Sample Question Paper - 18
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

OR

Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

2. How many two digits numbers are divisible by 3?

3. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

4. Draw a line segment of length 7.8 cm and divide it in the ratio 5 : 8. Measure the two parts.

5. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm. ?

6. The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

OR

The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then find the value of $(b - a)$.

Section B

7. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

8. Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct a pair of tangents to the circle.

9. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

10. Find the mean and median for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

OR

Prove that $\sum(x_i - \overline{x}) = 0$

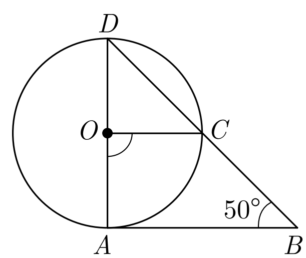
Section C

11. The two palm trees are of equal heights and are standing opposite each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees.

12. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10\,\text{cm}$, find the radius r of the circle.

OR

In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50^\circ$. Find $\angle AOC$.



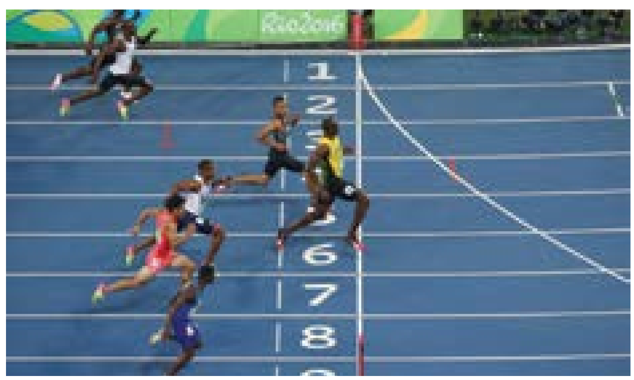
13. Computer Animations : The animation on a new computer game initially allows the hero of the game to jump a (screen) distance of 10 inch over booby traps and obstacles. Each successive jump is limited to $\frac{3}{4}$ inch less than the previous one.

(i) Find the length of the seventh jump

(ii) Find the total distance covered after seven jumps.

Figure is given on next Page.

14. 100 Metres Race : The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named "the fastest man or woman in the world".



A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

Based on the above information, answer the following questions.

(i) Estimate the mean time taken by a student to finish the race.

(ii) What will be the upper limit of the modal class ? What is the sum of lower limits of median class and modal class ?

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
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SECTION A

1. Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

Sol :

We have $kx^2 - 14x + 8 = 0$

Let one root be α and other root be 6α.

Sum of roots, $\alpha + 6\alpha = \frac{14}{k}$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots, $\alpha(6\alpha) = \frac{8}{k}$ or $6\alpha^2 = \frac{8}{k}$ ...(2)

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$

$$6 \times \frac{4}{k^2} = \frac{8}{k}$$

$$\frac{3}{k^2} = \frac{1}{k}$$

$$3k = k^2$$

$$3k - k^2 = 0$$

$$k[3 - k] = 0$$

$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

or

Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Sol :

We have $2x^2 - 4x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2$ $b = -4$, $c = 3$

Now $D = b^2 - 4ac$

$$= (-4)^2 - 4(2) \times (3)$$

$$= -8 < 0 \text{ or } (-\text{ve})$$

Hence, the given equation has no real roots.

2. How many two digits numbers are divisible by 3?

Sol :

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 96 and 99. Lowest two digit number divisible by 3 is 12 and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12, ends with 99 and common difference is 3.

So, the AP is 12, 15, 18, ..., 96, 99.

Here, $a = 12$, $d = 3$ and $a_n = 99$

$$a_n = a + (n-1)d$$

$$99 = 12 + (n-1)3$$

$$99 - 12 = 3(n-1)$$

$$n - 1 = \frac{87}{3} = 29 \Rightarrow n = 30$$

Therefore, there are 30, two digit numbers divisible by 3.

3. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Sol :

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$
$$a_{21} = 3 + (21-1) \times 12$$
$$= 3 + 20 \times 12 = 243$$

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore, $a_n = a + (n-1)d$

$$363 = 3 + (n-1)12$$
$$360 = 12(n-1)$$
$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

4. Draw a line segment of length 7.8 cm and divide it in the ratio 5 : 8. Measure the two parts.

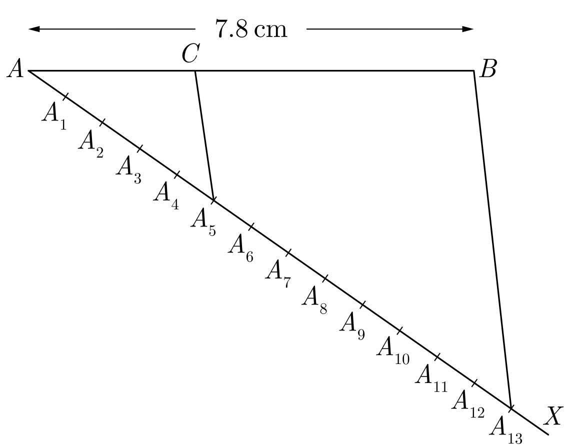
Sol :

Steps of Construction :

1. Draw a line segment AB of length 7.8 cm.
2. Draw any ray AX making an acute angle with AB.
3. Mark $5 + 8 = 13$ point $A_1, A_2, A_3, ..., A_{13}$ on AX such that $AA_1 = A_1A_2 = A_2A_3 = ..., A_{12}A_{13}$.
4. Join BA_{13}.
5. At point A_5, draw a line CA_5 parallel to BA_{13}. Hence $AC : CB = 5 : 8$

On measuring, we get

$$AC = 3\text{ cm and } CB = 4.8\text{ cm}$$



5. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

Sol :

Let t be the time in which the level of the water in the tank will rise by 21 cm.

Length of water that flows in 1 hour is 15 km or 15000 m.

Radius of pipe is $\frac{14}{2} = 7$ cm or 0.07 m.

Volume of water in 1 hour,

$$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000$$
$$= 231 \text{ m}^3$$

Volume of water in time t,

$$= 231t \text{ m}^3$$

This volume of water is equal to the water flowed into the cuboidal pond which is 50 m long, 44 m wide and 0.21 m high.

Thus $231t = 50 \times 44 \times 0.21$

$$t = \frac{50 \times 44 \times 0.21}{231} = 2 \text{ Hours}$$

6. The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

Sol :

Mode is 36 which lies in class 30-40, therefore this is model class.

Here, $f_0 = f$, $f_1 = 16$, $f_2 = 12$, $l = 30$ and $h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$36 = 30 + \frac{16 - f}{2 \times 16 - f - 12} \times 10$$
$$6 = \frac{16 - f}{20 - f} \times 10$$
$$120 - 6f = 160 - 10f$$
$$4f = 40 \Rightarrow f = 10$$

or

The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then find the value of $(b - a)$.

Sol :

Since, a, b and c and are in ascending order, therefore median is b i.e $b = 35$.

Mean $\frac{a+b+c}{3} = 50$

$$a + b + c = 150$$

$$a + c = 150 - 35 = 115 \quad (1)$$

Also, it is given that $c - a = 55$ ...(2)

Subtracting equation (2) and (1), we get

$$a = 30$$

Hence, $b - a = 35 - 30 = 5$

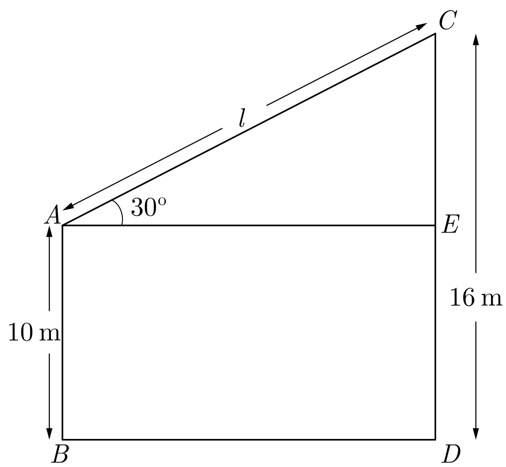
Section B

7. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

Sol :

Let AB and CD be two poles, where $AB = 10$ m, $CD = 16$ m.

As per given in question we have drawn figure below.



Length $CE = CD - CE = CD - AB$

$$= 16 - 10 = 6 \text{ m}.$$

From $\Delta AEC, \sin 30^\circ$

$$\frac{1}{2} = \frac{CE}{l}$$

$$l = 2CE$$

$$= 6 \times 2 = 12 \text{ m}.$$

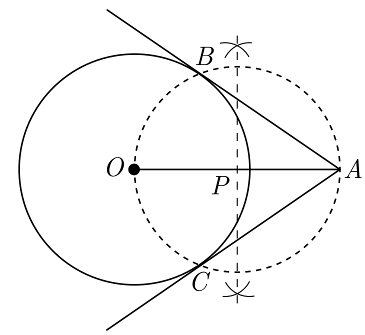
Hence, the value of l is 12 m.

8. Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct a pair of tangents to the circle.

Sol :

Steps of Construction :

1. Draw a line segment OA of length 8 cm.
1. Draw a circle with centre O and radius 5 cm.
3. Taking OA as diameter draw another circle which intersects the given circle at B and C.
4. Join A to B and A to C. Thus AB and AC are required tangents.
5. $AB = AC = 6.2$ cm.



AB and AC are required tangents.

$AB = AC = 6.2$ cm.

9. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

Sol :

Canal is the shape of cuboid where

$$\text{Breadth} = 6 \text{ m}$$

$$\text{Depth} = 1.5 \text{ m}$$

and speed of water $= 10$ km/hr

Length of water moved in 60 minutes i.e. 1 hour

$$= 10 \text{ km}$$

Length of water moved in 30 minutes i.e. $\frac{1}{2}$ hours,

$$= \frac{1}{2} \times 10 = 5 \text{ km} = 5000 \text{ m}$$

Now, volume of water moved from canal in 30 minutes

$$= \text{Length} \times \text{Breadth} \times \text{Depth}$$

$$= 5000 \times 6 \times 1.5 \text{ m}^3$$

Volume of flowing water in canal

$$= \text{volume of water in area irrigated}$$

$$5000 \times 6 \times 1.5 \text{m}^3 = \text{Area Irrigated} \times 8 \text{ cm}$$

$$5000 \times 6 \times 1.5 \text{m}^3 = \text{Area Irrigated} \times \frac{8}{100} \text{ m}$$

$$\text{Area Irrigated} = \frac{5000 \times 6 \times 1.5 \times 100}{8} \text{ m}^2$$

$= 5.625 \times 10^5 \text{ m}^2$

10. Find the mean and median for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

Sol :

We prepare following cumulative frequency table to find median class.

Class	x_i (class marks)	f_i	$f_i x_i$	*c.f.*
0-10	5	8	40	8
10-20	15	16	240	24
20-30	25	36	900	60
30-40	35	34	1190	94
40-50	45	6	270	100
Total		$\sum f_i = 100$	$\sum f_i x_i = 2640$	

Mean $\quad M = \frac{\sum f_i x_i}{\sum f_i} = \frac{2640}{100} = 26.4$

We have $\quad N = 100 \;;\; \frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 20-30. Thus median class is 20-30.

Now, $l = 20,\; f = 36,\; F = 24$ and $h = 10$

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 20 + \frac{50 - 24}{36} \times 10$$

$$= 20 + 7.22 = 27.22$$

or

Prove that $\sum(x_i - \overline{x}) = 0$

Sol :

We have $\quad \overline{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$

$$n\overline{x} = \sum_{i=1}^{n} x_i$$

Now, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 - \overline{x}) + (x_2 - \overline{x}) + \dots + (x_n - \overline{x})$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 + x_2 + \dots + x_n) - n\overline{x}$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = \sum_{i=1}^{n} x_i - n\overline{x} = \sum_{i=1}^{n}(x_i - \overline{x})$$

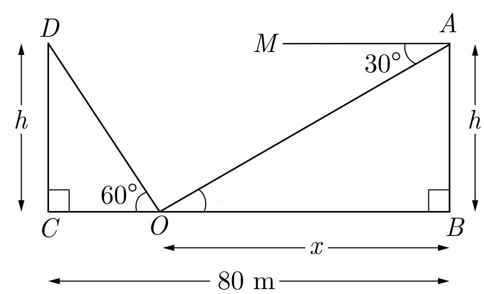
Hence, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = 0$

Section C

11. The two palm trees are of equal heights and are standing opposite each other on either side of the river, which is 80 m wide. From a point O between them on the river the angles of elevation of the top of the trees are 60° and 30°, respectively. Find the height of the trees and the distances of the point O from the trees.

Sol :

Let AB and CD be two palm trees, each of height h meters. Let the distance between palm tree AB and point O be x. As per given in question we have drawn figure below.



Here distance between palm tree CD and O is $80 - x$.

In right angle triangle ΔABO, $\angle AOB = 30°$

$$\tan 30° = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \quad \dots(1)$$

In angle triangle ΔCDO,

$$\tan 60° = \frac{CD}{CO} = \frac{CD}{CB - OB}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \quad \dots(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$$

Hence, height of the palm trees is 34.64 m

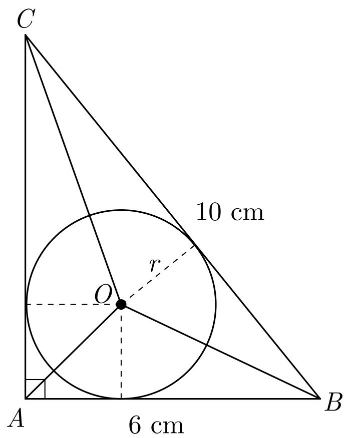
Distance of point O from tree AB is 20 m.

Distance of point O from tree CD is 60 m.

12. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10$ cm, find the radius r of the circle.

Sol :

As per question we draw figure shown below.



In triangle ΔABC,

$$AC = \sqrt{10^2 - 36^2} = 8 \text{ cm}$$

Area of triangle ΔABC,

$$\Delta ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Here we have joined AO, BO and CO.

For area of triangle we have

$$\Delta ABC = \Delta OBC + \Delta OCA + \Delta OAB$$

$$24 = \tfrac{1}{2}rBC + \tfrac{1}{2}rAC + \tfrac{1}{2}rAB$$

$$= \tfrac{1}{2}r(BC + AC + AB)$$

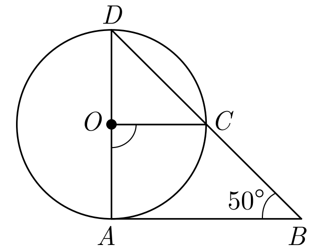
$$= \tfrac{1}{2}r(6 + 10 + 8) = 12r$$

or $\quad 12r = 24$

Thus $r = 2$ cm.

or

In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50°$. Find $\angle AOC$.



Sol :

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $\quad \angle A = 90°$

Now in ΔDAB we have

$$\angle D + \angle A + \angle B = 180°$$

$$\angle D + 90° + 50° = 180°$$

$$\angle D = 40°$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle AOC = 2\angle ADC = 2\angle D$$

$$= 2 \times 40° = 80°$$

13. Computer Animations : The animation on a new computer game initially allows the hero of the game to jump a (screen) distance of 10 inch over booby traps and obstacles. Each successive jump is limited to $\frac{3}{4}$ inch less than the previous one.

(i) Find the length of the seventh jump

(ii) Find the total distance covered after seven jumps.

Sol :

Successive jump of hero formed arithmetic sequence with $a_1 = 10$, and $d = -\frac{3}{4}$.

(i) Length of the seventh jump

$$a_n = a_1 + (n-1)d$$

$$a_7 = 10 + (7-1)\left(\frac{-3}{4}\right)$$

$$= 10 + 6\left(\frac{-3}{4}\right) = 10 - \frac{9}{2}$$

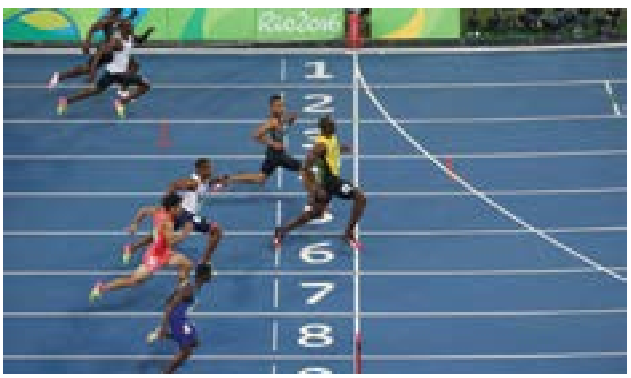
$$a_7 = 5.5 \text{ inches}$$

(ii) The total distance covered after seven jumps,

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{7(10+5.5)}{2}$$

$$= \frac{7(15.5)}{2} = 54.25 \text{ inches}$$

14. 100 Metres Race : The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named "the fastest man or woman in the world".



A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

Based on the above information, answer the following questions.

(i) Estimate the mean time taken by a student to finish the race.

(ii) What will be the upper limit of the modal class ? What is the sum of lower limits of median class and modal class ?

Sol :

(i) We prepare the following commutative frequency distribution table.

Time (in sec)	Number of students f_i	Mid-value x_i	$f_i x_i$	Cumulative frequency ***cf***
0-20	8	10	80	8
20-40	10	30	300	18
40-60	13	50	650	31
60-80	6	70	420	37
80-100	3	90	270	40
Total	$\sum f_i = 40$		$\sum f_i x_i = 1720$	

Mean $$M = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43$$

Mean time taken by a student to finish the race is 43 sec.

(ii) Since 40-60 has highest frequency i.e. 13 upper limit of modal class is 60.

Cumulative frequency just greater than $\frac{N}{2} = \frac{40}{2} = 20$ is 31 and the corresponding class is 40-60. Thus median class is 40-60 and lower limit is 40.

Sum of lower limits of median class and modal class $= 40 + 40 = 80$.