

Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 2

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ =$ [1]
a) $\frac{1}{2}$ b) $\frac{-1}{2}$
c) 1 d) 0
2. if $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ then $f(x) = ?$ [1]
a) $(x^2 - 2)$ b) $(x^2 + 1)$
c) $(x^2 - 1)$ d) x^2
3. The mean and S.D. of 1, 2, 3, 4, 5, 6 is [1]
a) $3, \frac{35}{12}$ b) 3, 3
c) $\frac{7}{2}, \sqrt{\frac{35}{12}}$ d) $\frac{7}{2}, \sqrt{3}$
4. $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m}, n > m > 0$ is equal to [1]
a) $\frac{m}{n}$ b) 0
c) 1 d) $\frac{n}{m}$
5. The lines $8x + 4y = 1, 8x + 4y = 5, 4x + 8y = 3, 4x + 8y = 7$ form a [1]
a) Trapezium b) Rhombus
c) Rectangle d) Square
6. The reflection of the point (α, β, γ) in the xy- plane is [1]
a) $(\alpha, \beta, -\gamma)$ b) $(0, 0, \gamma)$

- c) $(\alpha, \beta, 0)$ d) $(-\alpha, -\beta, \gamma)$
7. If $z = (3 + \sqrt{2}i)$ then $z \times z = ?$ [1]
 a) 11 b) 7
 c) $\sqrt{11}$ d) 5
8. The number of ways in which 5 + and 5 – signs can be arranged in a line such that no two – signs occur together is [1]
 a) $P(5, 5)$ b) $C(5, 5)$
 c) $P(6, 5)$ d) $C(6, 5)$
9. $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$ is equal to [1]
 a) 1 b) -1
 c) 2 d) -2
10. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B)$ is equal to [1]
 a) 2 b) 0
 c) 1 d) 3
11. Let $S = \{x \mid x \text{ is a positive multiple of 3 less than 100}\}$
 $P = \{x \mid x \text{ is a prime number less than 20}\}$. Then $n(S) + n(P)$ is [1]
 a) 41 b) 30
 c) 34 d) 33
12. $(\sqrt{5} + 1)^4 + (\sqrt{5} - 1)^4$ is [1]
 a) an irrational number b) a negative real number
 c) a rational number d) a negative integer
13. If $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$ then [1]
 a) $x < y$ b) $x > y$
 c) $x = y$ d) $x \geq y$
14. Solve the system of inequalities $(x + 5) - 7(x - 2) \geq 4x + 9$, $2(x - 3) - 7(x + 5) \leq 3x - 9$ [1]
 a) $\frac{-9}{4} \leq x \leq 1$ b) $-4 \leq x \leq 1$
 c) $-1 \leq x \leq 1$ d) $-4 \leq x \leq 4$
15. Which of the following is a null set? [1]
 a) $C = \phi$ b) $B = \{x : x + 3 = 3\}$
 c) $D = \{0\}$ d) $A = \{x : x > 1 \text{ and } x < 3\}$
16. $\sin 18^\circ = ?$ [1]
 a) $\frac{(\sqrt{3}+1)}{2}$ b) $\frac{(\sqrt{3}-1)}{2}$
 c) $\frac{(\sqrt{5}+1)}{4}$ d) $\frac{(\sqrt{5}-1)}{4}$
17. $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ is equal to: [1]

a) na^{n-1}

b) 1

c) na^n

d) na

18. In an examination, a candidate has to pass in each of the five subjects. In how many ways can he fail? [1]

a) 31

b) 10

c) 21

d) 5

19. **Assertion (A):** Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Then, $A \subset B$. [1]

Reason (R): If every element of X is also an element of Y, then X is a subset of Y.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of infinite terms of a geometric progression is given by $S_\infty = \frac{a}{1-r}$, provided $|r| < 1$. [1]

Reason (R): The sum of n terms of Geometric progression is $S_n = \frac{a(r^n - 1)}{r - 1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. f, g and h are three functions defined from R to R as follows: [2]

i. $f(x) = x^2$

ii. $g(x) = x^2 + 1$

iii. $h(x) = \sin x$

Then, find the range of each function.

OR

Find the domain and range of the function $f(x) = 1 - |x - 2|$

22. Differentiate the function with respect to x: $(3x^2 - x + 1)^4$. [2]

23. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be a black card (i.e., a club or, a spade). [2]

OR

An integer is chosen at random from the numbers ranging from 1 to 50. What is the probability that the integer chosen is a multiple of 2 or 3 or 10?

24. If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, e, f, g\}$, verify that: $(A \cap B) \cap C = A \cap (B \cap C)$ [2]

25. In what ratio is the line joining the points (2, 3) and (4, -5) divided by the line passing through the points (6, 8) and (-3, -2). [2]

Section C

26. The letters of the word **SURITI** are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word **SURITI**. [3]

27. If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a, b and c. [3]

28. Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients [3]

of middle terms in the expansion of $(1 + x)^{2n-1}$.

OR

Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively.

29. Differentiate $\frac{x^2-1}{x}$ from first principle. [3]

OR

Find the derivative of function $\frac{ax+b}{cx+d}$ (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers).

30. If the p^{th} and q^{th} terms of a GP are q and p respectively, then show that $(p + q)^{\text{th}}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$. [3]

OR

Find a G.P. for which sum of the first two term is -4 and the fifth term is 4 times the third term.

31. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed
- i. in English and Mathematics but not in Science
 - ii. in Mathematics and Science but not in English
 - iii. in Mathematics only
 - iv. in more than one subject only

Section D

32. Calculate the mean, median and standard deviation of the following distribution: [5]

Class-interval:	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70
Frequency:	2	3	8	12	16	5	2	3

33. Draw the shape of the ellipse $4x^2 + 9y^2 = 36$ and find its major axis, minor axis, value of c, vertices, directrices, foci, eccentricity and length of latusrectum. [5]

OR

Find the equation of the parabola whose focus is (1, -1) and whose vertex is (2, 1). Also find its axis and latus - rectum.

34. Solve the following system of linear inequalities [5]

$$-2 - \frac{x}{4} \geq \frac{1+x}{3} \text{ and } 3 - x < 4(x-3)$$

35. Prove that: $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$. [5]

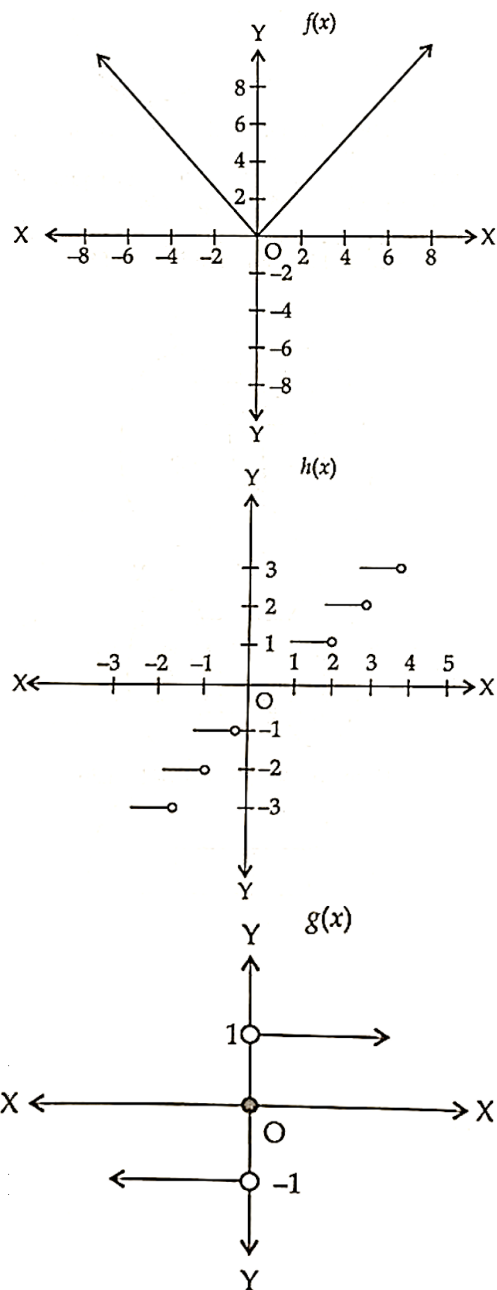
OR

$$\text{Prove that: } \cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x.$$

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Consider the graphs of the functions $f(x)$, $h(x)$ and $g(x)$.



- Find the range of $h(x)$. (1)
- Find the domain of $f(x)$. (1)
- Find the value of $f(10)$. (2)

OR

Find the range of $g(x)$. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

Four friends Dinesh, Yuvraj, Sonu, and Rajeev are playing cards. Dinesh, shuffling a cards and told to Rajeev choose any four cards.



- What is the probability that Rajeev getting all face card. (1)

ii. What is the probability that Rajeev getting two red cards and two black card. (1)

iii. What is the probability that Rajeev getting one card from each suit. (2)

OR

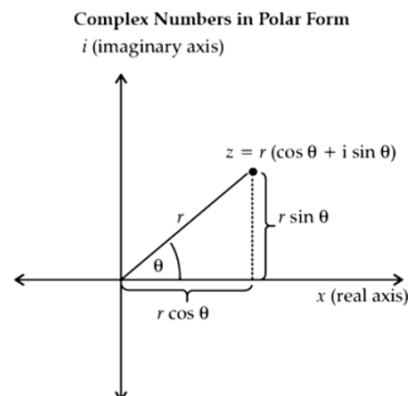
What is the probability that Rajeev getting two king and two Jack cards. (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

Consider the complex number $Z = 2 - 2i$.

Complex Number in Polar Form



i. Find the principal argument of Z . (1)

ii. Find the value of $\mathbf{z\bar{z}}$? (1)

iii. Find the value of $|Z|$. (2)

OR

Find the real part of Z . (2)

Solution

Section A

1.

(b) $\frac{-1}{2}$

Explanation: $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ$

$$= 2 \cos \left(\frac{40^\circ + 80^\circ}{2} \right) \cos \left(\frac{40^\circ - 80^\circ}{2} \right) + \cos 160^\circ + \cos (180^\circ + 60^\circ) \quad [\because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)]$$

$$= 2 \cos 60^\circ \cos (-20^\circ) + \cos 160^\circ - \frac{1}{2}$$

$$= 2 \times \frac{1}{2} \cos 20^\circ + \cos 160^\circ - \frac{1}{2}$$

$$= \cos (180^\circ - 20^\circ) + \cos 20^\circ - \frac{1}{2}$$

$$= -\cos 20^\circ + \cos 20^\circ - \frac{1}{2}$$

$$= -\frac{1}{2}$$

2. (a) $(x^2 - 2)$

Explanation: $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

Put, $\left(x + \frac{1}{x}\right) = t$

$$\Rightarrow f(t) = t^2 - 2$$

$$\therefore f(x) = x^2 - 2$$

3.

(c) $\frac{7}{2}, \sqrt{\frac{35}{12}}$

Explanation: Mean = $\frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$

$$S.D = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{36-1}{12}} = \sqrt{\frac{35}{12}}$$

4.

(b) 0

Explanation: $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m} \cdot \frac{x^{m+n}}{x^{m+n}}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x^n}{x^n} \cdot \frac{x^m}{(\sin x)^m} \cdot \frac{x^n}{x^m}$$

$$\Rightarrow 1 \cdot 1^m \cdot x^{n-m}$$

$$\Rightarrow 1(0) = 0$$

5.

(b) Rhombus

Explanation: On solving the equations $8x + 4y = 1$ and $4x + 8y = 3$, we get the point of intersection as $\left(\frac{-1}{2}, \frac{5}{12}\right)$

On solving the equations $8x + 4y = 5$ and $4x + 8y = 7$, we get the point of intersection as $\left(\frac{1}{4}, \frac{3}{4}\right)$

On solving the equations $8x + 4y = 1$ and $4x + 8y = 7$, we get the point of intersection as $\left(\frac{-5}{12}, \frac{13}{12}\right)$

On solving the equations $8x + 4y = 5$ and $4x + 8y = 3$, we get the point of intersection as $\left(\frac{7}{12}, \frac{1}{12}\right)$

Let the points $A\left(\frac{-1}{2}, \frac{5}{12}\right)$, $B\left(\frac{7}{12}, \frac{1}{12}\right)$, $C\left(\frac{1}{4}, \frac{3}{4}\right)$ and $D\left(\frac{-5}{12}, \frac{13}{12}\right)$ be the vertices of the quadrilateral

Since the slopes of the opposite sides are equal the quadrilateral is a parallelogram

The slope of the diagonal AC is $\frac{5/12 - 3/4}{-1/2 - 1/4} = 1$

The slope of the diagonal BD is $\frac{1/2 - 13/12}{7/12 - (-5/12)} = -1$

Since the product of the slopes is -1, the diagonals are perpendicular to each other.

Hence the parallelogram is a rhombus.

6. (a) $(\alpha, \beta, -\gamma)$

Explanation: In xy-plane, the reflection of the point (α, β, γ) is $(\alpha, \beta, -\gamma)$

7. (a) 11

Explanation: $zz = |z|^2 = \{3^2 + (\sqrt{2})^2\} = (9 + 2) = 11$

8.

(d) C(6, 5)

Explanation: Since all the plus signs are identical, we have number of ways in which 5 plus signs can be arranged = 1.

Now we will have 6 empty slots between these 5 identical + signs

Hence the number of possible places of - sign = 6

Therefore the number of ways in which the 5 minus sign can take any of the possible 6 places = C(6, 5)

9.

(b) -1

Explanation: Given, $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} = \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{-(\pi - x)}$
 $= -1 \left[\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \pi - x \rightarrow 0 \Rightarrow x \rightarrow \pi \right]$

10. (a) 2

Explanation: $\tan(A - B) = \tan \frac{\pi}{4}$

$$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$$

$$\Rightarrow \tan A - \tan B = 1 + \tan A \tan B \dots (i)$$

Now,

$$(1 + \tan A)(1 - \tan B) = 1 + \tan A - \tan B - \tan A \tan B$$

$$= 1 + 1 + \tan A \tan B - \tan A \tan B \text{ (Using eq. (i))}$$

$$= 2$$

11. (a) 41

Explanation: We have to find $n(S) + n(P)$

$S = \{x \mid x \text{ is a positive multiple of 3 less than 100}\}$

$$\Rightarrow S = \{3, 6, 9, 12, 15, \dots, 99\}$$

$$\Rightarrow n(S) = \frac{99}{3} = 33$$

$P = \{x \mid x \text{ is a prime number less than 20}\}$

$$\Rightarrow P = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$\Rightarrow n(P) = 8$$

$$n(S) + n(P) = 33 + 8 = 41$$

Therefore, answer is 41

12.

(c) a rational number

Explanation: We have $(a + b)^n + (a - b)^n$

$$= [{}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots + {}^nC_n b^n] +$$

$$[{}^nC_0 a^n - {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 - {}^nC_3 a^{n-3} b^3 + \dots + (-1)^n {}^nC_n b^n]$$

$$= 2[{}^nC_0 a^n + {}^nC_2 a^{n-2} b^2 + \dots]$$

Let $a = \sqrt{5}$ and $b = 1$ and $n = 4$

$$\text{Now we get } (\sqrt{5} + 1)^4 + (\sqrt{5} - 1)^4 = 2[{}^4C_0 (\sqrt{5})^4 + {}^4C_2 (\sqrt{5})^2 1^2 + {}^4C_4 (\sqrt{5})^0 1^4]$$

$$= 2[25 + 30 + 1] = 112$$

13. (a) $x < y$

Explanation: Given $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$

$$\text{Now } y = (101)^{50} = (100 + 1)^{50} = {}^{50}C_0 (100)^{50} + {}^{50}C_1 (100)^{49} + {}^{50}C_2 (100)^{48} + \dots + {}^{50}C_{50} \dots (i)$$

$$\text{Also } (99)^{50} = (100 - 1)^{50} = {}^{50}C_0 (100)^{50} - {}^{50}C_1 (100)^{49} + {}^{50}C_2 (100)^{48} - \dots + {}^{50}C_{50} \dots (ii)$$

Now subtract equation (ii) from equation (i), we get

$$(101)^{50} - (99)^{50} = 2[{}^{50}C_1 (100)^{49} + {}^{50}C_3 (100)^{47} + \dots]$$

$$= 2 \left[50(100)^{49} + \frac{50 \times 49 \times 48}{3 \times 2 \times 1} (100)^{47} + \dots \right]$$

$$= (100)^{50} + 2 \left(\frac{50 \times 49 \times 48}{3 \times 2 \times 1} (100)^{47} \right)$$

$$\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$$

$$\Rightarrow (101)^{50} > (100)^{50} + (99)^{50}$$

$$\Rightarrow y > x$$

14.

$$(b) -4 \leq x \leq 1$$

$$\text{Explanation: } (x+5) - 7(x-2) \geq 4x+9$$

$$\Rightarrow x+5-7x+14 \geq 4x+9$$

$$\Rightarrow -6x+19 \geq 4x+9$$

$$\Rightarrow -6x-4x \geq 9-19$$

$$\Rightarrow -10x \geq -10$$

$$\Rightarrow x \leq 1$$

$$\Rightarrow x \in (-\infty, 1]$$

$$2(x-3) - 7(x+5) \leq 3x-9$$

$$\Rightarrow 2x-6-7x-35 \leq 3x-9$$

$$\Rightarrow -5x-41 \leq 3x-9$$

$$\Rightarrow -5x-3x \leq 41-9$$

$$\Rightarrow -8x \leq 32$$

$$\Rightarrow -x \leq \frac{32}{8} = 4$$

$$\Rightarrow x \geq -4$$

$$\Rightarrow x \in [-4, \infty)$$

Hence the solution set is $[-4, \infty) \cap (-\infty, 1] = [-4, 1]$

Which means $-4 \leq x \leq 1$

15. (a) $C = \phi$

Explanation: ϕ is denoted as null set.

16.

$$(d) \frac{(\sqrt{5}-1)}{4}$$

$$\text{Explanation: Remember } \sin 18^\circ = \frac{(\sqrt{5}-1)}{4}$$

17. (a) na^{n-1}

$$\text{Explanation: } \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$$

$$= \lim_{x \rightarrow a^+} \frac{x^n - a^n}{x - a} \quad [\because f(x) \text{ exists, } \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^+} f(x)]$$

$$= \lim_{h \rightarrow 0} \frac{(a+h)^n - a^n}{a+h-a}$$

$$= \lim_{h \rightarrow 0} a^n \frac{\left[\left(1+\frac{h}{a}\right)^n - 1\right]}{h}$$

$$= a^n \lim_{h \rightarrow 0} \left[1 + n \cdot \frac{h}{a} + \frac{n(n-1)}{2!} \frac{h^2}{a^2} + \dots + \dots - 1\right]$$

$$= a^n \lim_{h \rightarrow 0} \left[\frac{n}{a} + \frac{h(h-1)}{2!} \frac{h}{a^2} + \dots\right]$$

$$= a^n \frac{n}{a}$$

$$= na^{n-1}$$

18. (a) 31

Explanation: The candidate can fail by failing in 1 or 2 or 3 or 4 or 5 subjects out of 5 in each case.

$$\therefore \text{required number of ways} = {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$

$$= {}^5C_1 + {}^5C_2 + {}^5C_{(5-3)} + {}^5C_{(5-4)} + 1$$

$$= {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + 1$$

$$= 2({}^5C_1 + {}^5C_2) + 1$$

$$= 2\left(5 + \frac{5 \times 4}{2 \times 1}\right) + 1 = (30 + 1) = 31.$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion Since, every element of A is in B, so $A \subset B$.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

Section B

21. i. Given: $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x^2$

since the value of x is squared, $f(x)$ will always be equal or greater than 0

\therefore the range is $[0, \infty)$

ii. Given: $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $g(x) = x^2 + 1$

since, the value of x is squared and also adding with 1, $g(x)$ will always be equal or greater than 1

\therefore Range of $g(x) = [1, \infty)$

iii. Given: $h : \mathbb{R} \rightarrow \mathbb{R}$ such that $h(x) = \sin x$

We know that, $\sin(x)$ always lies between -1 to 1

\therefore Range of $h(x) = (-1, 1)$

OR

According to the question, $f(x) = 1 - |x - 2|$

We observe that $f(x)$ is defined for all $x \in \mathbb{R}$. Therefore, $\text{Domain}(f) = \mathbb{R}$

$\therefore 0 \leq |x-2| < \infty$ for all $x \in \mathbb{R}$

$\Rightarrow -\infty < -|x - 2| \leq 0$ for all $x \in \mathbb{R}$

$\Rightarrow -\infty < 1 - |x - 2| \leq 1$ for all $x \in \mathbb{R}$

$\Rightarrow -\infty < f(x) \leq 1$ for all $x \in \mathbb{R}$

$\therefore \text{Range}(f) = (-\infty, 1]$

22. To Find : $\frac{d}{dx}(3x^2 - x + 1)^4$

Formula used: $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let $y = (3x^2 - x + 1)^4$

So, by using the above formula, we have

$$\frac{d}{dx}(3x^2 - x + 1)^4 = 4(3x^2 - x + 1)^3 \times \frac{d}{dx}(3x^2 - x + 1) = 4(3x^2 - x + 1)^3(6x - 1)$$

Differentiation of $y = (3x^2 - x + 1)^4$ is $4(3x^2 - x + 1)^3(6x - 1)$

23. When a card is drawn from a well shuffled deck of 52 cards, the number of possible outcomes is 52.

Let C be the event 'card drawn is black card'.

Since total number of black cards = 26

$$\text{So, } P(C) = \frac{26}{52} = \frac{1}{2}$$

Thus, probability of a black card = $\frac{1}{2}$

OR

We have to find the probability that the integer is chosen is a multiple of 2 or 3 or 10

Out of 50 integers an integer can be chosen in ${}^{50}C_1$ ways.

Total number of elementary events = ${}^{50}C_1 = 50$

Consider the following events:

A = Getting a multiple of 2, B = Getting a multiple of 3 and, C = Getting a multiple of 10

Clearly, $A = \{2, 4, \dots, 50\}$, $B = \{3, 6, \dots, 48\}$, $C = \{10, 20, \dots, 50\}$

$A \cap B = \{6, 12, \dots, 48\}$, $B \cap C = \{30\}$, $A \cap C = \{10, 20, \dots, 50\}$ and, $A \cap B \cap C = \{30\}$

$$\therefore P(A) = \frac{25}{50}, P(B) = \frac{16}{50}, P(C) = \frac{5}{50}, P(A \cap B) = \frac{8}{50}, P(B \cap C) = \frac{1}{50}$$

$$P(A \cap C) = \frac{5}{50} \text{ and } P(A \cap B \cap C) = \frac{1}{50}$$

Required probability = $P(A \cap B \cap C)$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$= \frac{25}{50} + \frac{16}{50} + \frac{5}{50} - \frac{8}{50} - \frac{5}{50} - \frac{1}{50} + \frac{1}{50} = \frac{33}{50}$$

24. Suppose x be any element of $(A \cap B) \cap C$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } x \in (B \cap C)$$

$$\Rightarrow x \in A \cap (B \cap C)$$

$$\Rightarrow (A \cap B) \cap C \subset A \cap (B \cap C) \dots (i)$$

Now, suppose x be an element of $A \cap (B \cap C)$

Then, $x \in A$ and $(B \cap C)$

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in C$$

$$\Rightarrow x \in (A \cap B) \cap C$$

$$\Rightarrow A \cap (B \cap C) \subset (A \cap B) \cap C \dots (ii)$$

Using (i) and (ii), we have $(A \cap B) \cap C = A \cap (B \cap C)$

[every set is a subset of itself]

Hence, proved.

25. Therefore required equation of the line joining the points (6, 8) and (-3, -2) is

$$y - 8 = \frac{-2-8}{-3-6}(x - 6)$$

$$\Rightarrow 10x - 9y + 12 = 0$$

Suppose $10x - 9y + 12 = 0$ divide the line joining the points (2, 3) and (4, -5) at point P in the ratio k : 1

$$\therefore P \equiv \left(\frac{4k+2}{k+1}, \frac{-5k+3}{k+1} \right)$$

P lies on the line $10x - 9y + 12 = 0$

$$\therefore 10 \left(\frac{4k+2}{k+1} \right) - 9 \left(\frac{-5k+3}{k+1} \right) + 12 = 0$$

$$\Rightarrow 40k + 20 + 45k - 27 + 12k + 12 = 0$$

$$\Rightarrow 97k + 5 = 0$$

$$\Rightarrow k = \frac{-5}{97}$$

Therefore, the line joining the points (2, 3) and (4, -5) is divided by the line passing through the points (6, 8) and (-3, -2) in the ratio -5 : 97 externally.

Section C

26. Given the word SURITI.

Arranging the permutations of the letters of the word SURITI in a dictionary:

To find: Rank of word SURITI in dictionary.

First comes, words starting with letter I = $5! = 120$

words starting from letter R = $\frac{5!}{2!} = 60$

words starting from SI = $4! = 24$ (4 letters no repetition)

words starting from SR = $\frac{4!}{2!} = 12$ (4 letters, one repetition of I)

words starting from ST = $\frac{4!}{2!} = 12$ (4 letters, one repetition of I)

words starting from SUI = $3! = 6$ (3 letters no repetition)

words starting from SUR; SURIIT = 1

SURITI = 1

Rank of the word SURITI = $120 + 60 + 24 + 12 + 12 + 6 + 1 + 1$

= 236

27. Here P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c) are vertices of triangle PQR.

\therefore Coordinates of centroid of ΔPQR is $\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3} \right)$

$$= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3} \right)$$

But it is given that coordinates of centroid is (0, 0, 0)

$$\frac{2a+4}{3} = 0 \Rightarrow 2a + 4 = 0 \therefore a = -2$$

$$\frac{3b+16}{3} = 0 \Rightarrow 3b + 16 = 0 \Rightarrow b = \frac{-16}{3}$$

$$\frac{2c-4}{3} = 0 \Rightarrow 2c - 4 = 0 \Rightarrow c = 2$$

28. As discussed in the previous example, the middle term in the expansion of $(1+x)^{2n}$ is given by $T_{n+1} = {}^{2n}C_n x^n$

So, the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is ${}^{2n}C_n$.

Now, consider the expansion of $(1+x)^{2n-1}$ Here, the index (2n-1) is odd.

So, $\left(\frac{(2n-1)+1}{2} \right)^{th}$ and $\left(\frac{(2n-1)+1}{2} + 1 \right)^{th}$ i.e., n^{th} and $(n+1)^{th}$ terms are middle terms.

$$\text{Now, } T_n = T_{(n-1)+1} = {}^{2n-1}C_{n-1} (1)^{(2n-1)-(n-1)} x^{n-1} = {}^{2n-1}C_{n-1} x^{n-1}$$

$$\text{and, } T_{n+1} = {}^{2n-1}C_n (1)^{(2n-1)-n} x^n = {}^{2n-1}C_n x^n$$

So, the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$ are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$.

$$\therefore \text{Sum of these coefficients} = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n$$

$$= ({}^{2n-1}C_n)^{+1} C_n [{}^nC_{n-1} + {}^nC_n = {}^{n+1}C_n]$$

$$= {}^{2n}C_n$$

= Coefficient of middle term in the expansion of $(1+x)^{2n}$.

OR

$$\text{We have } T_1 = {}^nC_0 a^n b^0 = 729 \dots \text{(i)}$$

$$T_2 = {}^nC_1 a^{n-1} b = 7290 \dots \text{(ii)}$$

$$T_3 = {}^nC_2 a^{n-2} b^2 = 30375 \dots \text{(iii)}$$

$$\text{From (i) } a^n = 729 \dots \text{(iv)}$$

$$\text{From (ii) } n a^{n-1} b = 7290 \dots \text{(v)}$$

$$\text{From (iii) } \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \dots \text{(vi)}$$

Multiplying (iv) and (vi), we get

$$\frac{n(n-1)}{2} a^{2n-2} b^2 = 729 \times 30375 \dots \text{(vii)}$$

Squaring both sides of (v) we get

$$n^2 a^{2n-2} b^2 = (7290)(7290) \text{(viii)}$$

Dividing (vii) by (viii), we get

$$\frac{n(n-1)a^{2n-2}b^2}{2n^2a^{2n-2}b^2} = \frac{729 \times 30375}{7290 \times 7290}$$

$$\Rightarrow \frac{(n-1)}{2n} = \frac{30375}{72900} \Rightarrow \frac{n-1}{2n} = \frac{5}{12} \Rightarrow 12n - 12 = 10n$$

$$\Rightarrow 2n = 12 \Rightarrow n = 6$$

$$\text{From (iv) } a^6 = 729 \Rightarrow a^6 = (3)^6 \Rightarrow a = 3$$

$$\text{From (v) } 6 \times 3^5 \times b = 7290 \Rightarrow b = 5$$

Thus $a = 3$, $b = 5$ and $n = 6$.

29. We need to find derivative of $f(x) = \frac{x^2-1}{x}$

Derivative of a function $f(x)$ from first principle is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ \{where h is a very small positive number\}}$$

\therefore Derivative of $f(x) = \frac{x^2-1}{x}$ is given as

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h}$$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{hx(x+h)}$$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{h} \times \lim_{h \rightarrow 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} \frac{\{(x+h)^2-1\}x - (x+h)(x^2-1)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} \frac{\{x^2+h^2+2xh-1\}x - \{x^3+hx^2-x-h\}}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} \frac{x^3+h^2x+2x^2h-x-x^3-hx^2+x+h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} \frac{h^2x+x^2h+h}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} \frac{h(hx+x^2+1)}{h}$$

$$\Rightarrow f'(x) = \frac{1}{x^2} \lim_{h \rightarrow 0} (hx + x^2 + 1)$$

$$\Rightarrow f'(x) = \frac{1}{x^2} (0 \times x + x^2 + 1) = \frac{x^2+1}{x^2} = 1 + \frac{1}{x^2}$$

$$\therefore f'(x) = 1 + \frac{1}{x^2}$$

Hence,

$$\text{Derivative of } f(x) = \frac{x^2-1}{x^2} = 1 + \frac{1}{x^2}$$

OR

$$\begin{aligned}
 \text{Here } f(x) &= \frac{ax+b}{cx+d} \\
 \therefore f'(x) &= \frac{d}{dx} \left[\frac{ax+b}{cx+d} \right] \\
 &= \frac{(cx+d) \frac{d}{dx}(ax+b) - (ax+b) \frac{d}{dx}(cx+d)}{(cx+d)^2} \\
 &= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} \\
 &= \frac{acx+ad-acx-bc}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}
 \end{aligned}$$

30. Let first term be A and common ratio be R of a GP.

Given, p^{th} term, $T_p = q$ and q^{th} term, $T_q = p$

Then, $AR^{p-1} = q$ and $AR^{q-1} = p \dots(i)$

$$\therefore \frac{AR^{p-1}}{AR^{q-1}} = \frac{q}{p}$$

$$\Rightarrow R^{p-q} = \frac{q}{p} \Rightarrow R = \left(\frac{q}{p} \right)^{\frac{1}{p-q}} \quad [\because \text{raise both sides to power } \frac{1}{p-q}]$$

On putting the value of R in Eq. (i), we get

$$A \cdot \left(\frac{q}{p} \right)^{\frac{p-1}{p-q}} = q$$

$$\Rightarrow A = q \cdot \left(\frac{p}{q} \right)^{\frac{p-1}{p-q}}$$

Now, $(p+q)^{\text{th}}$ term,

$$\begin{aligned}
 T_{p+q} &= AR^{p+q-1} = q \cdot \left(\frac{p}{q} \right)^{\frac{p-1}{p-q}} \times \left(\frac{q}{p} \right)^{\frac{p+q-1}{p-q}} \\
 &= q^{\frac{1 - \frac{p-1}{p-q} + \frac{p+q-1}{p-q}}{p-q}} = q^{\frac{p-q-p+1+p+q-1}{p-q}} \\
 &= q^{\frac{p}{p-q}} = \left(\frac{q^p}{p^q} \right)^{\frac{1}{p-q}}
 \end{aligned}$$

Hence proved.

OR

$$S_2 = -4, a_5 = 4, a_3$$

$$\frac{a(1-r^2)}{1-r} = -4$$

$$a(1+r) = -4$$

$$ar^4 = 4ar^2$$

$$r = \pm 2$$

$$\text{when } r = 2$$

$$a = -4/3$$

$$\text{sequence is } -\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, \dots$$

$$\text{when } r = -2$$

$$a = 4$$

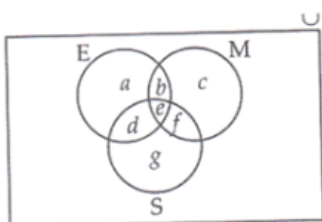
$$\text{sequence is}$$

$$4, 8, 16, 32, 64, \dots$$

31. Let the set of students who passed in Mathematics be M, the set of students who passed in English be E and the set of students who passed in Science be S.

$$\text{Then } n(U) = 100, n(M) = 12, n(E) = 15, n(S) = 8, n(E \cap M) = 6, n(M \cap S) = 7, n(E \cap S) = 4 \text{ and } n(E \cap M \cap S) = 4$$

Let us draw a Venn diagram



According to the Venn diagram,

$$n(E \cap S) = 4 \Rightarrow e = 4$$

$$n(E \cap M) = 6 \Rightarrow b + e = 6 \Rightarrow b + 4 = 6 \Rightarrow b = 2$$

$$n(M \cap S) = 7 \Rightarrow e + f = 7 \Rightarrow 4 + f = 7 \Rightarrow f = 3$$

$$n(E \cap S) = 4 \Rightarrow d + e = 4 \Rightarrow d + 4 = 4 \Rightarrow d = 0$$

$$n(E) = 15 \Rightarrow a + b + d + e = 15 \Rightarrow a + 2 + 0 + 4 = 15 \Rightarrow a = 9$$

$$n(M) = 12 \Rightarrow b + c + e + f = 12 \Rightarrow 2 + c + 4 + 3 = 12 \Rightarrow c = 3$$

$$n(S) = 8 \Rightarrow d + e + f + g = 8 \Rightarrow 0 + 4 + 3 + g = 8 \Rightarrow g = 1$$

Hence we get,

- i. Number of students who passed in English and Mathematics but not in Science, $b = 2$.
- ii. Number of students who passed in Mathematics and Science but not in English, $f = 3$.
- iii. Number of students who passed in Mathematics only, $c = 3$.
- iv. Number of students who passed in more than one subject $= b + e + d + f = 2 + 4 + 0 + 3 = 9$.

Section D

32. 1st of all we will prepare the below table with the help of given information.

Class Interval	f_i	Mid point x_i	$u_i = \frac{x_i - 53}{4}$	u_i^2	$f_i u_i$	$f_i u_i^2$
31-35	2	33	-5	25	-10	50
36-40	3	38	-3.75	14.06	-11.25	42.18
41-45	8	43	-2.5	6.25	-20	50
46-50	12	48	-1.25	1.56	-15	18.72
51-55	16	53	0	0	0	0
56-60	5	58	1.25	1.56	6.25	7.8
61-65	2	63	2.5	6.25	5	12.5
66-70	3	68	3.75	14.06	11.25	42.18
	$N = 51$					$\sum_{i=1}^n f_i u_i^2 = 223.38$

$$X = a + h \left(\frac{\sum_{i=1}^n f_i u_i}{N} \right)$$

$$= 53 + 4 \left(\frac{-33.75}{51} \right)$$

$$= 50.36$$

$$\sigma^2 = h^2 \left(\frac{\sum_{i=1}^n f_i u_i^2}{N} - \left(\frac{\sum_{i=1}^n f_i u_i}{N} \right)^2 \right)$$

$$= 16 \left(\frac{223.38}{51} - \frac{1139.06}{2601} \right)$$

$$= 63.07$$

$$\sigma = \sqrt{63.07}$$

$$= 7.94$$

f_i	Cumulative frequency
2	2
3	5
8	13
12	25
16	41
5	46
2	48
3	51

$$\sum f_i = 51 = N$$

$$\frac{N}{2} = 25.5$$

Median class interval is 51 - 55

$$L = 51$$

$$F = 25$$

$$f = 16$$

$$h = 4$$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 51 + \frac{25.5 - 25}{16} \times 4$$

$$= 51 + \frac{0.5}{4}$$

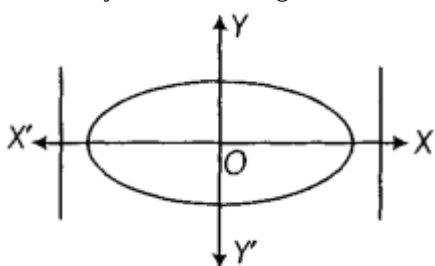
$$= 51.125$$

33. We have, equation of ellipse is $4x^2 + 9y^2 = 36$

$$\text{or } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Since, the denominator of $\frac{x^2}{9}$ is greater than denominator of $\frac{y^2}{4}$

So, the major axis lies along X-axis.



i. Shape is shown above.

ii. Major axis, $2a = 2 \times 3 = 6$

iii. Minor axis, $2b = 2 \times 2 = 4$

iv. Value of $c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$

v. Vertices = $(-a, 0)$ and $(a, 0)$ i.e., $(-3, 0)$ and $(3, 0)$

vi. Directrices, $x = \pm \frac{a^2}{c} = \pm \frac{9}{\sqrt{5}}$

vii. Foci = $(-c, 0)$ and $(c, 0)$ i.e., $(-\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$

viii. Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

ix. Length of latusrectum, $2l = \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

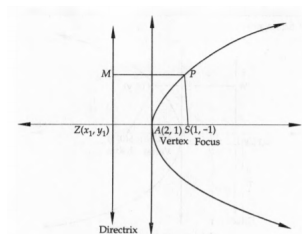
OR

Here, we are given the coordinates of the focus and vertex.

So, we require the equation of the directrix.

Let $Z(x_1, y_1)$ be the coordinates of the point of intersection of the axis and the directrix.

Then, the vertex $A(2, 1)$ is the mid-point of the line segment joining $Z(x_1, y_1)$ and the focus $S(1, -1)$



$$\therefore \frac{x_1 + 1}{2} = 2 \text{ and } \frac{y_1 + (-1)}{2} = 1 \Rightarrow x_1 = 3, y_1 = 3$$

Thus, the directrix meets the axis at $Z(3, 3)$

Let m_1 be the slope of the axis. Then,

$$m_1 = (\text{Slope of the line joining the focus } S \text{ and the vertex } A) = \frac{1+1}{2-1} = 2 \dots (i)$$

\therefore Slope of the directrix = $-\frac{1}{2}$ [\because Directrix is perpendicular to the axis]

Thus, the directrix passes through $(3, 3)$ and has slope $-1/2$.

So its equation is

$$y - 3 = -\frac{1}{2}(x - 3)$$

$$\text{or, } x + 2y - 9 = 0$$

Let P (x, y) be a point on the parabola.

Then,

Distance of P from the focus = Distance of P from the directrix

$$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \left| \frac{x+2y-9}{\sqrt{1^2+2^2}} \right|$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{(x+2y-9)^2}{5}$$

$$\Rightarrow 5x^2 + 5y^2 - 10x + 10y + 10 = x^2 + 4y^2 + 81 + 4xy - 18x - 36y$$

$$\Rightarrow 4x^2 + y^2 - 4xy + 8x + 46y - 71 = 0, \text{ which is the required equation of the parabola.}$$

The axis passes through the focus (1, -1), and its slope is $m_1 = 2$

Therefore, equation of the axis is $y + 1 = 2(x, -1)$ or, $2x - y - 3 = 0$

Now,

Latus-rectum = 2 (Length of the perpendicular from the focus on the directrix)

= 2 [Length of the perpendicular from (1, -1) on $x + 2y - 9 = 0$]

$$= 2 \left| \frac{1-2-9}{\sqrt{1+4}} \right| = 2 \times \frac{10}{\sqrt{5}} = 4\sqrt{5}$$

34. The given system of linear inequalities is

$$-2 - \frac{x}{4} \geq \frac{1+x}{3} \dots (i)$$

$$\text{and } 3 - x < 4(x - 3) \dots (ii)$$

From inequality (i), we get

$$-2 - \frac{x}{4} \geq \frac{1+x}{3}$$

$$\Rightarrow -24 - 3x \geq 4 + 4x \text{ [multiplying both sides by 12]}$$

$$\Rightarrow -24 - 3x - 4 \geq 4 + 4x - 4 \text{ [subtracting 4 from both sides]}$$

$$\Rightarrow -28 - 3x \geq 4x$$

$$\Rightarrow -28 - 3x + 3x \geq 4x + 3x \text{ [adding 3x on both sides]}$$

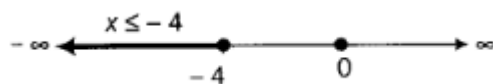
$$\Rightarrow -28 \geq 7x$$

$$\Rightarrow -\frac{28}{7} \geq \frac{7x}{7} \text{ [dividing both sides by 7]}$$

$$\Rightarrow -4 \geq x \text{ or } x \leq -4 \dots (iii)$$

Thus, any value of x less than or equal to -4 satisfied the inequality.

So, solution set is $x \in (-\infty, -4]$



From inequality (ii), we get

$$3 - x < 4(x - 3)$$

$$\Rightarrow 3 - x < 4x - 12$$

$$\Rightarrow 3 - x + 12 < 4x - 12 + 12 \text{ [adding 12 on both sides]}$$

$$\Rightarrow 15 - x < 4x$$

$$\Rightarrow 15 - x + x < 4x + x \text{ [adding x on both sides]}$$

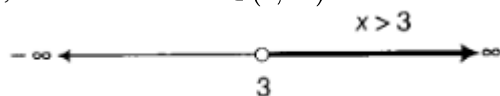
$$\Rightarrow 15 < 5x$$

$$\Rightarrow 3 < x \text{ [dividing both sides by 5]}$$

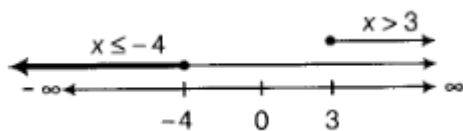
$$\text{or } x > 3 \dots (iv)$$

Thus, any value of x greater than 3 satisfies the inequality.

So, the solution set is $x \in (3, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



As no region is common, hence the given system has no solution.

$$35. \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$$

$$\text{LHS} = \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$$

$$= \cos 30^\circ \cos 10^\circ \cos 50^\circ \cos 70^\circ$$

$$= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ \cos 70^\circ)$$

$$= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ) \cos 70^\circ$$

$$= \frac{\sqrt{3}}{4} (2 \cos 10^\circ \cos 50^\circ) \cos 70^\circ \text{ [Multiplying and dividing by 2]}$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \{ \cos (50^\circ + 10^\circ) + \cos (10^\circ - 50^\circ) \} \text{ [Using } 2 \cos A \cos B = \cos (A + B) + \cos (A - B)]$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \{ \cos 60^\circ + \cos (-40^\circ) \}$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \left[\frac{1}{2} + \cos 40^\circ \right] \left[\because \cos 60^\circ = \frac{1}{2} \text{ and } \cos (-x) = \cos x \right]$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{4} \cos 70^\circ \cos 40^\circ$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} (2 \cos 70^\circ \cos 40^\circ)$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos (70^\circ + 40^\circ) + \cos (70^\circ - 40^\circ)]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos 110^\circ + \cos 30^\circ]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos (180^\circ - 70^\circ) + \frac{\sqrt{3}}{2}] \left[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \right]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2}] \left[\because \cos (180^\circ - x) = -\cos x \right]$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$$

$$= \text{RHS}$$

Hence proved.

OR

We have to prove that $\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$.

We know that,

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow 4\cos^3\theta = \cos 3\theta + 3\cos\theta$$

$$\Rightarrow \cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4} \dots (i)$$

And similarly

$$\Rightarrow \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\Rightarrow 4\sin^3\theta = 3\sin\theta - \sin 3\theta$$

$$\Rightarrow \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4} \dots (ii)$$

Now,

$$\text{LHS} = \cos^3 x \sin 3x + \sin^3 x \cos 3x$$

Substituting the values from equation (i) and (ii), we get

$$\Rightarrow \left(\frac{\cos 3x + 3\cos x}{4} \right) \sin 3x + \left(\frac{3\sin x - \sin 3x}{4} \right) \cos 3x$$

$$= \frac{1}{4} (\sin 3x \cos 3x + 3 \sin 3x \cos x + 3 \sin x \cos 3x - \sin 3x \cos 3x)$$

$$= \frac{1}{4} [3(\sin 3x \cos x + \sin x \cos 3x) + 0]$$

$$= \frac{1}{4} (3 \sin(3x + x))$$

$$(\text{as } \sin(x+y) = \sin x \cos y + \cos x \sin y)$$

$$\Rightarrow \frac{3}{4} \sin 4x$$

$$\text{LHS} = \text{RHS}$$

Hence Proved

Section E

36. i. $h(x) = [x]$ is the greatest integer function. Its range is \mathbb{Z} (set of integers)

ii. $f(x) = |x|$. The domain of $f(x)$ is \mathbb{R} .

iii. Since $10 > 0$, $f(10) = 1$.

OR

$g(x)$ is the signum function. Its range is $\{-1, 0, 1\}$.

37. i. Total number of possible outcomes = ${}^{52}C_4$

We know that there are 12 face cards

\therefore Number of favourable outcomes = ${}^{12}C_4$

\therefore Required probability = $\frac{{}^{12}C_4}{{}^{52}C_4}$

ii. Total number of possible outcomes = ${}^{52}C_4$

We know that there are 26 red and 26 black cards.

\therefore Number of favourable outcomes = ${}^{26}C_2 \times {}^{26}C_2$

\therefore Required probability = $\frac{({}^{26}C_2)^2}{{}^{52}C_4}$

iii. Total number of possible outcomes = ${}^{52}C_4$

\therefore Number of favourable outcomes = $({}^{13}C_1)^4$

\therefore Required probability = $\frac{(13)^4}{{}^{52}C_4}$

OR

Total number of possible outcomes = ${}^{52}C_4$

In playing cards there are 4 king and 4 jack cards.

\therefore Number of favourable outcomes = $({}^4C_2 \times {}^4C_2)$

$= 6 \times 6 = 36$

\therefore Required probability = $\frac{36}{{}^{52}C_4}$

38. i. $r = |Z| = 2\sqrt{2}$

$x = 2, y = -2$

$\cos\theta = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{y}{r} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\text{Arg}(Z) = \frac{-\pi}{4}$

ii. $z\bar{z} = |z|^2 = (2\sqrt{2})^2 = 8$

iii. $|Z| = \sqrt{2^2 + (-2)^2}$

$= \sqrt{8} = 2\sqrt{2}$

OR

Real part of $2 - 2i = 2$