JEE MAIN 2024

Sample Paper - 4

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

- All questions are compulsory.
- There are three parts and each part carries 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer. However, there is no negative marking for Section-B (Numerical Questions)

PHYSICS (Section-A)

1. Correct one is given below: **[4]**

a) Both $\beta = -V\frac{dP}{dV}$ and $\gamma = \frac{MgL}{\pi r^2 l}$

b) $\gamma = \frac{MgL}{\pi r^2 l}$

c) $\beta = V\frac{dP}{dV}$

d) $\beta = -V\frac{dP}{dV}$

2. Which of the following is not an example of linear motion? **[4]**

a) A car at rest

b) A ball in uniform circular motion

c) A body rolling down an inclined plane

d) Wheel rotating at uniform speed on road

3. Which of the following statements is false for a particle moving in a circle with a constant angular speed? **[4]**

a) The velocity and acceleration vectors are perpendicular to each other.

b) The acceleration vector points to the centre of the circle.

c) The acceleration vector is tangent to the circle.

d) The velocity vector is tangent to the circle.

4. In a game of angry birds, the bluebird is projected with an angle 60°. with a velocity of 6 m/s. After reaching the highest point, the bird splits up into three birds of masses in ratio 2:1:1. Amongst the three birds, heaviest bird falls vertically downward with velocity 15.22 m/s and one bird travels straight. The velocity of the third bird will be: **[4]**

a) $9\hat{i} + 30.44\hat{j}$

b) $3\hat{i} - 6\hat{j}$

c) $33.44\hat{i} - 15.22\hat{j}$

d) $9(\hat{i} + 4\hat{j})$

5. A force applied by an engine of a train of mass 2.05×10^6 kg changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is: **[4]**

a) 1.025 MW

b) 6 MW

c) 5 MW

d) 2.05 MW

6. A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is: **[4]**

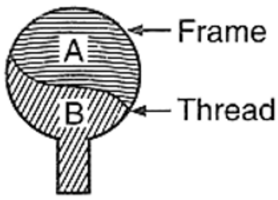
a) 2/7

b) 3/5

c) 3/7

d) 2/5

7. A thread is tied slightly loose to a wireframe as shown in the figure. And the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread: **[4]**



a) becomes concave towards A

b) either becomes convex towards A or becomes concave towards A depending on the size of A w.r.t. B

c) becomes convex towards A

d) remains in the initial position

8. A wire 3 m in length and 1mm in diameter at 30°C is kept in a low temperature at -170°C and is stretched by hanging a weight of 10 kg at one end. The change in length of the wire is: ($Y = 2 \times 10^{11}$ N/m^2, g = 10 m/s^2 and $\alpha = 1.2 \times 10^5/^oC$ **[4]**

a) 52 mm

b) 2.5 mm

c) 5.2 mm

d) 25 mm

9. Which of the following is not a thermodynamic co-ordinate? **[4]**

a) V

b) P

c) T

d) R

10. Total energy of a particle performing SHM depends on: **[4]**

a) amplitude and time period

b) amplitude and displacement

c) amplitude and time period and displacement

d) time period and displacement

[4]

11. An electron microscope is used to probe arrangements to a resolution of 5 $\mathring{A}$. What should be the electric potential to which the electrons need to be accelerated?

a) 5 kV

b) 2.5 V

c) 2.5 kV

d) 5.76 V

12. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to **[4]**

a) 10 V

b) 15 V

c) 20 V

d) 40 V

13. Which one of the following is correct? Wrist watches may be made anti-magnetic by shielding their machinery with: **[4]**

a) a magnetic substance of low permeability

b) a magnetic substance of high permeability

c) a metal of high conductivity

d) an insulator

14. In the inductive circuit given in the figure, the current rises after the switch are off. At the instant when the current is 15 mA, then the potential difference across the inductor will be: **[4]**

a) 60 V

b) 240 V

c) Zero

d) 180 V

15. The primary winding of a transformer has 100 turns and its secondary winding has 200 turns. The primary is connected to an AC supply of 120 V and the current flowing in it is 10 A. The voltage and the current in the secondary are: **[4]**

a) 240 V, 5 A

b) 120 V, 20 A

c) 60 V, 20 A

d) 240 V, 10 A

16. The intensity of X-rays depends upon the number of: **[4]**

a) Neutron

b) Protons

c) Positrons | d) Electrons

17. Monochromatic light incident on a metal surface emits electrons with kinetic energies from zero to 2.6 eV. What is the least energy of the incident photon if the tightly bound electron needs 4.2 eV to remove? **[4]**

a) From 1.6 eV to 6.8 eV
b) 1.6 eV
c) More than 6.8 eV
d) 6.8 eV

18. An electron of a hydrogen like atom, having Z = 4, jumps from 4^{th} energy state to 2^{nd} energy state. The energy released in this process, will be: **[4]**
(Given Rch = 13.6 eV)
Where R = Rydberg constant, c = Speed of light in vacuum, h = Planck's constant

a) 3.4 eV
b) 10.5 eV
c) 40.8 eV
d) 13.6 eV

19. The equivalent energy of 1 g of substance is: **[4]**

a) 6×10^{13} J
b) 3×10^{13} J
c) 9×10^{13} J
d) 6×10^{12} J

20. The truth table given in fig. represents **[4]**

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

a) NOR - Gate
b) AND - Gate
c) NAND - Gate
d) OR - Gate

PHYSICS (Section-B)

Attempt any 5 questions

21. Two identical cells each of emf 1.5V are connected in series across a 10 Ω resistance. An ideal voltmeter connected across 10 Ω resistance reads 1.5 V. The internal resistance of each cell is ________ Ω. **[4]**

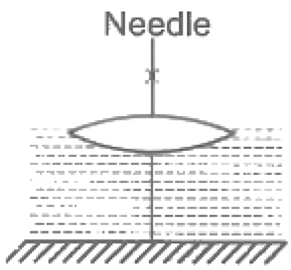
22. Two spherical conductors of radii r and 2r having surface charge densities $-\sigma$ and $+\sigma$ respectively are connected with each other. Final surface charge density of the smaller sphere is found to be K times that of σ. What is the value of K? **[4]**

23. A conducting circular loop is placed in X - Y plane in presence of magnetic field $\vec{B} = \left(3t^3\hat{j} + 3t^2\hat{k}\right)$ in SI unit. If the radius of the loop is 1 m, the induced emf in the loop, at time, t = 2s is $n\pi$V. The value of n is ________. **[4]**

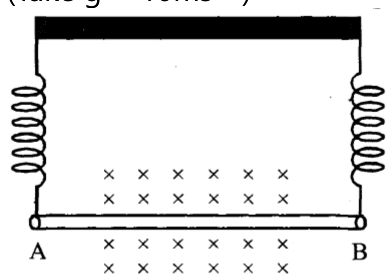
24. Planet A of mass M has radius R. Planet B of mass 4M has radius 4R. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_B}{v_A} = \frac{4}{n}$. The value of n is ________. **[4]**

25. A clock which keeps correct time at 20° C is subjected to 40° C. If the coefficient of linear expansion of the pendulum is 12×10^{-6} per ^{o}C, how much will it gain or lose in time in seconds/day? **[4]**

26. Figure shows an equi-convex lens of refractive index 1.5, is in contact with a liquid layer on the top of a plane mirror. A small needle is moved along the axis of lens until the inverted image of the needle is just coinciding with the needle. This distance is found to be 45 cm from mirror. Now, the liquid is removed and the needle is moved again in similar fashion. This time the corresponding distance is 30 cm. The refractive index of liquid is $\frac{4\alpha}{9}$. Determine the value of α. Assume layer of liquid to be thin. **[4]**



27. A straight wire AB of mass 40 g and length 50 cm is suspended by a pair of flexible leads in uniform magnetic field of magnitude 0.40 T as shown in the figure. The magnitude of the current required in the wire to remove the tension in the supporting leads is ________ A. (Take $g = 10ms^{-2}$) **[4]**



28. Two waves are simultaneously passing through a string and their equations are: $y_1 = A_1 \sin k(x - vt)$, $y_2 = A_2 \sin k(x - vt + x_0)$. Given amplitudes $A_1 = 12$ mm and $A_2 =$ **[4]**

5mm, x_0 = 3.5 cm and wave number k = 6.28cm^{-1}. The amplitude of resulting wave will be ________ mm.

29. One mole of an ideal gas is adiabatically compressed so that its temperature rises from 25^{o}C to 75^{o}C. The change in the internal energy of the gas is ________ J. (R = 8 J/mol.K, γ = 1.4) **[4]**

30. A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. Show that the fractional change in the radius of the sphere is (Mg/xAB) when a mass M is placed on the piston to compress the liquid. Find the value of x. **[4]**

CHEMISTRY (Section-A)

31. A 0.66 kg ball is moving with a speed of 100 m/s. The associated wavelength will be : **[4]**

a) 1.0×10^{-32} m
b) 1.0×10^{-35} m
c) 6.6×10^{-34} m
d) 6.6×10^{-32} m

32. Indicate the CORRECT decreasing order of 2nd ionization energies for Si, P, S and Cl from the options below. **[4]**

a) Cl > P > S > Si
b) Cl > S > P > Si
c) Si > Cl > S > P
d) Si > P > S > Cl

33. Select molecule in which following type of hybridization and overlapping is observed in molecule formation. **[4]**

a) BeF_2
b) OF_2
c) BeH_2
d) ICl_2^-

34. If 100 mole of H_2O_2 decomposes at 1 bar and 300 K, the work done (kJ) by one mole of O_2(g) as it expands against 1 bar pressure is: **[4]**
$2H_2O_2(l) \rightleftharpoons H_2O(l) + O_2(g)$
(R = 83 JK^{-1} mol^{-1})

a) 249.00
b) 62.25
c) 124.50
d) 498.00

35. For the reaction 2HI $\rightleftharpoons$ $H_2 + I_2$, if the standard free energy $\triangle G^o$ < 0, the equilibrium constant K_c would be ________. **[4]**

a) $K_c = 1$ b) $K_c = 0$

c) $K_c < 1$ d) $K_c > 1$

36. Which substance serves as a reducing agent in the following reaction, **[4]**
$14H^+ + Cr_2O_4^{2-} + 3Ni \longrightarrow 2Cr^{3+} + 7H_2O + 3Ni^{2+}$?

a) Ni b) $Cr_2O_7^{2-}$

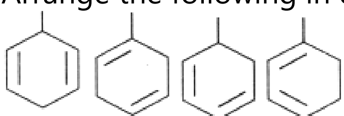
c) H_2O d) H^+

37. CS_2 reacts with Cl_2 to produce: **[4]**

a) CCl_4 and SCl_2 b) CCl_2 and SCl_4

c) CCl_4 and SCl_6 d) CCl_4 and S_2Cl_2

38. Arrange the following in decreasing order of heat of hydrogenation: **[4]**



a) S > R > Q > P b) P > Q > R > S

c) Q > P > S > R d) R > Q > P > S

39. $CH_3CH_3 + HNO_3 \xrightarrow{675K} ?$ **[4]**

a) $2CH_3NO_2$ b) $CH_2 = CH_2$

c) $CH_3CH_2NO_2$ d) $CH_3CH_2NO_2 + CH_3NO_2$

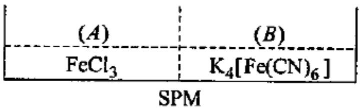
40. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is (R = 0.08206 L atm K^{-1} mol^{-1}) **[4]**

a) 2.46 atm b) 8.2 atm

c) 1.64 atm d) 4.92 atm

41. Two solutions (A) containing $FeCl_3$ (aq) and (B) containing $K_4[Fe(CN)_6]$ are separated by a semipermeable membrane as shown below. If $FeCl_3$ on reaction with $K_4[Fe(CN)_6]$, **[4]**

produces the blue color of $Fe_4[Fe(CN)_6]$, the blue color will be noticed in:



a) (B)

b) both (A) and (B)

c) neither in (A) nor in (B)

d) (A)

42. A 0.01 M KCl solution has specific conductance and resistance values of 0.00141 Ω^{-1} cm^{-1} and 4.2156 Ω, respectively. If the same cell is filled with 1 M HCl solution having a resistance of 1.0326 Ω, the specific conductance of HCl solution is _______. **[4]**

a) 0.00575 Ω^{-1} cm^{-1}

b) 0.00141 Ω^{-1} cm^{-1}

c) 0.00594 Ω^{-1} cm^{-1}

d) 0.00282 Ω^{-1} cm^{-1}

43. An increase in the concentration of the reactants of a reaction leads to change in **[4]**

a) threshold energy.

b) activation energy.

c) heat of reaction.

d) collision frequency.

44. Which of the following statement is correct for the complex $K_4[Fe(CN)_5O_2]$? [Fe having $t_{2g}^6 e_g 0$ configuration] **[4]**

a) d^2sp^3 and diamagnetism

b) sp^3d^2 and diamagnetism

c) d^2sp^3 and paramagnetism

d) sp^3d^2 and paramagnetism

45. Shielding constant o for Ne is 4.15. The effective nuclear charge on Na^+ and F^- are respectively: **[4]**

a) 6.85, 4.85

b) 5.85, 6.85

c) 4.85, 6.85

d) 4.85, 4.85

46. Which of the following has an unchanged oxidation number? **[4]**

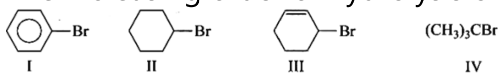
a) Fe

b) O

c) Na

d) P

47. The increasing order of hydrolysis of the following compounds is: **[4]**



a) I < II < IV < III
b) I < IV < II < III
c) I < II < III < IV
d) IV < III < II < I

a *b* **[4]**

48. Phenol → X → Toluene. Identify reagents **a** and **b**.

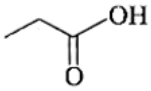
a) a = $CHCl_3$/NaOH
b = NaOH/H^+

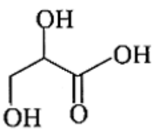
b) a = Zn dust, Δ
b = CH_3Cl, anhydrous $AlCl_3$

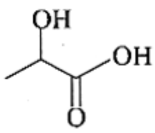
c) a = NaOH
b = CO_2/H^+

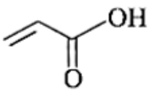
d) a = $Na_2Cr_2O_7$/H^+
b = Raney Ni

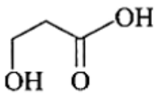
49. The end product of reaction is: **[4]**

 $\xrightarrow[H_2O]{P\ +\ Cl_2}$ $\xrightarrow[\Delta]{KOH\ (alc.)}$

a) 

b) 

c) 

d) 

50. Which statement is NOT correct for p-toluenesulphonyl chloride? **[4]**

a) On treatment with secondary amine, it leads to a product, that is soluble in alkali.
b) It doesn't react with tertiary amines.
c) It is used to distinguish primary and secondary amines.
d) It is known as Hinsberg's reagent.

CHEMISTRY (Section-B)

Attempt any 5 questions

51. An accelerated electron has speed of 5 × 10^6 ms^{-1} with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is x × 10^{-9} m. The value of x is ________. (Nearest integer) (Use mass of electron = 9.1 × 10^{-31} kg, h = 6.63 × 10^{-34}Js, π = 3.14) **[4]**

52. If the solubility product of AB_2, is 3.20 × 10^{-11} M^3, then the solubility of AB_2 in pure water is _______ × 10^{-4} mol L^{-1}. **[4]**
[Assuming that neither kind of ion reacts with water]

53. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _______. **[4]**

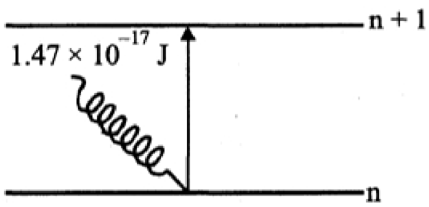
54. Find the total number of cations for which I.P. of cation is lower than corresponding atom. **[4]**
K^+, O^+, Ne^+, P^+, Be^+, Na^{2+}, Fe^+, Sn^{2+}

55. Calculate crystal field stabilization energy in kJ/mol of $[Ti(H_2O)_6]Cl_3$. Given that Δ_0 of above complex is 243 kJ/mol. **[4]**

56. Consider the following reactions: **[4]**
$NaCl + K_2Cr_2O_7 + H_2SO_4$ (Conc.) → (A) + Side products
(A) + NaOH → (B) + Side products
(B) + H_2SO_4 (dilute) + H_2O_2 → (C) + Side products
The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _______.

57. Two elements A and B which form 0.15 moles of A_2B and AB_3 type compounds. If both A_2B and AB_3 weigh equally, then the atomic weight of A is _______ times of atomic weight of B. **[4]**

58. The electron in the nth orbit of Li^{2+} is excited to (n + 1) orbit using the radiation of energy 1.47 × 10^{-17} J (as shown in the diagram). The value of n is _______. Given R_H = 2.18 × 10^{-18} J **[4]**



59. Find the number of σ-bonds in the nodal plane of π-bonds in $B_3N_3H_6$. **[4]**

60. A gas absorbs 0.2 kJ of heat and undergoes isothermal irreversible expansion against the external pressure of 2.0 atm from a volume of 0.5 L to 1.0 L. The change in internal energy of the system is _______ J. **[4]**

MATHEMATICS (Section-A)

61. The range of function f (x) = sgn (sin x) + sgn (cos x) + sgn (tan x) + sgn (cot x), $x \neq \frac{n\pi}{2} (n \in I)$ is : **[4]**

[Note: sgn k denotes signum function of k.]

a) {-2, 0, 4}
b) {-4, -2, 0, 4}

c) {-2, 4}

d) {0, 2, 4}

62. If z and w are two complex numbers such that |zw| = 1 and arg(z) - arg(w) = $\frac{\pi}{2}$, then **[4]**

a) $z\bar{w} = \frac{1-i}{\sqrt{2}}$

b) $\bar{z}w = i$

c) $\bar{z}w = -i$

d) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

63. The number of arrangements of all 52 cards in a deck such that the red and black cards are alternate, is **[4]**

a) $2(26!)^2$

b) $(26!)^2$

c) 2(52!)

d) $(52!)^2$

64. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}$: 1, then the third term from the beginning is: **[4]**

a) $30\sqrt{2}$

b) $60\sqrt{2}$

c) $30\sqrt{3}$

d) $60\sqrt{3}$

65. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{10^{99}} a_{2r} = 10^{100}$ and $\sum_{r=1}^{10^{99}} a_{2r-1} = 10^{99}$, then the common difference of A.P. is: **[4]**

a) 10^{99}

b) 9

c) 10 d) 1

66. Let a function f be given by **[4]**

f(x) = 1 + x, 0 $\le$ x $\le$ 2

= 3 - x, 2 < x $\le$ 3

The number of points at which fof is not continuous is

a) 0 b) 3

c) 1 d) 2

67. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = -1, 0, 1, then the set S = {x $\in$ R : f(x) = f(0)} contains exactly **[4]**

a) Four irrational numbers

b) Two irrational and two rational numbers

c) Four rational numbers

d) Two irrational and one rational number

68. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where [t] denotes the greatest integer less than or equal to t, is **[4]**

a) $\frac{3}{10}(4\pi - 3)$

b) $\frac{1}{12}(7\pi - 5)$

c) $\frac{3}{20}(4\pi - 3)$

d) $\frac{1}{12}(7\pi + 5)$

69. The x-intercept of angle bisector of angle between the lines 2x + y - 2 = 0 and 2x + 4y + 7 = 0 which contains the fixed point on the family of lines (2cosα + 3 sinα)x + (3 cosα - 5 sin α)y = 5 cosα - 2 sinα for different values of α, is equal to : **[4]**

a) $\frac{1}{2}$

b) $-\frac{1}{2}$

c) $\frac{11}{2}$

d) $-\frac{11}{2}$

70. Length of the intercept cut off by the circle $x^2 + y^2 - 5x - 4y - 6 = 0$ on x-axis is **[4]**

a) 7 b) 6

c) 8 d) 5

71. The focal distance of a point on the parabola $y^2 = 16x$ whose ordinate is twice the abscissa, is **[4]**

a) 10 b) 8

c) 12 d) 6

72. The general solution of the differential equation $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$ is: **[4]**

a) $\sin^{-1}y = x\sqrt{1 - x^2} + c$

b) $2\sin^{-1}y = x\sqrt{1 - x^2} + \sin^{-1}x + c$

c) $\cos^{-1}y = x\sqrt{1 - x^2} + c$

d) $2\cos^{-1}y = x\sqrt{1 - x^2} + \sin^{-1}x + c$

73. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ **[4]**

a) is parallel to z-axis b) lies in yz plane

c) lies in xy plane d) lies in zx plane

74. If the vector $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = 3\hat{i} - \hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\left|\vec{b}_1 \times \vec{b}_2\right|$ is equal to **[4]**

a) $\frac{5\sqrt{5}}{2}$

b) $\frac{\sqrt{115}}{4}$

c) $\sqrt{115}$

d) $\frac{\sqrt{115}}{2}$

75. Two teams A and B have the same mean and their coefficients of variation are 4, 2 respectively. If σ_A, σ_B are the standard deviations of teams A, B respectively then the relation between them is. **[4]**

a) $\sigma_B = 2\sigma_A$

b) $\sigma_A = 2\sigma_B$

c) $\sigma_B = 4\sigma_A$

d) $\sigma_A = \sigma_B$

76. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2 + 3p}{6}$, $P(E_2) = \frac{2 - p}{8}$ and $P(E_3) = \frac{1 - p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to: **[4]**

a) $\frac{5}{3}$

b) 1

c) $\frac{5}{4}$

d) $\frac{2}{3}$

77. If $\sin x + \sin y = \frac{1}{2}$ and $\cos x + \cos y = 1$, then $\tan (x + y) =$ **[4]**

a) $\frac{-8}{3}$

b) $\frac{4}{3}$

c) $-\frac{3}{4}$

d) $\frac{8}{3}$

78. The equation of the transverse and conjugate axes of a hyperbola are respectively x + 2y - 3 = 0, 2x - y + 4 = 0 and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is: **[4]**

a) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$

b) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

c) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$

d) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

79. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The values of m and n are **[4]**

a) 6, 4

b) 6, 3

c) 7, 4

d) 7, 6

80. For which of the following ordered pairs (μ, δ), the system of linear equations **[4]**

$x + 2y + 3z = 1$
$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent?

a) (4, 6)

b) (3, 4)

c) (4, 3)

d) (1, 0)

MATHEMATICS (Section-B)

Attempt any 5 questions

81. Let f(x) be a continuous and differentiable function satisfying the following conditions **[4]**

i. $\prod_{r=1}^{6} f(r) < 0, \prod_{r=1}^{3} f(2r) < 0, \prod_{r=1}^{2} f(3r) < 0$

ii. f(6) > 0, and

iii. f(x) is monotonic in (n, n + 1), n $\in$ I

Let A denotes the set consisting of number of distinct possible roots of f (x) = 0 in x $\in$ (1, 6). Find the sum of all the elements of set A.

82. If $\lim_{x \to 0} \left\{ \frac{1}{x^8} \left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is ________. **[4]**

83. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes $P_1 : \vec{r}(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and $P_2 : \vec{r}(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P is ________. **[4]**

84. Let for x $\in$ R $f(x) = \frac{x + |x|}{2}$ and $g(x) = \begin{cases} x, & x < 0 \\ x^2 & x \geq 0 \end{cases}$. **[4]**

Then area bounded by the curve y = (fog)(x) and the lines y = 0, 2y - x = 15 is equal to ________.

85. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2,3, - 4) and parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is ________ units. **[4]**

86. A die is tossed. If the die shows a 1 or a 2 then one coin is tossed. If the die shows a 3 then two coins are tossed. Otherwise, three coins are tossed. Given that the resulting coin tosses produced no heads. If the probability that the die showed a 1 or 2 can be expressed in lowest rational as $\left(\frac{m}{n}\right)$, find the value of (m + n). **[4]**

87. Let $A_1, A_2, A_3, \ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ________. **[4]**

88. Incircle of radius 4cm of a triangle ABC touches the side BC at D. If BD = 6 cm, DC = 8 cm and area of triangle ABC is k cm^2, then find characteristic of log k to the base 7. **[4]**

89. Let A = $\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and B = $\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$, $\alpha, \beta \in$ R. Let α_1 be the value of a which satisfies $(A + B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of a which satisfies $(A + B)^2 = B^2$. Then $|\alpha_1 - \alpha_2|$ is equal to _______. **[4]**

90. The number of relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _______. **[4]**

JEE MAIN 2024

Sample Paper - 4

Solution

PHYSICS (Section-A)

1. **(a)** Both $\beta = -V\frac{dP}{dV}$ and $\gamma = \frac{MgL}{\pi r^2 l}$

   Explanation: Both $\beta = -V\frac{dP}{dV}$ and $\gamma = \frac{MgL}{\pi r^2 l}$

2.

   (b) A ball in uniform circular motion

   Explanation: A body in uniform circular motion is moving in a plane and is a two-dimensional motion.

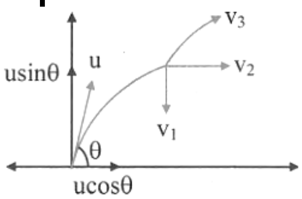
3.

   (c) The acceleration vector is tangent to the circle.

   Explanation: The acceleration vector acts along the radius of the circle. The given statement is false.

4. **(a)** $9\hat{i} + 30.44\hat{j}$

   Explanation:



By law of conservation of linear momentum

$m_1v_1 + m_2v_2 + m_3v_3 = MV$

Given: $m_1 : m_2 : m_3 = 2 : 1 : 1$

i.e., $M = m_1 + m_2 + m_3 = 4m$ kg

and $V = u\cos\theta = 6 \times \cos(60) = 3$ m/s

At highest point, let particle m_2 travel straight

$\Rightarrow \overrightarrow{v_2} = u \cos\theta\hat{i}$

$\therefore -2mv_1\hat{j} + mu\cos\theta\hat{i} + m\vec{v}_3 = 4mu\cos\theta\hat{i}$

i.e., $u\cos\theta\hat{i} - 2v_1\hat{j} + \vec{v}_3 = 4u\cos\theta\hat{i}$

$v_3 = 3(u\cos\theta)\hat{i} + 2v_1\hat{j}$

$= 3 \times 3\hat{i} + (2 \times 15.22)\hat{j}$

$= 9\hat{i} + 30.44\hat{j}$

5.

   (d) 2.05 MW

   Explanation: As we know that,

Power = $\frac{\text{Workdone}}{\text{time}}$

$= \frac{\frac{1}{2}m(v^2 - u^2)}{t}$

$P = \frac{1}{2} \times \frac{2.05 \times 10^6 \times \left[(25)^2 - (5)^2\right]}{5 \times 60}$

$P = 2.05 \times 10^6$ W

= 2.05 MW

6. **(a)** 2/7

   Explanation: Total energy,

$K = K_R + K_T = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mr^2\omega^2$
$= \frac{1}{5}mr^2\omega^2 + \frac{1}{2}mr^2\omega^2 = \frac{7}{10}mr^2\omega^2$
Now, rotational kinetic energy
$K_R = \frac{1}{2}I\omega^2 = \frac{1}{5}mr^2\omega^2$
$\therefore \quad \frac{K_R}{K} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$

7. **(a)** becomes concave towards A
Explanation: We know that when the portion A is punctured with a pin, then the film on the portion B is contracted to occupy minimum surface area due to surface tension. Therefore, the thread becomes concave towards A.

8.
(c) 5.2 mm
Explanation: The contraction in the length of the wire due to change in temperature
= αLT = (1.2 × 10^{-5}) × 3 × (-170 - 30)
= -7.2 × 10^{-3}m
The expansion in the length of wire due to stretching force
$= \frac{FL}{YA} = \frac{(10\times10)\times3}{(2\times10^{11})(0.75\times10^{-6})} = 2\times10^{-3}$ m
The resultant change in length
= -7.2 × 10^{-3} + 2 × 10^{-3}m
= -5.2 × 10^{-3}m = -5.2 mm
A negative sign shows a contraction.

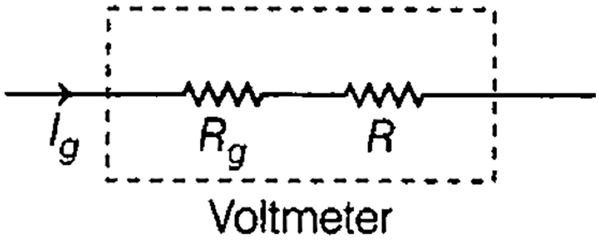
9.
(d) R
Explanation: We know that the physical quantities which are used to specify the state of a system, are called the thermodynamic coordinates, e.g., pressure (P), volume (V) and temperature (T). Therefore, R (gas constant) is not a thermodynamic co-ordinate.

10. **(a)** amplitude and time period
Explanation: amplitude and time period

11.
(d) 5.76 V
Explanation: We have, d sinφ = nλ
For $\phi = 90°$ and n = 1, we get d = λ
But $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{h^2}{2meV}}$
$= \sqrt{\left(\frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times V}\right)}$
$= \sqrt{\frac{1.5}{V}} \times 10^{-9}m$
$\therefore d = \sqrt{\frac{1.5}{V}} \times 10^{-9}$
or $5\times10^{-10} = \sqrt{\frac{1.5}{V}} \times 10^{-9}$ or $0.5 = \sqrt{\frac{1.5}{V}}$
or $0.5\times0.5 = \frac{1.5}{V} \Rightarrow V = \frac{1.5}{0.5\times0.5} = 6$ V
6 V $\approx$ 5.76 V

12.
(c) 20 V

Explanation:
Given, resistance of galvanometer, R_g = 50Ω
Current, I_g = 4mA = 4×10^{-3} A
Resistance used in converting a galvanometer in voltmeter,
R = 5 kΩ = 5×10^3Ω



∴ Maximum current in galvanometer is
$I_g = \frac{E}{R+R_g}$
$\therefore E = I_g(R+R_g)$
$= 4 \times 10^{-3} \times (5 \times 10^3 + 50)$
$= 5050 \times 4 \times 10^{-3}$
$= 20.2\text{V} \simeq 20\text{V}$

13.
(c) a metal of high conductivity
Explanation: a metal of high conductivity

14.
(d) 180 V
Explanation: At any instant
$V_R + V_L = 240$
$iR + V_L = 240$
$V_L = 240 - iR$
$= 240 - 15 \times 10^{-3} \times 400 = 180\text{V}$

15. **(a)** 240 V, 5 A
Explanation: We know that,
$\frac{N_s}{N_p} = \frac{V_s}{V_p}$
$\Rightarrow \frac{200}{100} = \frac{V_s}{120}$
$\Rightarrow V_s = \frac{200 \times 120}{100} = 240V$
Also,
$\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{240}{120} = \frac{10}{i_s}$
$i_s = \frac{10 \times 120}{240} = 5A$
Thus V_S = 240V, i_S = 5A

16.
(d) Electrons
Explanation: The intensity of X−rays depends upon the number of electrons striking the target.

17.
(d) 6.8 eV
Explanation: According to Einstein's equation, E = W_0 + KE
Given,
W_{0max} = 4.2 eV

KE = 2.6 eV

$E_{min} = W_{0max}$ = 4.2 eV + KE = (4.2 + 2.6) eV = 6.8 eV

18.

(c) 40.8 eV

Explanation: Energy released = $E_4 - E_2$

= $-13.6\left(\frac{1}{4^2} - \frac{1}{2^2}\right) \times z^2$ eV

= $-13.6\left(\frac{1}{16} - \frac{1}{4}\right) \times 16$ eV = $-13.6\left(-\frac{12}{64}\right) \times 16$ eV

= $13.6 \times \frac{3}{16} \times 16$ eV = 40.8 eV

19.

(c) 9×10^{13} J

Explanation: Using, $E = mc^2$

Here,

m = 1

g = 1×10^{-3} kg

c = 3×10^{8} ms^{-1}

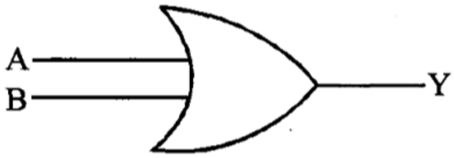
Then,

E = $10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$ J

20.

(d) OR - Gate

Explanation: it represents OR - Gate.



A	B	A + B = Y
0	0	0
0	1	1
1	0	1
1	1	1

PHYSICS (Section-B)

21. 5.0

Explanation:

Reading of voltmeter, V = 1.5 V

Net emf, E = 1.5 V + 1.5 V = 3 V

External resistance, R = 10 Ω

Let r be the internal resistance of each cell.

Using, V = IR

V = I × 10 ⇒ 1.5 = $\left(\frac{3}{10+2r}\right) \times 10$ ⇒ r = 5Ω

22. 1

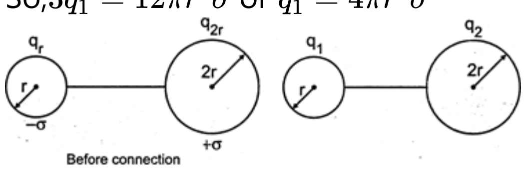
Explanation:

$q_r = -4\pi r^2\sigma, q_{2r} = 16\pi r^2\sigma, q_{\text{net}} = 12\pi r^2\sigma$

$\frac{q_1}{4\pi\varepsilon_0 r} = \frac{q_2}{4\pi\varepsilon_0 2r}$ or $q_1 = \frac{q_2}{2}$

and $q_1 + q_2 = q_{net} = 12\pi r^2\sigma$

So,$3q_1 = 12\pi r^2\sigma$ or $q_1 = 4\pi r^2\sigma$



So, surface charge density of smaller sphere after connection would be

$\sigma_1 = \frac{q_1}{4\pi r^2} = 1\sigma$

23. 12.0
Explanation:
Given,
Magnetic field, B $= \left(3t^3\hat{j} + 3t^2\hat{k}\right)$
Magnetic flux, ϕ = $\vec{B}.\vec{A}$
$= \left(3t^3\hat{j} + 3t^2\hat{k}\right) \cdot \left(\pi(1)^2\hat{k}\right)$ = $3t^2\pi$
Induced emf, $\varepsilon = \left|\frac{\mathrm{d}\phi}{\mathrm{dt}}\right| = \frac{\mathrm{d}\left(3\mathrm{t}^2\pi\right)}{\mathrm{dt}} = 6\mathrm{t}\pi$
$\therefore \varepsilon_t$ = 2 = 6 × 2 × π = 12π
24. 4
Explanation:
Escape velocity, v_e = $\sqrt{\frac{2GM}{R}}$
$\therefore \frac{v_B}{v_A} = \sqrt{\frac{2GM_p}{R_e}} \times \sqrt{\frac{R_A}{2GM_A}}$

= $\sqrt{\frac{2G(4M)}{(4R)}} \times \sqrt{\frac{R}{2GM}}$
= 1
$\therefore \frac{4}{n}$ = 1
$\therefore$ n = 4
25. 10.3
Explanation:
$\frac{\Delta t}{t} = \frac{1}{2}\frac{\Delta L}{L} = \frac{1}{2}\alpha\Delta\theta$
= $\frac{1}{2}$ × 12 × 10^{-6} × (40 - 20) = 12 × 10^{-5}
Δt = $t \times 12 \times 10^{-5}$ = 86400 × 12 × 10^{-5} = 10.3 sec/day.
26. 3
Explanation:
For object and image to coincide, the rays must be incident on the mirror normally, which would be the case when needle (object) is located at focal point (focus) of lens combination.
Case I: When liquid is present:
Let R be the radius of curvature of lens and jit be the refractive index of liquid. The liquid layer can be considered as a plane concave lens. It can be considered that these two lenses are placed in contact and equivalent focal length of this system is given by:
$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$
where $\frac{1}{f_1}$ = (1.5 - 1) $\times \frac{2}{R} = \frac{1}{R}$ and $\frac{1}{f_2} = (\mu - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = \frac{1-\mu}{R}$
$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{2-\mu}{R}$
For given condition, F = 45 cm = $\frac{R}{2-\mu}$
Case II: When liquid is removed:
f_1 = 30 cm, R = 30 cm
So, $\frac{R}{2-\mu}$ = 45 cm
or, 2 = 6 - 3μ or μ = $\frac{4}{3}$, so α = 3
27. 2.0
Explanation:

For equilibrium, force Mg = $I\ell B$

$\therefore I = \frac{mg}{\ell B} = \frac{40\times10^{-3}\times10}{50\times10^{-2}\times0.4}$ = 2 A

28. 7

Explanation:

$y_1 = A_1 \sin k(x - vt)$

y_1 = 12 sin 6.28 (x -vt)

y_2 = 5 sin 6.28 (x - vt + 3.5)

Phase difference, $\Delta\phi = \mathrm{k}(\Delta \mathrm{x}) = 6.28 \times 3.5 = 7\pi$

$\mathrm{A}^2 = \mathrm{A}_1^2 + \mathrm{A}_2^2 + 2\ \mathrm{A}_1\ \mathrm{A}_2 \cos\phi$

$\Rightarrow \mathrm{A} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$

$= \sqrt{144 + 25 - 120} = \sqrt{49}$ = 7 mm

29. 1000

Explanation:

For adiabatic change, change in internal energy of the gas,

$\Delta U = -\Delta W = \frac{R(T_2 - T_1)}{\gamma - 1}$

$= \frac{8}{(1.4-1)}$ (348 - 298)

= 20 $\times$ 50

= 1000 J

30. 3

Explanation:

As for a spherical body,

$V = \frac{4}{3}\pi R^3, \frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V}$ ....(i)

Now, by definition of bulk modulus,

$B = -V\frac{\Delta P}{\Delta V}$, i.e., $\left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} = \frac{Mg}{AB}\ \left[\because \Delta P = \frac{Mg}{A}\right]$

So, $\frac{\Delta R}{R} = \frac{1}{3}\left|\frac{\Delta V}{V}\right| = \frac{Mg}{3AB}$

Hence, x = 3

CHEMISTRY (Section-A)

31.

(b) 1.0×10^{-35} m

Explanation: $\lambda = \frac{h}{mu} = \frac{6.626\times10^{-34}}{0.66\times100} = 1\times10^{-35}\ \mathrm{m}$

32.

(b) Cl > S > P > Si

Explanation: For the given 3rd period of elements, the Ionization Enthalpy is expected to show an increasing trend from left to right in a period due to (a) decrease in the atomic size and (b) increase in the number of protons in the nucleus. The expected trend may be disturbed by S^{+1} as it exhibits a stable half-filled 3p-orbital from which it may be difficult to pull out an electron to form S^{+2} ion. However, the shielding effect and larger ionic radius of 3rd-period elements cancel the slight increase expected from the stable configuration of S^{+1}. Thus, the correct order is Cl > S > P > Si.

33. **(a)** BeF_2

Explanation: sp-hybrid orbital overlap with p-orbital so correct answer is BeF_2.

34.

(c) 124.50

Explanation: $2H_2O_2(l) \rightleftharpoons q + 2H_2O(l) + O_2(g)$
w = $-P_{ext}(\Delta V) = -n_{O_2}RT$
∵ 100 mole H_2O_2 on decomposition give 50 mole O_2
∴ w = -(50) (8.3) (300) = -124500 J = -124.5 kJ

35.
(d) $K_c > 1$

Explanation: $\triangle G^o$ and are related as

$\triangle G^o$ = -RT lnK_c

Given: $\triangle G^o < 0$
∴ K_c should be positive, i.e., $K_c > 1$

36. **(a)** Ni
Explanation: Ni

37.
(d) CCl_4 and S_2Cl_2
Explanation: CCl_4 and S_2Cl_2

38.
(b) P > Q > R > S
Explanation: S is most stabilise so, has minimum value.

39.
(d) $CH_3CH_2NO_2 + CH_3NO_2$
Explanation: $CH_3CH_2NO_2 + CH_3NO_2$

40.
(d) 4.92 atm
Explanation: For the relation, $\pi = CRT = \frac{n}{V}RT$
Given, mass of urea = 0.6 g
Molar mass of urea = 60 g mol^{-1}
Mass of glucose = 1.8 g
Molar mass of glucose = 180 g mol^{-1}

$$\pi = \frac{[n_2(\text{ urea }) + n_2(\text{ glucose })]}{V}RT$$
$$= \frac{\left(\frac{0.6}{60} + \frac{1.8}{180}\right)}{100} \times 1000 \times 0.0821 \times 300$$
$$= (0.01 + 0.01) \times 10 \times 0.0821 \times 300$$

π = 4.92 atm

41.
(c) neither in (A) nor in (B)
Explanation: The blue color is of the complex of Ferro ferricyanide and not of the solvent. But in osmosis, only solvent particles move.

42. **(a)** 0.00575 Ω^{-1} cm^{-1}
Explanation: Cell constant
= specific conductance × resistance
= 0.00141 × 4.2156 = 0.005944 cm^{-1}
The cell constant value remains same as the same cell is used.
∴ Specific conductance of HCl solution

$= \frac{cell\ constant}{resistance} = \frac{0.005944}{1.0326}$

$= 0.00575\ \Omega^{-1}\ cm^{-1}$

43.

(c) heat of reaction.

Explanation: Heat of reaction is an extensive property. Hence, on change of amount/concentration of reactants heat of reaction changes.

44.

(c) d^2sp^3 and paramagnetism

Explanation: d^2sp^3 and paramagnetism

45. **(a)** 6.85, 4.85

Explanation: 6.85, 4.85

46.

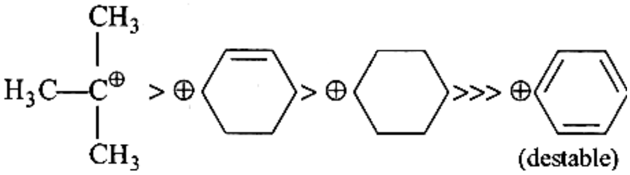
(c) Na

Explanation: Na has an unchanged oxidation number. Its oxidation number is +1 and is not variable. All alkali metals are always univalent.

47. **(a)** I < II < IV < III

Explanation:

Hydrolysis (S_N1) occur according to carbocations stability

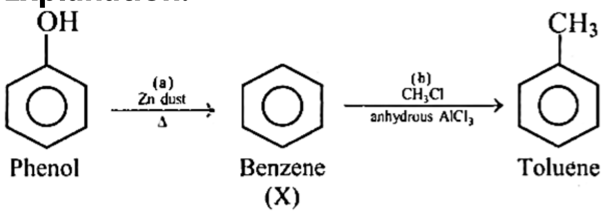


48.

(b) a = Zn dust, Δ

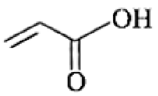
b = CH_3Cl, anhydrous $AlCl_3$

Explanation:

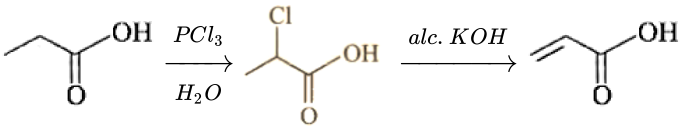


49.

(c)

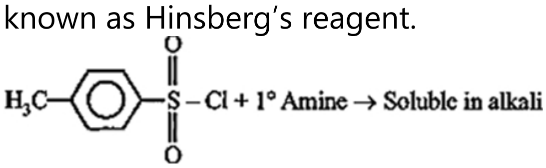


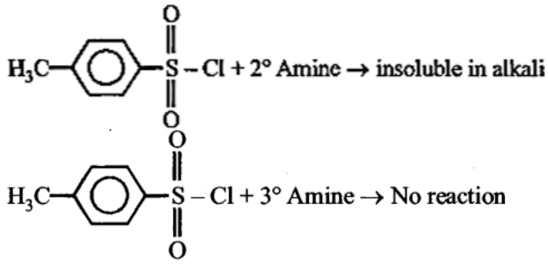
Explanation:



50. **(a)** On treatment with secondary amine, it leads to a product, that is soluble in alkali.

Explanation: p-toluenesulphonyl chloride is the Derivative of Benzenesulphonyl chloride also known as Hinsberg's reagent.





CHEMISTRY (Section-B)

51. 58

    Explanation:

    $\Delta x \cdot \Delta p = \frac{h}{4\pi}$

    $\Delta x \cdot m\Delta v = \frac{h}{4\pi}$

    $\Delta v = 5 \times 10^6 \times \frac{0.02}{100}$

    $\Delta v = 10^3$ m/s

    $\Delta x = \frac{h}{4\pi \times m \cdot \Delta v} = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 10^3}$

    $= 5.8 \times 10^{-8}$ m

    $= 58 \times 10^{-9}$ m

52. 2

    Explanation:

    $AB_2 \rightleftharpoons \underset{s}{A^{2+}}(aq) + \underset{2s}{2B^-}(aq)$

    $K_{sp} = 4s^3 = 3.2 \times 10^{-11}$

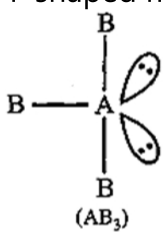
    $\Rightarrow s^3 = 8 \times 10^{-12}$

    $\Rightarrow s = 2 \times 10^{-4}$

53. 2

    Explanation:

    T-shaped molecule means central atom has 3 sigma bond and 2 lone pairs of electron.



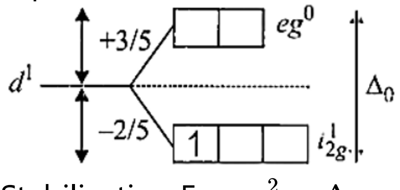
54. 0

    Explanation:

    All the cations have higher I.E. than the corresponding atom.

55. -97.20

    Explanation:



    Stabilization E = $-\frac{2}{5} \times \Delta_0$ = -97.20

56. 18

    Explanation:

$$4NaCl + K_2Cr_2O_4 + \underset{+2NaCO_4+3H_2O}{\underset{(Conc.)}{3H_2SO_4}} \rightarrow \underset{(A)}{2CrO_2Cl_2} + K_2SO_4$$

$$\underset{(A)}{CrO_2Cl_2} + NaOH \rightarrow \underset{(B)}{Na_2CrO_4} + H_2O + NaCl$$

$$\underset{(B)}{Na_2CrO_4} + \underset{(dilute)}{H_2SO_4} + H_2O_2 \rightarrow \underset{(C)}{CrO_5} + NaSO_4 + H_2O$$

The sum of total no. of atoms in one molecules each of A, B & C = 5 + 7 + 6 = 18

57. 2

Explanation:

Moles of A_2B= Moles of AB_3 = 0.15

$\frac{w}{2a+b} = \frac{w}{a+3b} \Rightarrow 2a + b = a + 3b \Rightarrow a = 2b$

58. 1

Explanation:

$\Delta E = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$1.47 \times 10^{-17} = 2.18 \times 10^{-18} \times 9\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$

$\frac{1.47}{1.96} = \frac{3}{4} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

So, n = 1

59. 3

Explanation:

3

60. 98.7

Explanation:

For isothermal irreversible expansion,

$w = -p_{ex} \times \Delta V = -2.0 \times (1.0 - 0.5)$

= -1.0 L. atm

= -1.0×101.3 = -101.3 J

For first law of thermodynamics,

$\Delta U = q + w$

= 200 - 101.3 = +98.7 J

MATHEMATICS (Section-A)

61. **(a)** {-2, 0, 4}

Explanation: $f(x) = \begin{cases} 4; 0 < x < \frac{\pi}{2} \\ -2; \frac{\pi}{2} < x < \pi \\ 0; \pi < x < \frac{3\pi}{2} \\ -2; \frac{3\pi}{2} < x < 2\pi \end{cases}$

$\therefore$ Range of f(x) = {-2, 0, 4}

62.

(c) $\bar{z}w = -i$

Explanation: It is given that, there are two complex numbers z and w, such that | z w | = 1 and arg(z) - arg(w) = $\pi/2$

$\therefore$ |z| |w| = 1 [$\because$ $|z_1 z_2| = |z_1| |z_2|$] and arg(z) = $\frac{\pi}{2}$ + arg(w)

Let |z| = r, then |w| = $\frac{1}{r}$ ...(i)

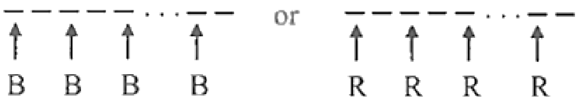
and let arg(w) = θ, then arg(z) = $\frac{\pi}{2} + \theta$ ...(ii)

So, we can assume z = $re^{i(\pi/2 + \theta)}$

[$\because$ if z = x + ry is a complex number, then it can be written as z = re$^{i\theta}$ where, r = |z| and θ = arg (z)]
and $w = \frac{1}{r}e^{i\theta}$ ...(iv)
Now, $\bar{z} \cdot w = re^{-i(\pi/2+\theta)} \cdot \frac{1}{r}e^{i\theta}$
$= e^{i(-\pi/2-\theta+\theta)} = e^{-i(\pi/2)} = -i$ [$\because$ e$^{i\theta}$ = cosθ - i sinθ]
and $z\bar{w} = re^{i(\pi/2+\theta)} \cdot \frac{1}{r}e^{-i\theta}$
$= e^{i(\pi/2+\theta-\theta)} = e^{i(\pi/2)} = i$

63. **(a)** 2(26!)^2

Explanation:



Black cards in 26 places are arranged in 26! ways and the red cards in 26 gaps are arranged in 26! ways.
The deck may start with a black or a red card.
Required number = 2(26!) (26!)
= 2(26!)^2

64.

(d) $60\sqrt{3}$

Explanation: $\frac{\text{Fifth term from begining}}{\text{Fifth term from end}}$

$= \frac{{}^nC_4 2^{\frac{n-4}{4}} \cdot \left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4 3^{-\left(\frac{n-4}{4}\right)} \cdot \left(2^{\frac{1}{4}}\right)^4} = \frac{\sqrt{6}}{1} \Rightarrow$ n = 10

So $T_3 = T_{2+1}$ = ${}^{10}C_2 \left(2^{\frac{1}{4}}\right)^8 \cdot \left(3^{-\frac{1}{4}}\right)^2 = \frac{45.4}{\sqrt{3}} = 60\sqrt{3}$

65.

(b) 9

Explanation: $\sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} (a_{2r-1} + d) = \sum_{r=1}^{10^{99}} a_{2r-1} + (10^{99})\, d$

$\Rightarrow 10^{100} = 10^{99} + 10^{99} d \Rightarrow 10^{99} (10 - 1) = 10^{99} d$
$\Rightarrow$ d = 9

66.

(d) 2

Explanation: (fof) (x) = f(1 + x), $0 \leq x \leq 2$
= f(3 - x), 2 < x $\leq$ 3
To get f(1 + x), when 0 $\leq$ x $\leq$ 2 ($\Leftrightarrow$ 1 $\leq$ 1 + x $\leq$ 3)
Let 1 + x = t, then we find f(t) for 1 $\leq$ t $\leq$ 3.
f(t) = 1 + t, 1 $\leq$ t $\leq$ 2
= 3 - t, 2 < t $\leq$ 3
$\Leftrightarrow$ f(1 + x) = 2 + x, 1 $\leq$ 1 + x $\leq$ 2
= 2 - x, 2 < 1 + x $\leq$ 3
$\Leftrightarrow$ f(1 + x) = 2 + x, 0 $\leq$ x $\leq$ 1
= 2 - x, 1 < x $\leq$ 2 ...(i)
To get f(3 - x), 2 < x $\leq$ 3 ($\Leftrightarrow$ 0 $\leq$ 3 - x < 1)
Let 3 - x = u, then we find f(u) for 0 $\leq$ u < 1.
f(u) = 1 + u, 0 $\leq$ u < 1

⇔ f(3 - x) = 1 + 3 - x, 2 < x ≤ 3
⇔ f(3 - x) = 4 - x, 2 < x ≤ 3 ...(ii)
From (i) and (ii), we get
(fof)(x) = 2 + x , 0 ≤ x ≤ 1
= 2 - x, 1 < x ≤ 2
= 4 - x, 2 < x ≤ 3
fof is continuous everywhere except possibly at x = 1 and x = 2

$\left.\begin{array}{l}\lim\limits_{x \rightarrow 1^{-}} fof(x)=\lim\limits_{x \rightarrow 1^{-}}(2+x)=3 \\ \lim\limits_{x \rightarrow 1^{+}} fof(x)=\lim\limits_{x \rightarrow 1^{+}}(2-x)=1\end{array}\right\}$ ⇒ fof is not continuous at x = 1

$\left.\begin{array}{l}\lim\limits_{x \rightarrow 2^{-}} f^{\circ} f(x)=\lim\limits_{x \rightarrow 2^{-}}(2-x)=0 \\ \lim\limits_{x \rightarrow 2^{+}} f^{\circ} f(x)=\lim\limits_{x \rightarrow 2^{+}}(4-x)=2\end{array}\right\}$ ⇒ fof is not continuous at x = 2

67.
(d) Two irrational and one rational number
Explanation: The non-zero four degree polynomial f(x) has extremum points at x = -1, 0, 1, so we can assume f'(x) = a(x + 1)(x - 0)(x - 1) = ax(x^2 - 1)
Where, a is non-zero constant.
f'(x) = ax^3 - ax
$\Rightarrow \quad f(x)=\frac{a}{4} x^{4}-\frac{a}{2} x^{2}+C$ [integrating both sides]
where, C is constant of integration.
Now, since f(x) = f(0)
$\Rightarrow \frac{a}{4} x^{4}-\frac{a}{2} x^{2}+C=C \Rightarrow \frac{x^{4}}{4}=\frac{x^{2}}{2}$
$\Rightarrow \quad x^{2}\left(x^{2}-2\right)=0 \Rightarrow x=-\sqrt{2}, 0, \sqrt{2}$
Thus, f(x) = f(0) has one rational and two irrational roots.

68.
(c) $\frac{3}{20}(4 \pi-3)$
Explanation: Let $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{d x}{[x]+[\sin x]+4}$
$=\int_{-\frac{\pi}{2}}^{-1} \frac{d x}{[x]+[\sin x]+4}+\int_{-1}^{0} \frac{d x}{[x]+[\sin x]+4}+\int_{0}^{1} \frac{d x}{[x]+[\sin x]+4}+\int_{1}^{\frac{\pi}{2}} \frac{d x}{[x]+[\sin x]+4}$
$\because[x]=\left\{\begin{array}{ll}-2, & \pi / 2<x<-1 \\ -1, & -1 \leq x<0 \\ 0 & 0 \leq x<1 \\ 1 & 1 \leq x<\pi / 2\end{array}\right.$
and $[\sin x]=\left\{\begin{array}{ll}-1, & -\pi / 2<x<-1 \\ -1, & -1<x<0 \\ 0, & 0<x<1 \\ 0, & 1<x<\pi / 2\end{array}\right.$
[∵ For x < 0, -1 ≤ sin x < 0 and for x > 0, 0 < sin x ≤ 1]
So, $I=\int_{-\frac{\pi}{2}}^{-1} \frac{d x}{-2-1+4}+\int_{-1}^{0} \frac{d x}{-1-1+4}+\int_{0}^{1} \frac{d x}{0+0+4}+\int_{1}^{\frac{\pi}{2}} \frac{d x}{1+0+4}$
$=\int_{-\frac{\pi}{2}}^{-1} \frac{d x}{1}+\int_{-1}^{0} \frac{d x}{2}+\int_{0}^{1} \frac{d x}{4}+\int_{1}^{\frac{\pi}{2}} \frac{d x}{5}$
$=\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}(1-0)+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$
$=\left(-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{\pi}{2}+\frac{\pi}{10}\right)$

$= \frac{-20+10+5-4}{20} + \frac{5\pi+\pi}{10}$
$= -\frac{9}{20} + \frac{3\pi}{5} = \frac{3}{20}(4\pi - 3)$

69.

(c) $\frac{11}{2}$

Explanation: $\frac{11}{2}$

70. **(a)** 7

Explanation: Length of intercept on x-axis
$= 2\sqrt{\left(\frac{-5}{2}\right)^2 - (-6)}$
$= 2\left(\frac{7}{2}\right)$ = 7

71.

(b) 8

Explanation: Let point be (h, k). But 2h = k, then k^2 = 16h ⇒ $4h^2$ = 16h ⇒ h = 0, h = 4 ⇒ k = 0, k = 8
Points are (0, 0), (4, 8). Hence focal distances are respectively 0 + a = 4, 4 + 4 = 8. (∵ a = 4)

72.

(b) 2 $\sin^{-1}$y = x $\sqrt{1-x^2}$ + $\sin^{-1}$ x + c
Explanation: $\frac{dy}{dx} = \sqrt{1-x^2-y^2+x^2y^2}$
$\Rightarrow \frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$
$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}$ dx (Variables separable)
Integrating on both sides, we get
$\sin^{-1}$ y = $\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}$ $\sin^{-1}$ x + c_1
⇒ 2 $\sin^{-1}$y = x $\sqrt{1-x^2}$ + $\sin^{-1}$ x + c, where c = $2c_1$

73.

(c) lies in xy plane
Explanation: The direction ratios of the given line are 3, 1, 0.
But line makes angles α, β, γ with the coordinate axes.
⇒ n = cosγ
⇒ cos γ = 0
⇒ γ = 90^o
⇒ The given line is perpendicular to the z-axis.
⇒ The given line lies in xy plane.

74.

(d) $\frac{\sqrt{115}}{2}$

Explanation: $\vec{b}_1$ is parallel to $\vec{a}$.
$\Rightarrow \vec{b}_1 = \lambda\vec{a}, \lambda \neq 0$
$\Leftrightarrow \vec{b}_1 = \lambda(3\hat{i} - \hat{j})$
Given, $\overrightarrow{b_2} = \vec{b} - \vec{b}_1$
$= (2\hat{i} + \hat{j} - 3\hat{k}) - \lambda(3\hat{i} - \hat{j})$
$= (2-3\lambda)\hat{i} + (1+\lambda)\hat{j} - 3\hat{k}$
Also, $\vec{a} \cdot \vec{b}_2 = 0$ ...[∵ $\vec{b}_2$ is perpendicular to $\vec{a}$]
$\Leftrightarrow 3(2-3\lambda) + (-1)(1+\lambda) + 0(-3) = 0$

$\Leftrightarrow 6-9\lambda-1-\lambda=0$

$\Leftrightarrow \lambda=\frac{1}{2}$

$\Rightarrow \vec{b}_1=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}$ and $\overrightarrow{b_2}=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

$$\vec{b}_1\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{3}{2} & \frac{-1}{2} & 0\\ \frac{1}{2} & \frac{3}{2} & -3\end{vmatrix}$$

$=\hat{k}\left(\frac{3}{2}\times\frac{3}{2}+\frac{1}{2}\times\frac{1}{2}\right)-3\left(-\frac{1}{2}\hat{i}-\frac{3}{2}\hat{j}\right)$

$=\frac{5}{2}\hat{k}+\frac{3}{2}\hat{i}+\frac{9}{2}\hat{j}$

$\Rightarrow\left|\vec{b}_1\times\overrightarrow{b_2}\right|=\sqrt{\left(\frac{5}{2}\right)^2+\left(\frac{3}{2}\right)^2+\left(\frac{9}{2}\right)^2}$

$=\sqrt{\frac{25+9+81}{4}}=\frac{\sqrt{115}}{2}$

75.

(b) $\sigma_{\mathrm{A}}=2\sigma_{\mathrm{B}}$

Explanation: C.V. of A = $\frac{\sigma_A}{\bar{x}}\times$ 100

$\therefore$ 4 = $\frac{\sigma_{\mathrm{A}}}{\bar{x}}\times$ 100

$\Rightarrow\sigma_{\mathrm{A}}=\frac{4\bar{x}}{100}$ ...(i)

and C.V. of B = $\frac{\sigma_{\mathrm{B}}}{\bar{x}}\times$ 100

$\therefore$ 2 = $\frac{\sigma_{\mathrm{B}}}{x}\times$ 100

$\Rightarrow\sigma_{\mathrm{B}}=\frac{2\bar{x}}{100}$ ...(ii)

From (i) and (ii),

$\sigma_{\mathrm{A}}=2\sigma_{\mathrm{B}}$

76. **(a)** $\frac{5}{3}$

Explanation: Given

P(E_1) = $\frac{2+3P}{6}$, P(E_2) = $\frac{2-P}{8}$ & P(E_3) = $\frac{1-P}{2}$.

According to question,

P(E_1) + P(E_2) + P(E_3) $\leq$ 1

$\frac{2+3P}{6}+\frac{2-P}{8}+\frac{1-P}{2}\leq 1$

26 - 3P $\leq$ 24 $\Rightarrow$ 2 $\leq$ 3P $\Rightarrow$ P $\geq\frac{2}{3}$

So, $\frac{2}{3}\leq$ P $\leq$ 1. Then, P_1 = 1 and P_2 = $\frac{2}{3}$

P_1 + P_2 = $\frac{5}{3}$

77.

(b) $\frac{4}{3}$

Explanation: sin x + sin y = $\frac{1}{2}$

$\Rightarrow$ 2 sin $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)=\frac{1}{2}$ ...(i)

cos x + cos y = 1

$\Rightarrow$ 2 sin $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$ = 1 ...(ii)

Dividing (i) by (ii), we get

tan $\left(\frac{x+y}{2}\right)=\frac{1}{2}$

Now, tan (x + y) = $\frac{2\tan\left(\frac{x+y}{2}\right)}{1-\tan^2\left(\frac{x+y}{2}\right)}$

= $\frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{4}{3}$

78.

(d) $\frac{2}{5}$(2x - y + 4)^2 - $\frac{3}{5}$(x + 2y - 3)^2 = 1

Explanation: Given, 2a = $\sqrt{2}$

$\Rightarrow$ a = $\frac{1}{\sqrt{2}}$

Also, 2b = $\frac{2}{\sqrt{3}}$

$\Rightarrow$ b = $\frac{1}{\sqrt{3}}$

If we take the two axes as the new coordinate system, and the point of intersection of the axes as the new origin, then in the new coordinate system, equation of the hyperbola will be:

$\frac{\mathbf{X}^2}{\mathbf{a}^2} - \frac{\mathbf{Y}^2}{\mathbf{b}^2}$ = 1

$\Rightarrow$ 2X^2 - 3Y^2 = 1

Let P(x, y) be the coordinates of a point on the hyperbola in original x-y system, then

X = $\frac{|2x-y+4|}{\sqrt{5}}$, Y = $\frac{|\mathbf{x+2y-3}|}{\sqrt{\mathbf{5}}}$ ($\because$ X is the distance of a point on hyperbola from 2x - y + 4 = 0 and Y is the distance of a point on hyperbola from x + 2y - 3 = 0)

So, the required equation is

$\frac{2(2x-y+4)^2}{5} - \frac{3(x+2y-3)^2}{5}$ = 1

79. **(a)** 6, 4

Explanation: Number of elements of A = m and number of elements of B = n

Now, 2^m - 2^n = 48

$\Rightarrow$ 2^n (2^{m-n} - 1) = 2^4 $\times$ 3

$\therefore$ n = 4 and 2^{m-n} = 4 = 2^2

$\Rightarrow$ m - n = 2 $\Rightarrow$ m - 4 = 2 $\Rightarrow$ m = 6

$\Rightarrow$ m = 6 and n = 4

80.

(c) (4, 3)

Explanation: From the given linear equation, we get

D = $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix}$ ($R_3 \rightarrow R_3$ - 2R_2 + 3R_3)

$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix}$ = 0

Now, let P_3 = 4x + 4y + 4z - δ = 0. If the system has solutions it will have infinite solution

So, $P_3 \equiv \alpha P_1 + \beta P_2$

Hence, $3\alpha + \beta$ = 4 and $4\alpha + 2\beta$ = 4

$\Rightarrow$ α = 2 and β = -2

So, for infinite solution 2μ - 2 = δ

$\Rightarrow$ For $2\mu \neq \delta$ + 2 system is inconsistent.

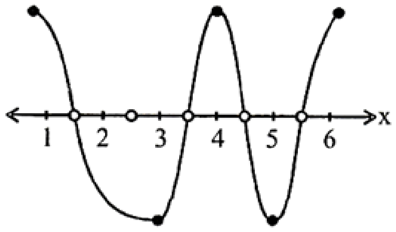
MATHEMATICS (Section-B)

81. 10

Explanation:

f(6) > 0, f(3) < 0, f(2), f(4) will be of opposite sign. f(1), f(5) will be opposite sign. Following 4 possibilities are as

i.



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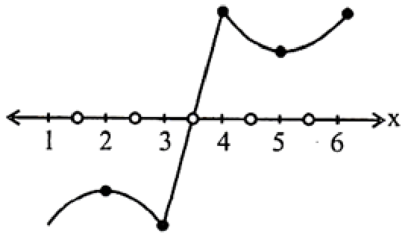

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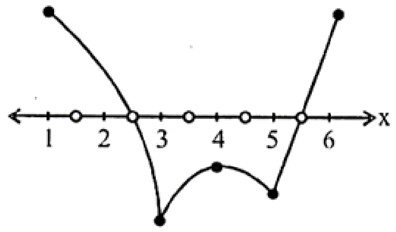
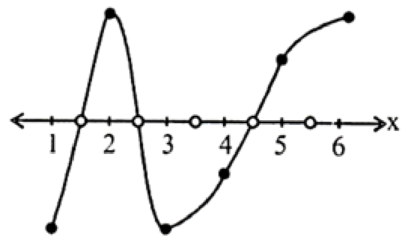


iv.



Number of possible roots 1, 2, 3, 4

A = {1, 2, 3, 4}

Hence, Σn(A) = 10

82. 8

Explanation:

$\lim\limits_{x\to 0} \frac{\left(1-\cos \frac{x^2}{2}\right)}{x^4} \frac{\left(1-\cos \frac{x^2}{4}\right)}{x^4}$ = 2^{-k}

$\Rightarrow \lim\limits_{x\to 0} \frac{2\sin^2 \frac{x^2}{4}}{\frac{x^4}{16}\times 16} \times \frac{2\sin^2 \frac{x^2}{8}}{\frac{x^4}{64}\times 64}$ = 2^{-k}

$\Rightarrow \frac{4}{16\times 64}$ = 2^{-8} = 2^{-k} [$\because \lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta}$ = 1]

$\therefore$ k = 8

83. 315.0

Explanation:

Given, $P_1 = \vec{r}\cdot(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

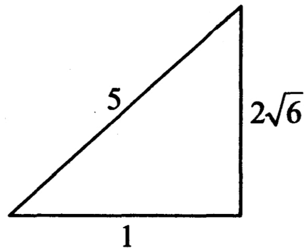
and $P_2 = \vec{r}\cdot(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$

Let $\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right) \Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$

So, $\cos\theta = \frac{1}{5}$

$\cos\theta = \frac{\vec{r}_1 \cdot \vec{r}_2}{|\vec{r}||\vec{r}_2|} = \frac{(3i-5j+K)(\lambda i+j-3K)}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}$

Now, $\frac{1}{5} = \left|\frac{3\lambda-8}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}\right| \Rightarrow \frac{1}{25} = \frac{9\lambda^2+64-48\lambda}{35(\lambda^2+10)}$



$\Rightarrow 19\lambda^2 - 120\lambda + 125 = 0$

$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$

$\Rightarrow x = 5, \frac{25}{19}$

Now, perpendicular distance of point

$(38\lambda_1, 10\lambda, 2) = (50, 50, 2)$ from plane P_1

$\frac{|3\times50-5\times50+2-7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$

Square of perpendicular distance = $\frac{105\times105}{35} = 315$

84. 72.0

Explanation:

Given, $f(x) = \frac{x+|x|}{2} = \begin{bmatrix} x & x \geq 0 \\ 0 & x < 0 \end{bmatrix}$

and g(x) = $\begin{bmatrix} x^2 & x \geq 0 \\ x & x < 0 \end{bmatrix}$

Now, fog(x) = f[g(x)] = $\begin{bmatrix} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{bmatrix}$

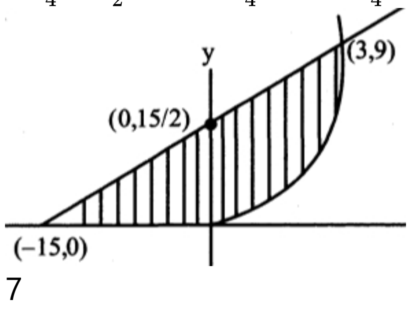
$\Rightarrow fog(x) = \begin{bmatrix} x^2 & x \geq 0 \\ x & x < 0 \end{bmatrix}$

Now, 2y - x = 15

So, area is A = $\int_0^3 \left(\frac{x+15}{2} - x^2\right) dx + \frac{1}{2} \times \frac{15}{2} \times 15$

$= \frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3}\Big|_0^3 + \frac{225}{4}$

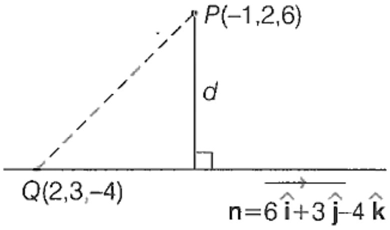
$= \frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4} = \frac{99-36+225}{4} = \frac{288}{4} = 72$



85. 7

Explanation:

Let point P whose position vector is $(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ and a straight line passing through Q(2 ,3 ,-4) parallel to the vector $\mathbf{n} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$



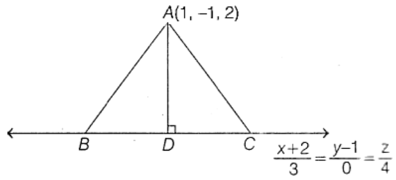
∴ Required distance d = Projection of line segment PQ perpendicular to vector n.

$= \frac{|\mathbf{PQ}\times\mathbf{n}|}{|\mathbf{n}|}$

Now, $\mathbf{PQ} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 10\hat{\mathbf{k}},$ so

$$\mathbf{PQ}\times\mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -10 \\ 6 & 3 & -4 \end{vmatrix} = 26\hat{\mathbf{i}} - 48\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

So, $d = \frac{\sqrt{(26)^2+(48)^2+(3)^2}}{\sqrt{(6)^2+(3)^2+(4)^2}}$



$\Rightarrow d = \sqrt{\frac{676+2304+9}{36+9+16}} = \sqrt{\frac{2989}{61}}$

$\Rightarrow d = \sqrt{49}$ = 7 units

86. 21
Explanation:
21

87. 9.0
Explanation:
Since, diagonal of square = $\sqrt{2}$ $\times$ side
$\Rightarrow$ side = $\frac{1}{\sqrt{2}}$ diagonal
Given that side of A_1 = 12 ∴ Side of A_2 = $\frac{12}{\sqrt{2}}$
Side of A_3 = $\frac{12}{2}$ = 6
∴ Sequence 12, $\frac{12}{\sqrt{2}}$, 6, $\frac{6}{\sqrt{2}}$... are in G.P.
Since area of square less than 1
∴ $(T_n)^2 < 1 \Rightarrow T_n < 1 \Rightarrow 12 \cdot \left(\frac{1}{\sqrt{2}}\right)^{n-1} < 1$
$\Rightarrow 2^{n-1} > 144 \Rightarrow n - 1 \geq 8$
$\Rightarrow$ n = 9

88. 2
Explanation:
2

89. 2.0
Explanation:
Now A + B = $\begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix}$
$\Rightarrow (A + B)^2 = \begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix} \begin{bmatrix} \beta+1 & 0 \\ 3 & \alpha \end{bmatrix}$

= $\begin{bmatrix} (\beta+1)^2 & 0 \\ 3(\beta+1)+3\alpha & \alpha^2 \end{bmatrix}$

Now A^2 = $\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ = $\begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2 \end{bmatrix}$

$\therefore$ $\begin{bmatrix} 1 & -\alpha+1 \\ 2\alpha+4 & \alpha^2 \end{bmatrix}$ = $\begin{bmatrix} (\beta+1)^2 & 0 \\ 3(\alpha+\beta+1) & \alpha^2 \end{bmatrix}$

$\Rightarrow \alpha_1$

Now B^2 = $\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$ = $\begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix}$

= $\begin{bmatrix} (\beta+1)^2 & 0 \\ 3(\beta+1)+3\alpha & \alpha^2 \end{bmatrix}$

$\therefore$ β = 0, α = -1 = α_2

$\therefore$ $|\alpha_1$ - $\alpha_2|$ = |1 - (-1)| = 2

90. 3.0

Explanation:

Given, A = {1, 2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R

For transitive: (1, 2) and (2, 3) $\in$ R $\Rightarrow$ (1, 3) $\in$ R

Not symmetric: (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 2), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (3, 2)}

Total number of relation = 3