ace. **(U)**
31. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls find the probability that both are red. **(U)**

32. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls find the probability that 1<sup>st</sup> ball is black and second is red. **(U)**
33. A die is tossed thrice. Find the probability of getting an odd number at least once. **(U)**
34. Probability of solving specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently. Find the probability that the problem is solved. **(U)**

35. Find the probability distribution of number of heads in two tosses of a coin. **(K)**
36. Find the probability distribution of number of tails in three tosses of a coin. **(K)**
37. Find the probability distribution number of success in 2 tosses of die where success is defined as number > 4. **(K)**
38. Probability distribution of X is

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> + k

Find i) k ii) P(X<3). **(K)**

39. The probability distribution of the random variable X is given

x	0	1	2	3
P(x <sub>1</sub> )	1/8	3/8	3/8	1/8

Find E(X). **(K)**

40. If  $P(E_1) = \frac{1}{2}$ ,  $P(E_2) = \frac{1}{2}$  and  $P(A/E_1) = \frac{1}{2}$ ,  $P(A/E_2) = \frac{1}{4}$ . Find  $P(E_1/A)$ . **(K)**
41. If  $E(X) = \frac{21}{6}$  and  $E(X^2) = \frac{91}{6}$ , find the standard deviation of X. **(K)**
42. If  $\text{Var}(X) = 9$  and  $E(X^2) = 25$ , find E(X). **(K)**
43. The probability distribution of X is ,

X	0.5	1	1.5	2
P(X)	K	K <sup>2</sup>	2K <sup>2</sup>	K

Find the mean of X. **(K)**

### Three mark questions.

- A die is thrown twice and sum of the numbers appeared is observed to be six.  
What is the conditional probability that the number 4 has appeared at least once. **(U)**
- A die is thrown 3 times events A and B are defined as follows. Event A : 4 on first throw and Event B : 6 and 5 on second and third throw. Find the probability of 'A' given that 'B' has already occurred. **(U)**
- A pair of die are thrown, an event A and B are as follows, A : the sum of 2 numbers on the die is 8 and B : there is an even number on the first die. Find the conditional probability  $P(B|A)$ . **(U)**

4. 10 cards numbered from 1 to 10 are placed in a box mix up thoroughly and 1 card is drawn random, if it is known that the number on the drawn card is more than 3. What is the probability that it is an even number. **(U)**
5. An instructor has question bank consisting of 300 easy true/false questions, 200 difficult true/false questions, 500 easy multiple choice questions and use 400 difficult MCQ's. If a question is selected at random from the question bank. What is the probability that it will be a easy question given that it's a MCQ. **(U)**
6. One card is drawn at random from a well shuffled deck of 52 cards. Find where events E and F are independent. E : the card drawn is a spade and F: the card drawn is an ace.**(U)**
7. A die is marked 1,2,3 in red and 4,5,6 in green is tossed. Let 'A' be an event that 'the number is even' and B be an event that 'the number is red. Are A and B independent.
8. An unbiased die is thrown twice, let A be an event 'odd number on the first throw' let 'B' be an event odd number on the 2<sup>nd</sup> throw check the independence of the events A and B . **(U)**
9. A die is thrown, if E be an event, the number appearing is a multiple of 3 and F be an event the number appearing is even, then find whether E and F are independent. **(U)**
10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag. What is the probability that the ball is red. **(U)**
11. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ? (A)
12. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II . The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. **(U)**
13. A bag contains 3 red and 4 black balls, another bag contains 5 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. **(U)**
14. There are three coins, one is a two headed coin, another is a biased coin that comes up head 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed it shows head. What is the probability that it was the two headed coin. **(U)**
15. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold? **(U)**
16. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver? **(U)**

17. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A? **(U)**
18. A doctor is to visit a patient from the past experience it is known that the probabilities that he will come by train, bus scooter or by other means of transportation are  $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$  and  $\frac{2}{5}$  respectively. The probability that he will be late are  $\frac{1}{4}, \frac{1}{3}$  &  $\frac{1}{12}$ . If he comes by train, bus scooter respectively. But he comes by the means of transport he will not be late. When he arrive, is late. What is the probability be will come by trains. **(U)**
19. Of the students in a college it is known that 60% reside in hotel and 40% are day scholar (not residing in hostel) previous year results report that 30% of the student who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year one student is chosen at random from college and he has 'A' grade. What is the probability that the student is a hosteller? **(U)**
20. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 & 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B? **(U)**
21. In answering a question on a multiple choice test a student either knows the answer or guesses. Let  $\frac{3}{4}$  be the probability that he knows the answer and  $\frac{1}{4}$  be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability  $\frac{1}{4}$ . What is the probability that a student knows the answer given that he answered it correctly. **(U)**
22. A men is know to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually six. **(U)**
23. Probability that a person speaks truth is  $\frac{4}{5}$ , A coin is tossed a person reports that head appears. Find the probability that it is actually head. **(U)**
24. Suppose that 5% of men and 0.25% of women have grey hair, A grey haired person is selected at random, what is the probability of this person being male? Assume that there are equal number of males and females. **(U)**
25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond. **(U)**
26. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? **(U)**

27. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ve but 1% are diagnosed as showing HIV+ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV? **(U)**
28. Suppose a girl throws a die if she gets 5 or 6 she tosses a coin 3 times and notes the no of head . If she gets one, 2, 3 or 4 she tosses a coin once and notes whether head or tail is obtained. If she obtained exactly one head what is the probability that she got 1,2,3 or 4 with a die. **(U)**
29. Suppose a girl throws a die if she gets 5 or 6 she tosses a coin 3 times and notes the no of head . If she gets one, 2, 3 or 4 she tosses a coin once and notes whether head or tail is obtained. If she obtained exactly one head what is the probability that she got 1,2,3 or 4 with a die. **(U)**
30. If A,B,C and D are 4 boxes containing coloured marbles as given below. **(U)**

Box	Marbles		
	Red	White	Black
A	1	3	6
B	6	2	2
C	8	1	1
D	0	6	4

31. One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C? **(U)**
32. Two dice are thrown simultaneously. If  $X$  denotes the number of sixes. Find the mean (expectation) of  $X$ . **(U)**
33. Find the mean number of heads in 3 tosses of a fair coin. **(U)**
34. Find the variable of the number of obtained on a thrown of unbiased die. **(U)**
35. Two cards are drawn simultaneously without replacement from a pack of 52 cards. Find the mean of the number of kings. **(U)**
36. A class has 15 students whose ages are 15,17,15,14,21,17,19,20,16,18,20,17,16,19 and 20 years. One student is selected in such a manner that each has same chance of being chosen and the age  $X$  of the selected student is recorded. Find mean of  $X$ . **(U)**
37. Two cards are drawn simultaneously without replacement from a pack of 52 cards. Let  $X$  be the number of aces obtained. Find the value of  $E(X)$ . **(U)**
38. Find the mean of the numbers obtained on throwing a die having writing 1 on three faces.2 on two faces and 5 on one face. **(U)**

### Five mark questions.

1. A die is thrown six time. If getting an odd number is a success, what is the probability of  
i ) exactly five success    ii)at least five successes. **(A)**
2. Find the probability of getting at most two sixes and at least three sixes in six throws of a single die. **(A)**
3. If a fair coin is tossed 10 times. Find the probability of (i) exactly six heads. and (ii) at least six heads. **(A)**
4. A fair coin is tossed 8 times find the probability of a) At most five heads b) At least five heads . **(A)**
5. A pair of dice thrown 4 times. If getting a doublet is considered as success. Find the probability of two successes and at most three successes. **(A)**
6. Find the probability of getting five exactly twice in 7<sup>th</sup> throws of a die. **(A)**
7. Five cards are drawn successively with replacement from a well shuffled deck of 52 playing cards. What is the probability that i) all the five cards are spades. ii) only 3 cards are spades, and iii) none is a spade. **(A)**
8. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is  $\frac{1}{100}$ . What is the probability that he will win a prize at least once and exactly once. **(A)**
9. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs i) none will fuse after 150 days ii) not more than one fuse after 150 days. **(A)**
10. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than 1 defective item. **(A)**
11. In an examination 20 questions of true false type are asked suppose a student tosses a fair coin to determine his answer to each question. If the coin falls head. He answers true if it false he answers false. Find the probability that he answers at least 1 questions correctly. **(A)**
12. On a multiple choice examination with 3 possible answer for each of the 5 questions. What is the probability that a candidate would getting 4 or more correct answers just by guessing. **(A)**
13. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective and at most one defective egg. **(A)**
14. It is known that 10% of certain articles manufacture are defective. What is the probability that in a random sample of 12 such articles 9 are defective. **(A)**
15. Suppose  $x$  has binomial distribution  $B\left(6, \frac{1}{2}\right)$  show that  $x = 3$  is the most likely outcome. **(A)**
16. The probability that a student is not swimmer is  $\frac{1}{5}$ . Then Find the probability that out of five students, four are swimmer. **(A)**



17. If 90% of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right handed. **(A)**
18. An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn its mark noted down and it is replaced ,6 balls are drawn in this way. Find the probability that i) all will bear X mark, ii) not more than 2 will bear Y mark. **(A)**

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